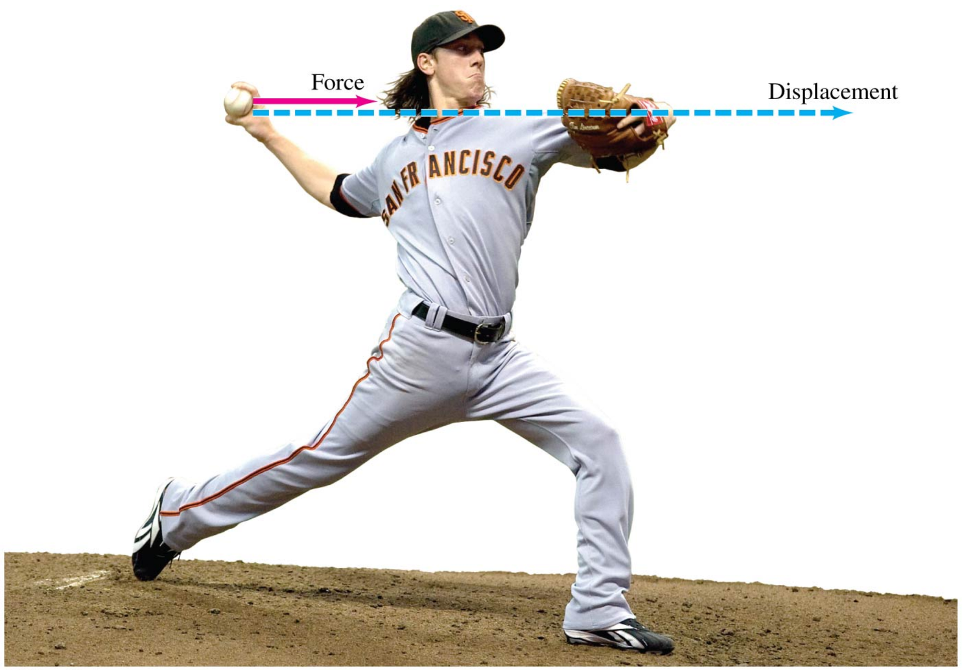
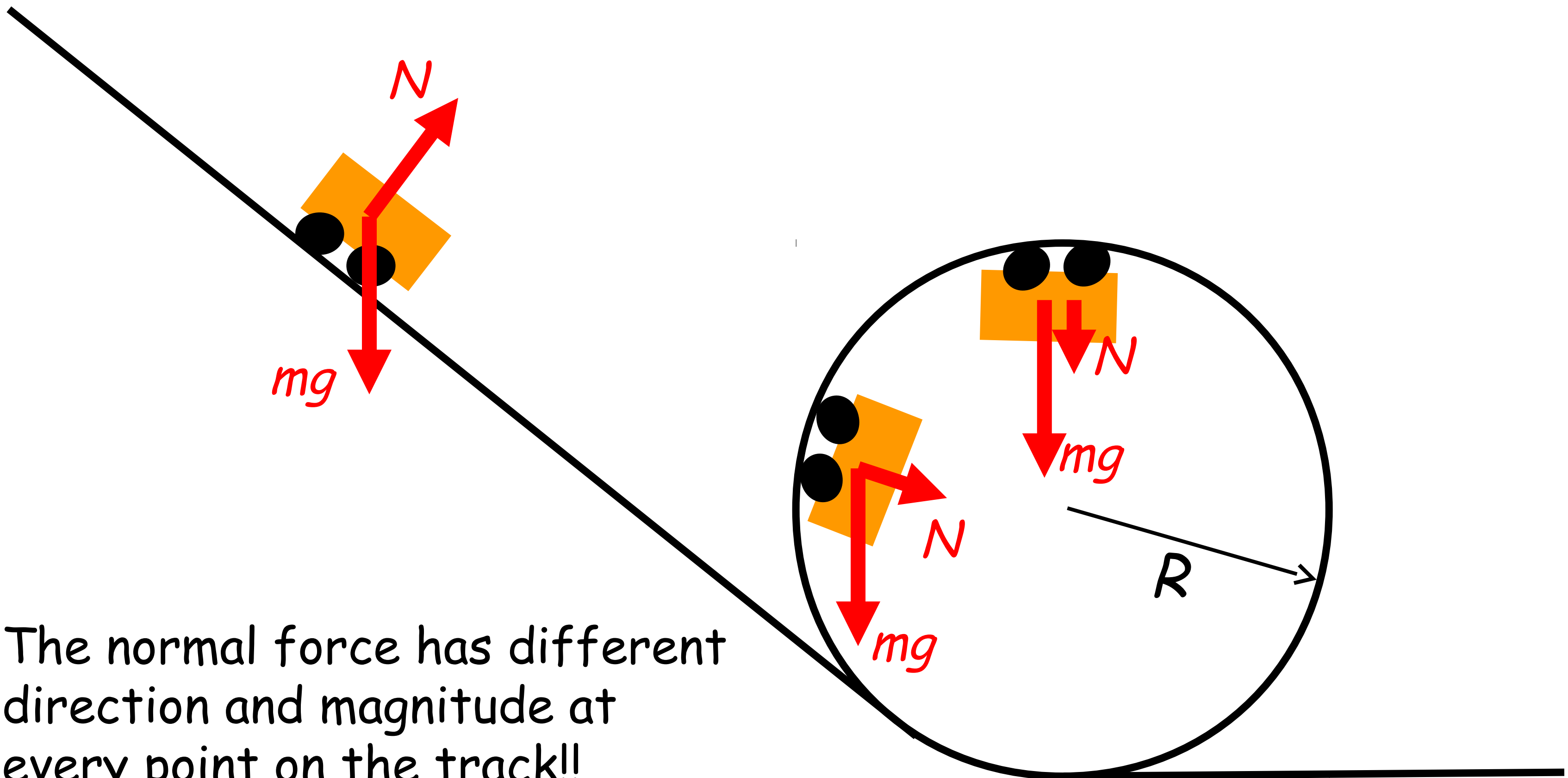
Lecture 9

Work and the Work/Kinetic Energy Theorem

Potential Energy



Motivation to go beyond Newton's laws

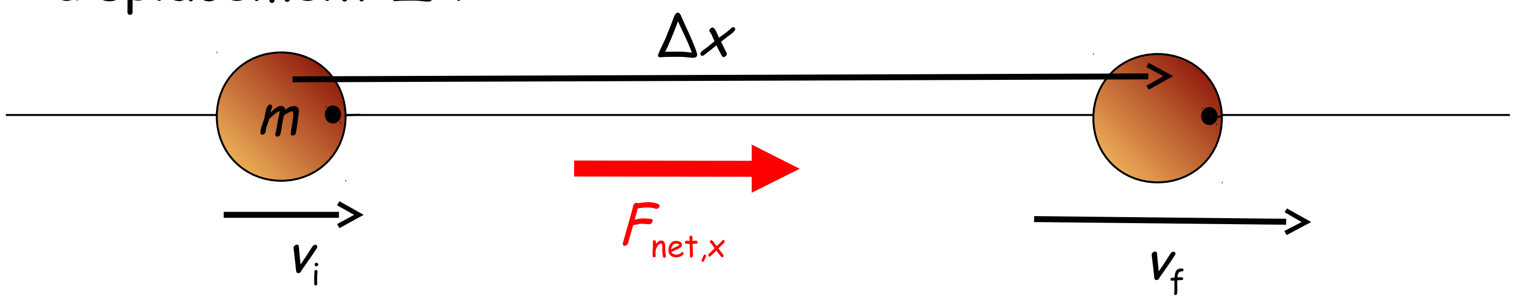


The normal force has different direction and magnitude at every point on the track!!

Writing and solving Newton's laws can be a nasty experience …

Playing around with Newton's 2nd law

Consider the motion of a bead of mass m on a straight wire pushed by a constant net force F_x parallel to the wire, along a displacement Δx :



Net force produces acceleration: $F_x = m a_x$

Acceleration produces change in speed: $2 a_x \Delta x = v_f^2 - v_i^2$

$$2 \frac{F_x}{m} \Delta x = v_f^2 - v_i^2$$

WORK (W) done by force F over displacement Δx

$$F_{net,x} \Delta x = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

Change in KINETIC ENERGY (K) of the bead

This is an expression of the "effectiveness" of the force.

$$F_{net,x}\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{net} = \Delta K$$

Work/Kinetic Energy Theorem

Of how a force applied over a distance…

…changes something in the system

Work

The "external agent" that changes the amount of kinetic energy (the state) in the system

$$W = F_x \Delta x$$

Kinetic energy

The "internal" quantity (state) of the system

$$K = \frac{1}{2}mv^2$$

Units for work and energy:

SI:

Joule	$1\,J = 1\,N \cdot m$

Other common units:

Kilowatt-hour	KWh
calorie	1 cal = 4.184 J
Not to be confused with:	
Calorie (or food calorie)	1 Cal = 1000 cal

Energy

Many types of energy:

- kinetic energy
- electric energy
- internal –thermal- energy
- elastic energy
- chemical energy
- Etc.

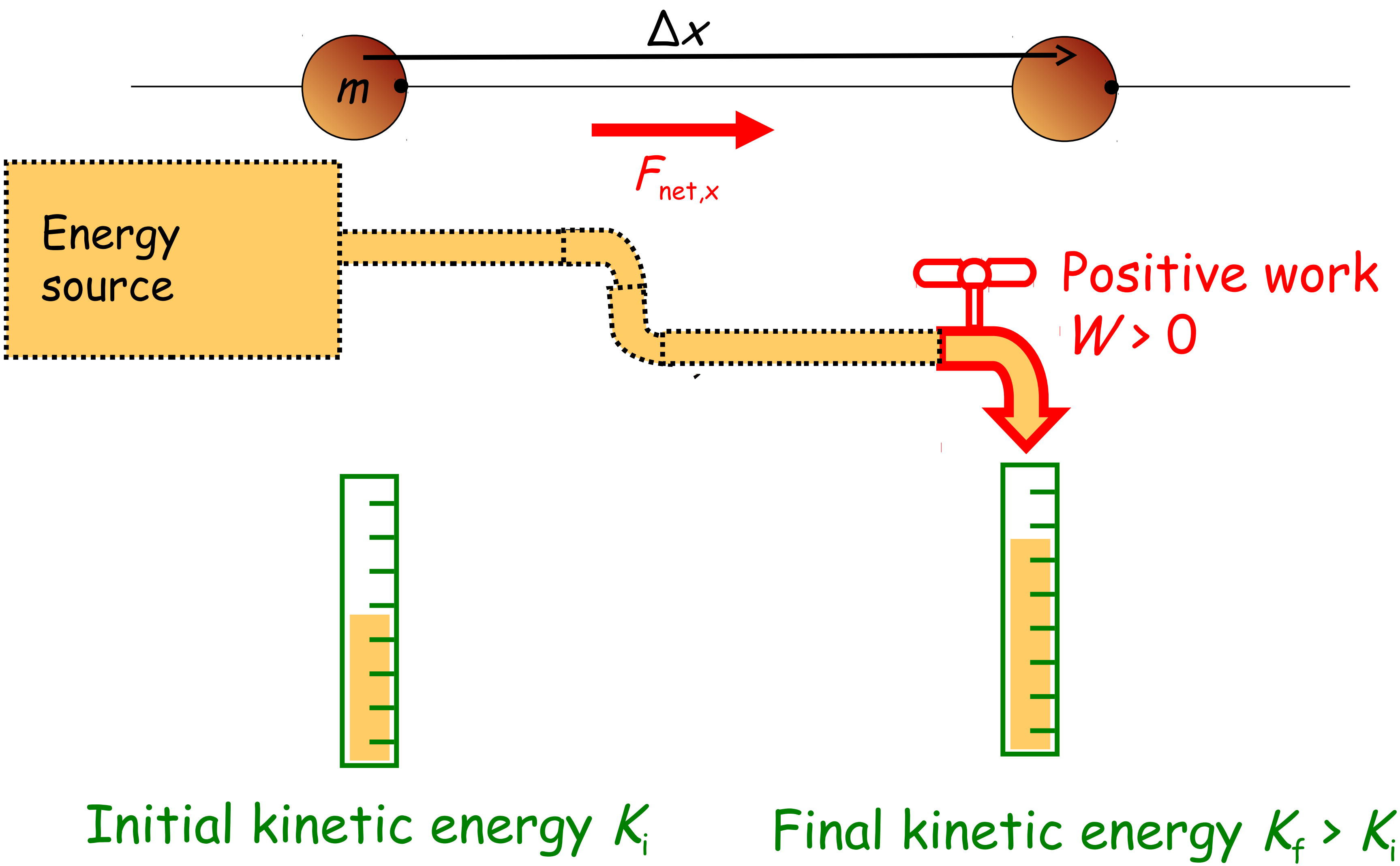
Energy is transferred and **transformed** from one type to another and is never destroyed or created.

What is work?

The definition of work $W = F \Delta x$ corresponds to the intuitive idea of effort:

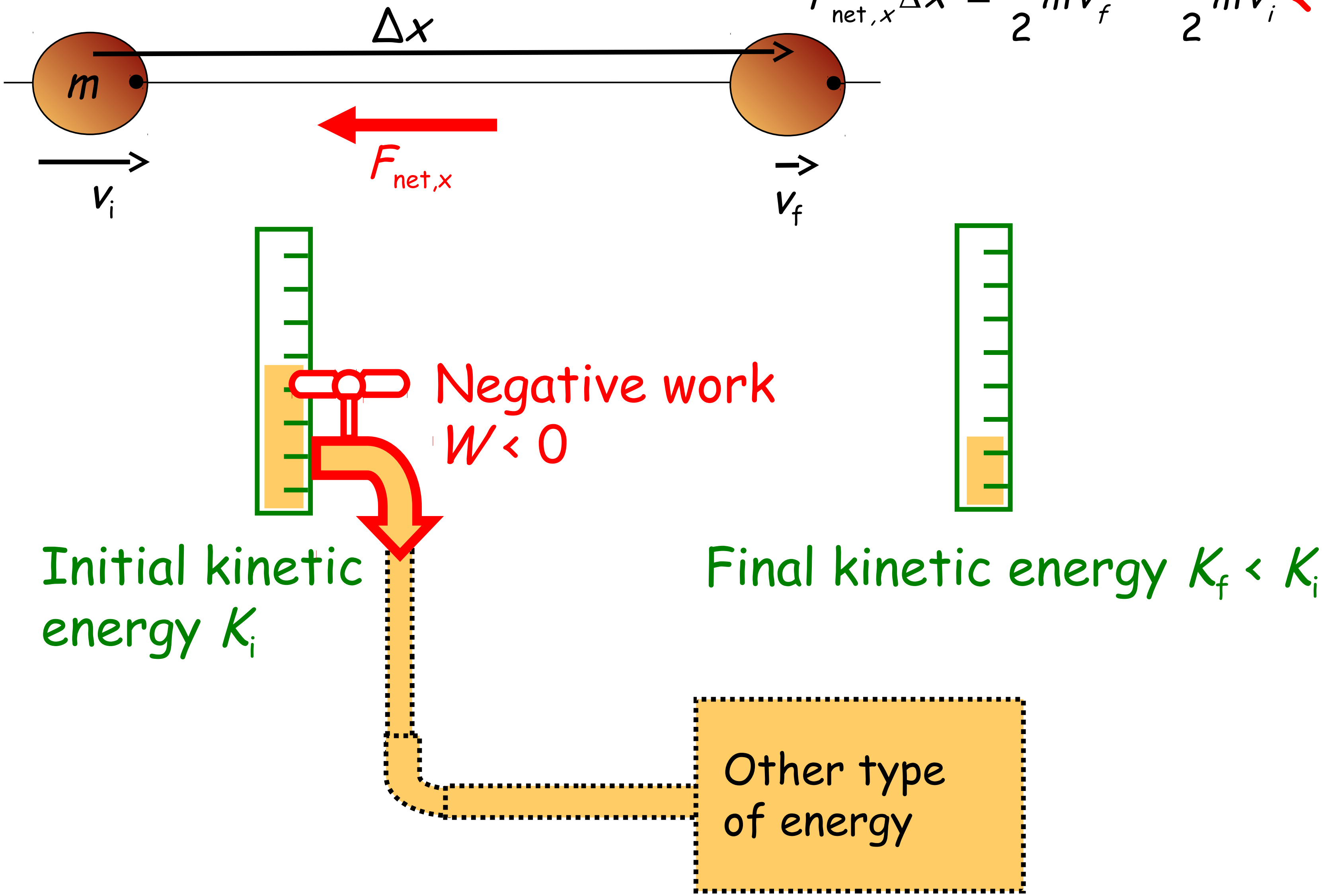
- More massive object will require more work to get to the same speed (from rest)
- For a given mass, getting to a higher speed (from rest) requires more work
- If we push for a longer distance, it's more work.
- It takes the same work to accelerate the object to the right as to the left (both displacement and force reverse)

Work is energy being transferred



Consider the bead again. This time, the force points in the opposite direction:

$$F_{net,x}\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 < 0$$



Example: Pushing a box with friction

Paul pushes a box along 10 m across the floor at a constant velocity by exerting a force of 200N.

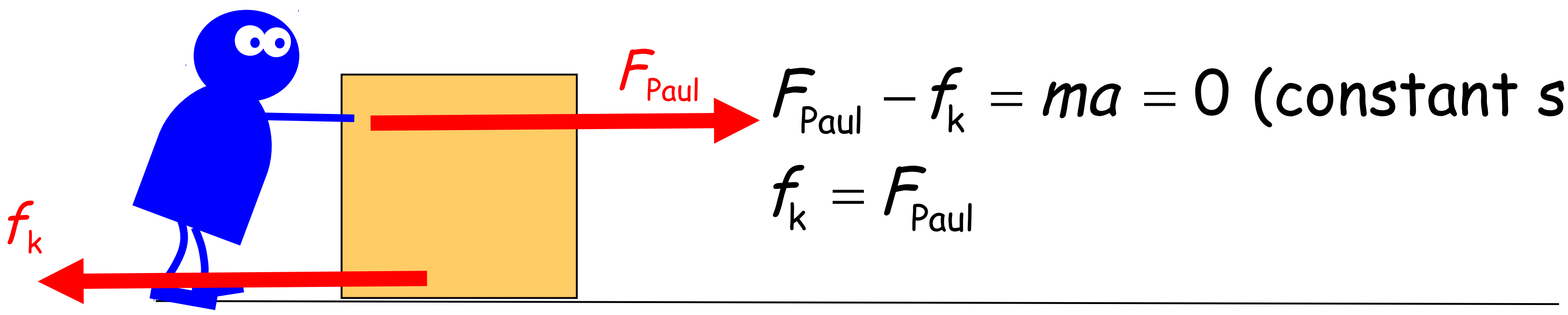
Example: Pushing a box with friction

Paul pushes a box along 10 m across the floor at a constant velocity by exerting a force of 200N.

Work by Paul on the box: W_{Paul} = (200 N)(10 m) = 2000 J
(energy coming from the biochemical reactions in his muscles)

Work by friction on the box: W_f = (-200 N)(10 m) = -2000 J
(released as thermal energy into the air and the floor)

WKE theorem: $W_{Paul} + W_f = 0$ $K_f - K_i = 0$ ✓



$$F_{Paul} - f_k = ma = 0 \text{ (constant speed)}$$

$$f_k = F_{Paul}$$

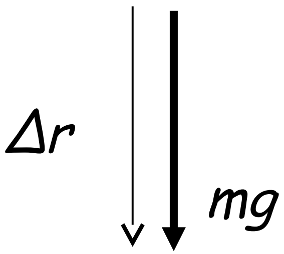
Example: Free fall with WKE

A ball is dropped and hits the ground 50 m below. If the initial speed is 0 and we ignore air resistance, what is the speed of the ball as it hits the ground?

Example: Free fall with WKE

A ball is dropped and hits the ground 50 m below. If the initial speed is 0 and we ignore air resistance, what is the speed of the ball as it hits the ground?

We can use kinematics or… the WKE theorem.



Work done by gravity: mgh

Change in K: $\Delta K = K_{\text{final}} - K_{\text{initial}} = \frac{1}{2} m v^2 - 0$

$$W = \Delta K$$

$$mgh = \frac{1}{2} m v^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(50\ \text{m})} = 31\ \text{m/s}$$

ACT: Two blocks pushed by equal forces

Two blocks ($m_1=2m_2$) are pushed by identical forces, each starting at rest at the blue vertical line (start).
Which object has the greater kinetic energy when it reaches the green vertical line (finish)?

A. Box 1
B. Box 2
C. They both have the same kinetic energy.

ACT: Two blocks pushed by equal forces

Two blocks ($m_1 = 2m_2$) are pushed by identical forces, each starting at rest at the blue vertical line (start).
Which object has the greater kinetic energy when it reaches the green vertical line (finish)?

Same force, same distance ⟶ Same work

↓

Same change in kinetic energy

A. Box 1
B. Box 2
C. They both have the same kinetic energy.

What if the force does not point in the direction of motion?

Work by a constant force along a straight path:

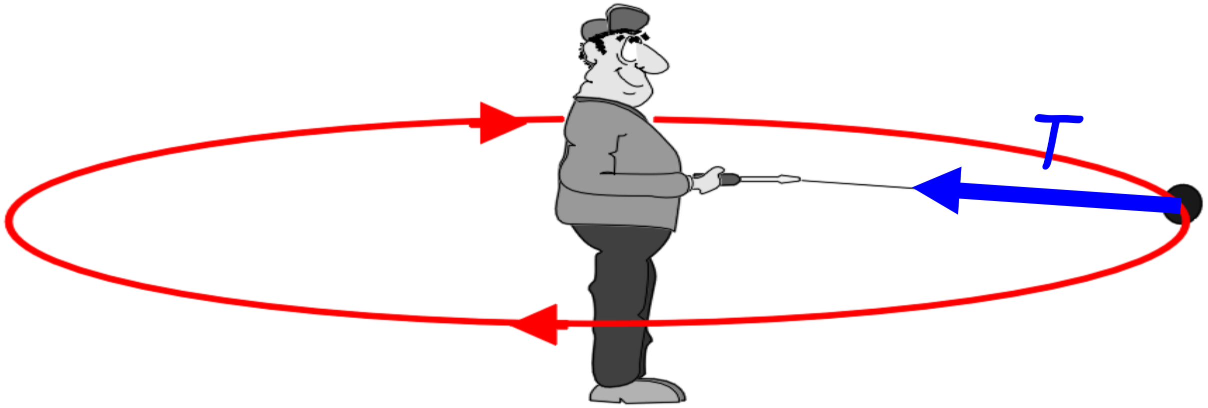
$$W = \mathbf{F} \cdot \Delta \mathbf{r} = F \, \Delta r \cos\theta = F_{\parallel} \Delta r = F \Delta r_{\parallel}$$

For work, only the part of the force that is in the direction of displacement "matters" (= can change speed)

(= can change kinetic energy)

Examples of Non-Work

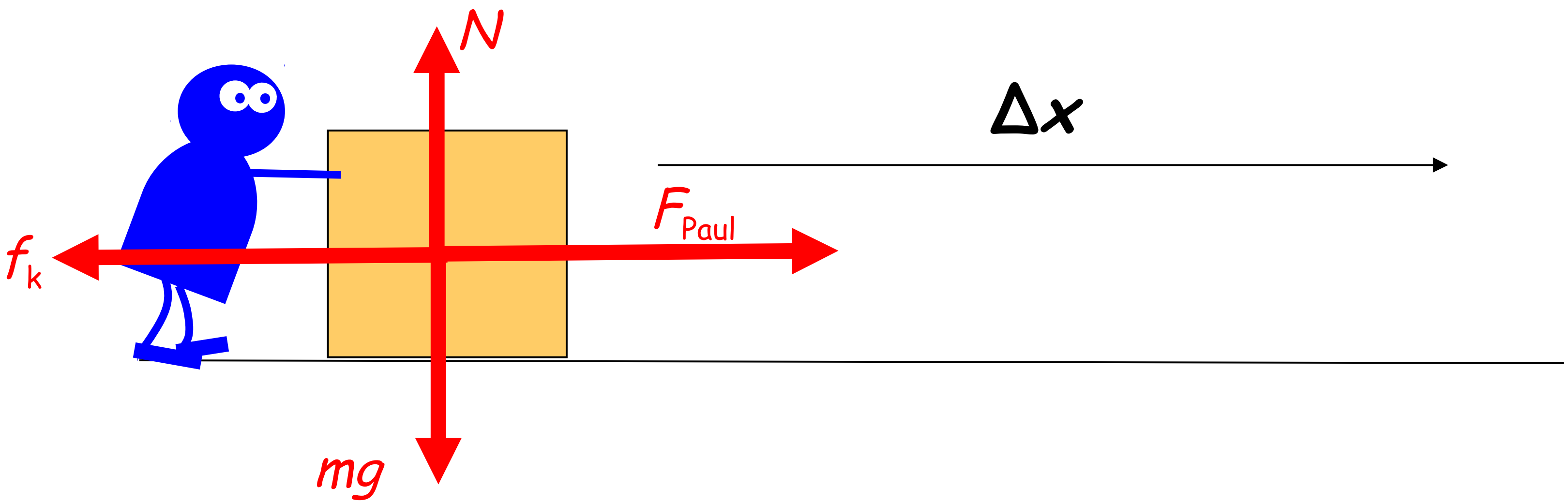
This person is not doing any work on the rock since the centripetal force is perpendicular to the direction that the stone is moving.



Examples of Non-Work

The weight of the box that Paul pushes along a horizontal surface does no work since the weight is perpendicular to the motion

The normal by the floor does no work either.



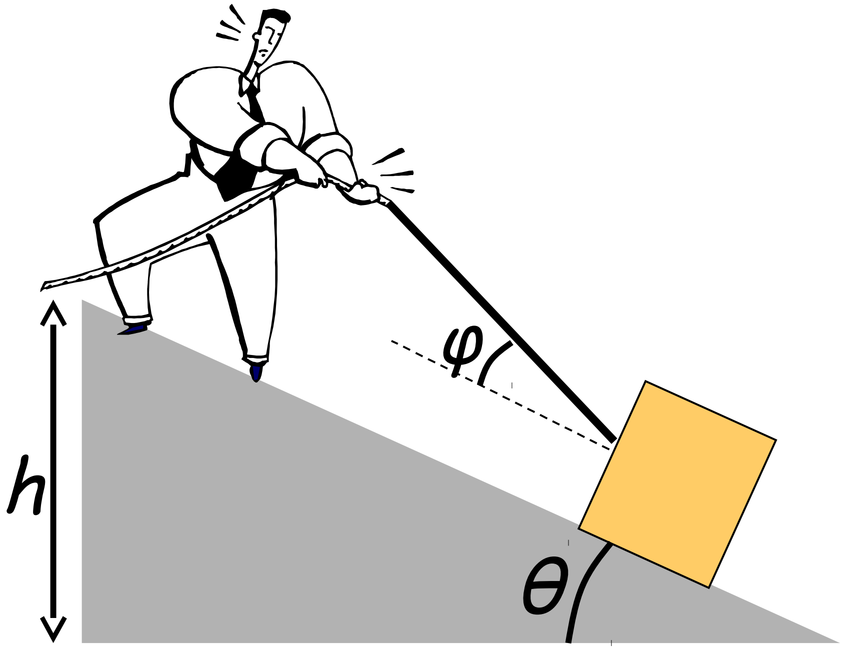
ACT: Ramp

A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Draw FBD.

A. 2

B. 3

C. 4



ACT: Ramp

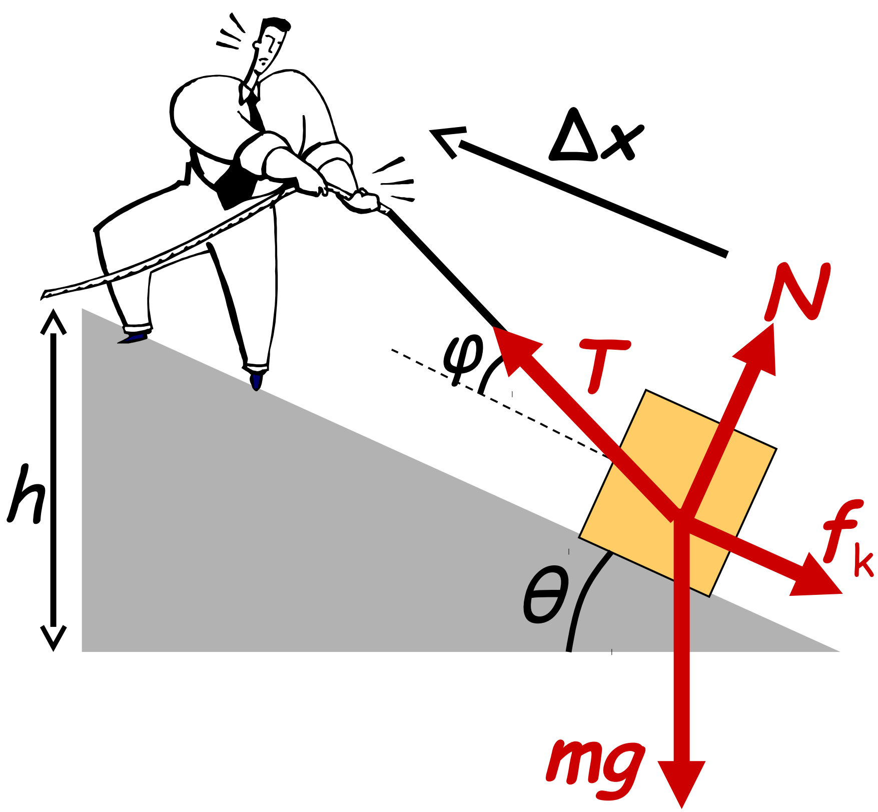
A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Draw FBD.

How many forces are doing non zero work on the box?

A. 2

B. 3

C. 4



$W_T > 0$

$W_f < 0$

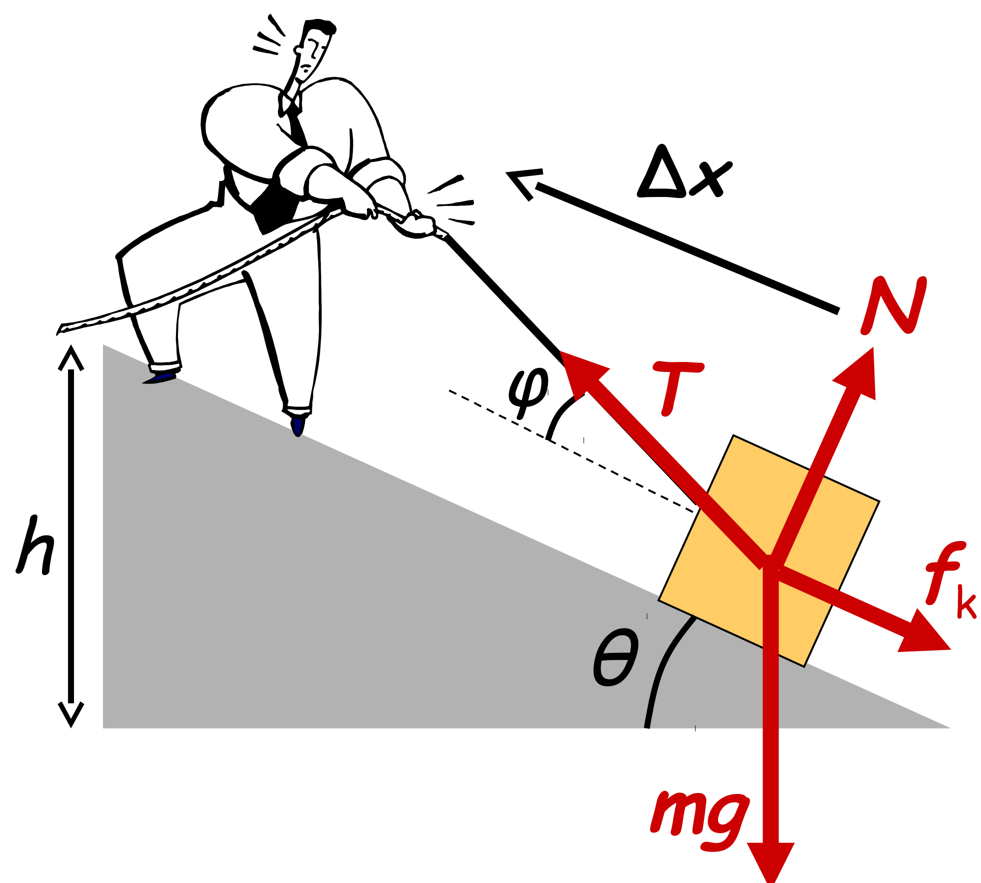
$W_g < 0$

$W_N = 0$

$W_{Net} = 0$

(Constant speed)

A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.



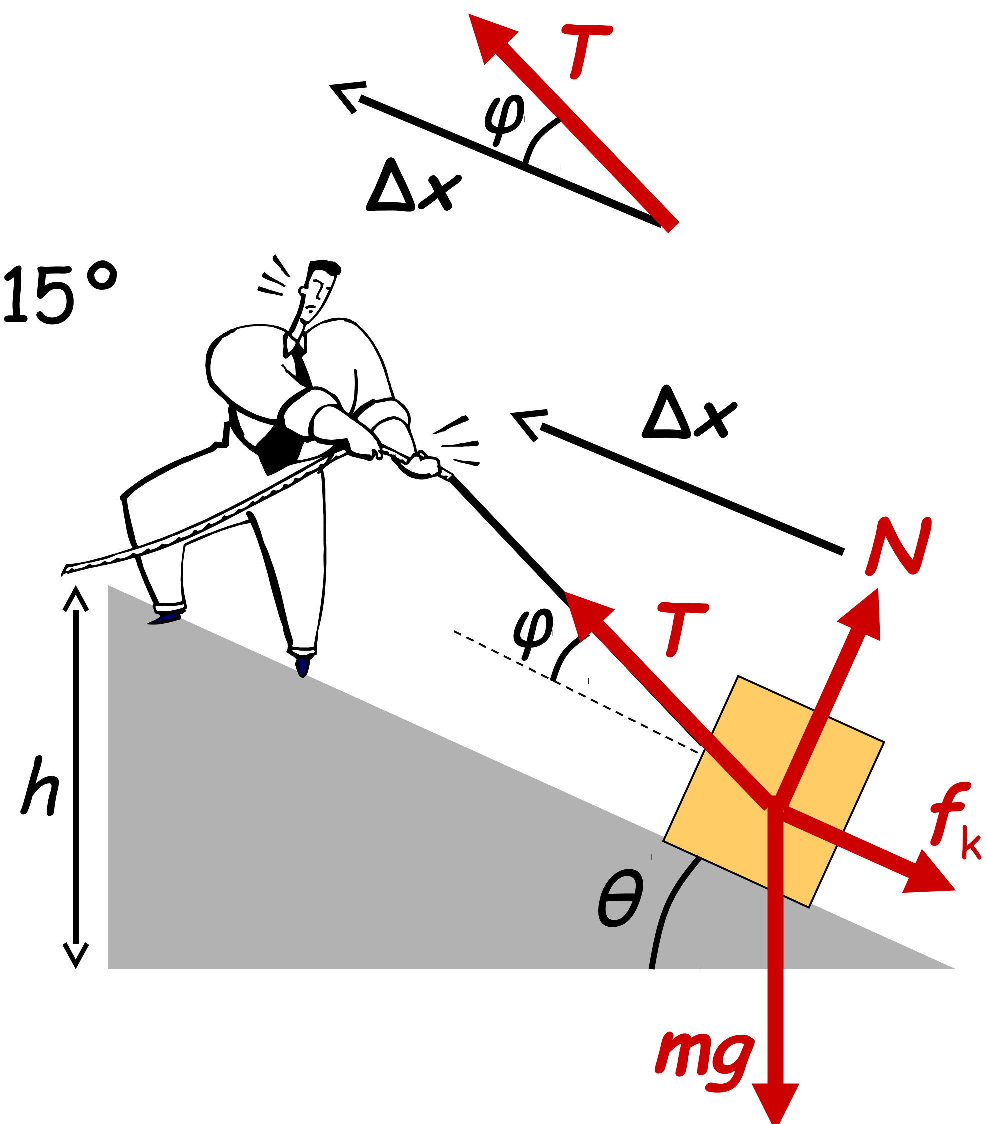
A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.

1. Work by the tension:

$$W_T = T\Delta x \cos\varphi$$
$$= (50\,\text{N})(57.6\,\text{m})\cos 15°$$
$$= 2780\,\text{J}$$

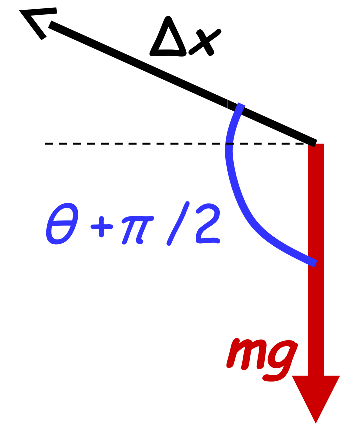
but:

$$\Delta x = \frac{h}{\sin\theta} = \frac{10\,\text{m}}{\sin 10°} = 57.6\text{ m}$$

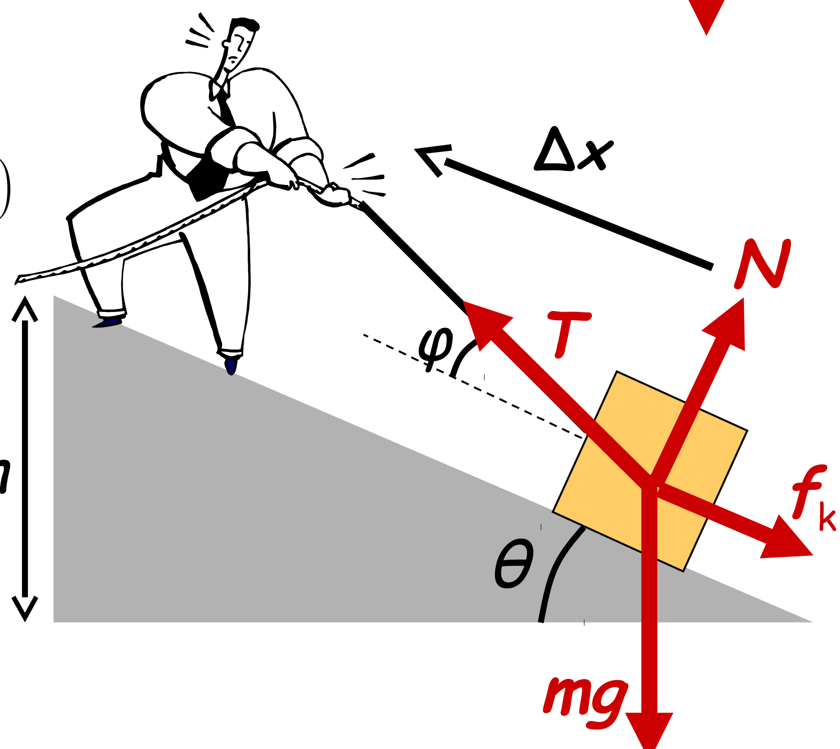


A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.

2. Work by gravity:



$$W_g = mg\Delta x \ \cos\left(\theta + \frac{\pi}{2}\right)$$
$$= mg\Delta x \ \left(-\sin\theta\right)$$
$$= -mgh$$
$$= -(10\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(10\,\mathrm{m})$$
$$= -980\,\mathrm{J}$$

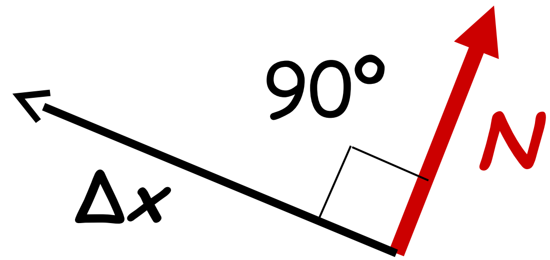


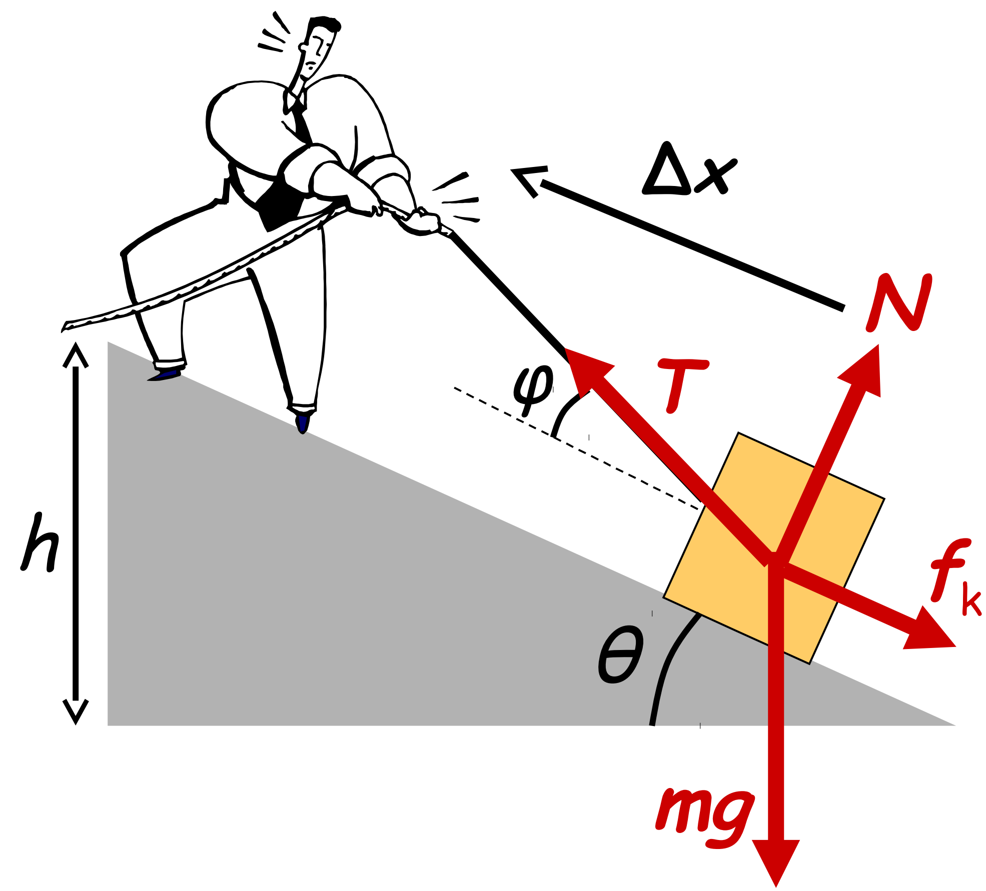
Of course!

$-h$ is the component of the displacement in the direction of weight!

3. Work by the normal:

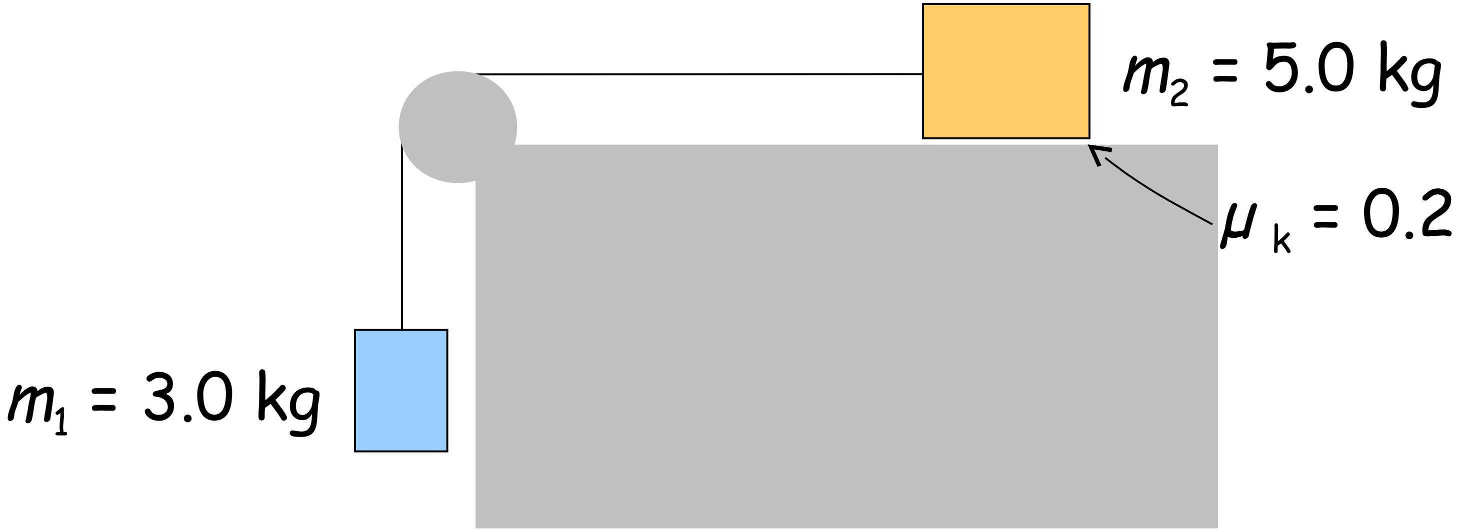
$$W_N = 0$$





Example: Systems with several objects

What is the speed of the system after box 1 has fallen for 30 cm?

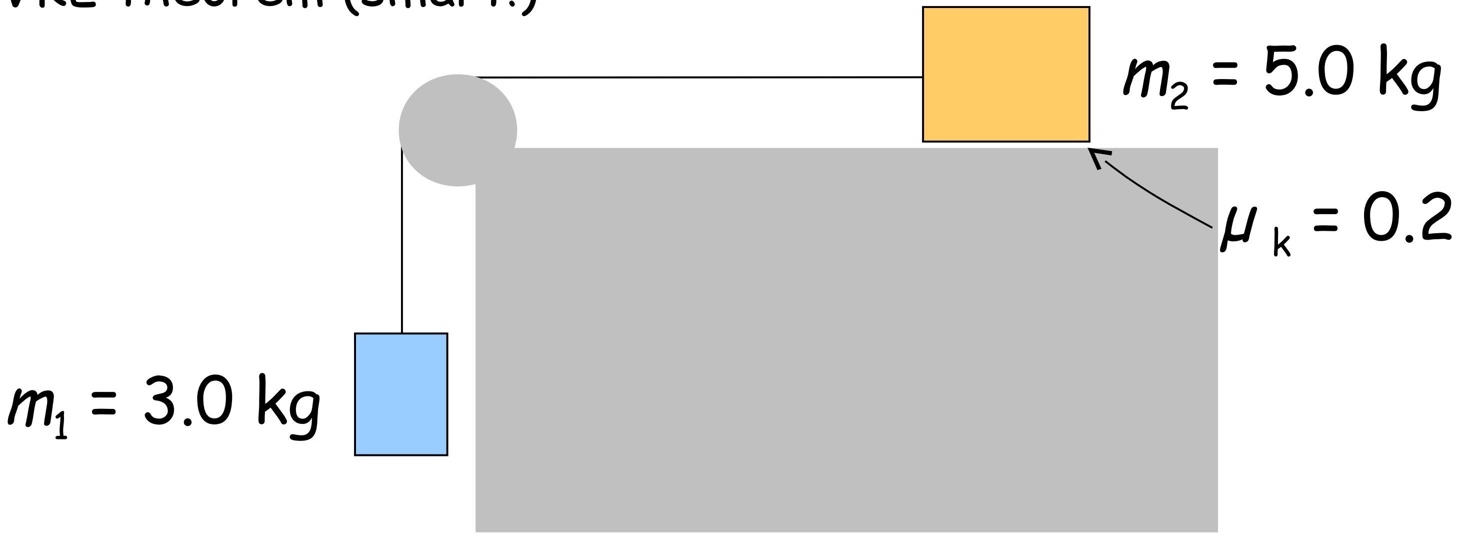


Example: Systems with several objects

What is the speed of the system after box 1 has fallen for 30 cm?

Two approaches:

1. Use Newton's laws to find acceleration; use kinematics to find speed. (long!!)
2. Use WKE theorem (smart!)



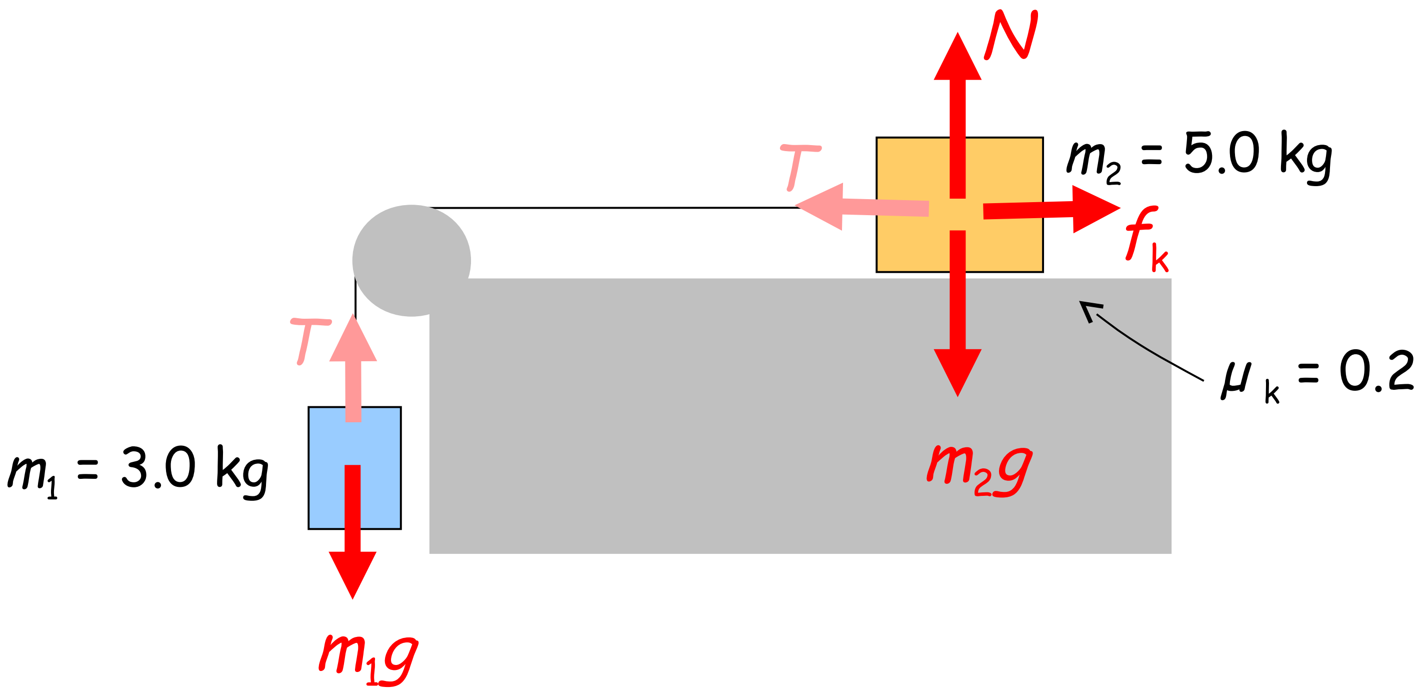
What is the speed of the system after box 1 has fallen for 30 cm? = x

External forces doing work: $m_1 g$, f_k

(The tensions are internal forces: their net work is zero)

$$W_{net} = m_1 g x - f_k x$$
$$= m_1 g x - \mu_k m_2 g x$$

$$\Delta KE = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - 0$$



What is the speed of the system after box 1 has fallen for 30 cm?

$$W_{net} = m_1 g x - \mu_k m_2 g x \qquad \Delta KE = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - 0$$

But these must be equal: Work-Kinetic-Energy theorem

$$W_{net} = \Delta KE$$

$$\left(m_1 - \mu_k m_2\right) g x = \frac{1}{2}\left(m_1 + m_2\right) v^2$$

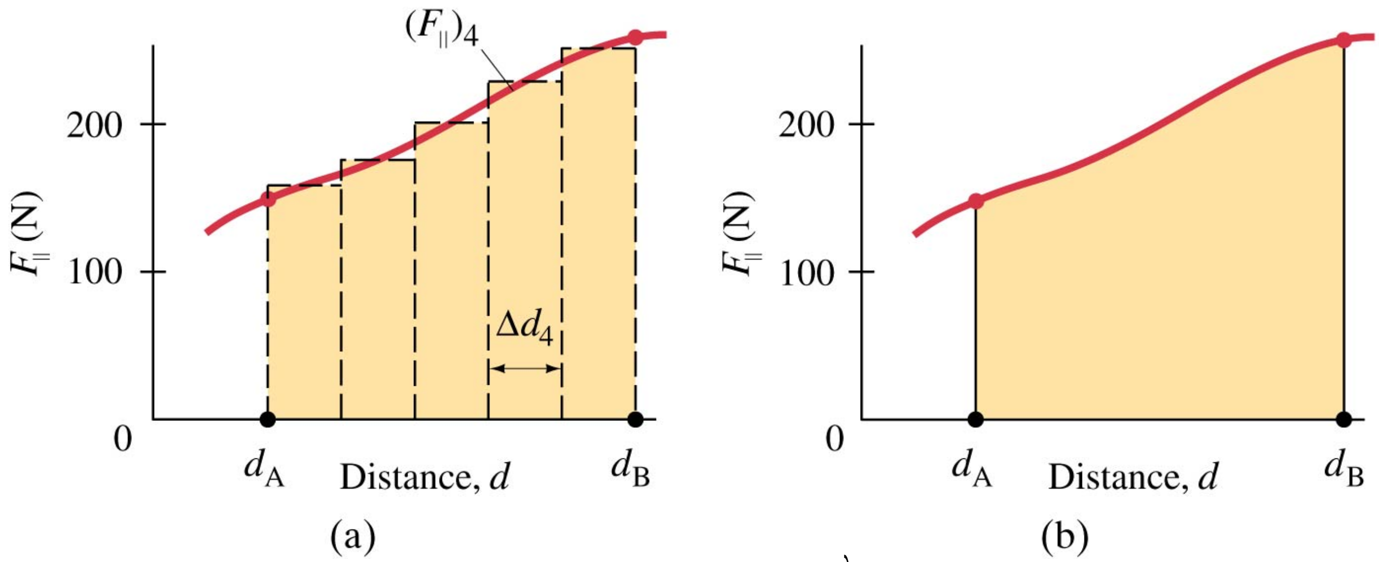
$$v = \sqrt{\frac{m_1 - \mu_k m_2}{m_1 + m_2}\left(2 g x\right)} = \sqrt{\frac{3.0\ \text{kg} - 0.2\left(5.0\ \text{kg}\right)}{3.0\ \text{kg} + 5.0\ \text{kg}} 2\left(9.8\ \text{m/s}^2\right)\left(0.3\ \text{m}\right)} = 1.2\ \text{m/s}$$

m_1 = 3.0kg m_2 = 5.0 kg

μ_k = 0.2 x = 0.3 m

Work Done by a Varying Force

For a force that varies, the work can be approximated by dividing the distance up into small pieces, finding the work done during each, and adding them up. As the pieces become very narrow, the work done is the area under the force vs. distance curve.

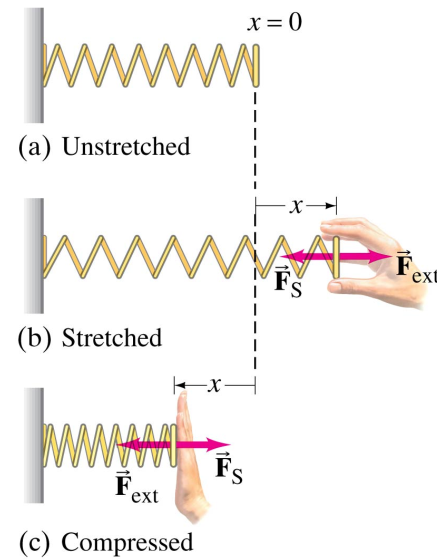


$$W = (F_{||})_1 \Delta d_1 + (F_{||})_2 \Delta d_2 + (F_{||})_3 \Delta d_3 + (F_{||})_4 \Delta d_4 + (F_{||})_5 \Delta d_5 = \text{area under curve}$$

Example of varying force
Hook's Force
Also named: spring force or elastic force

The force required to compress or stretch a spring is:

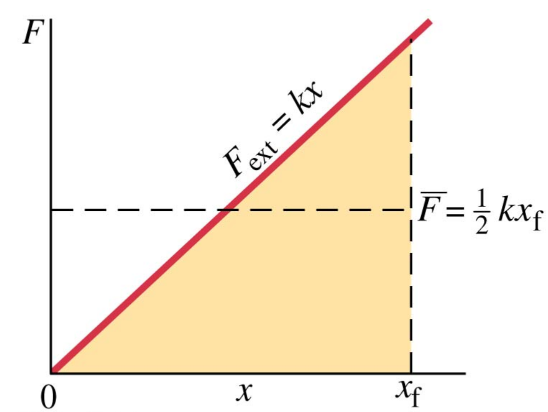
$$F_s = -kx$$



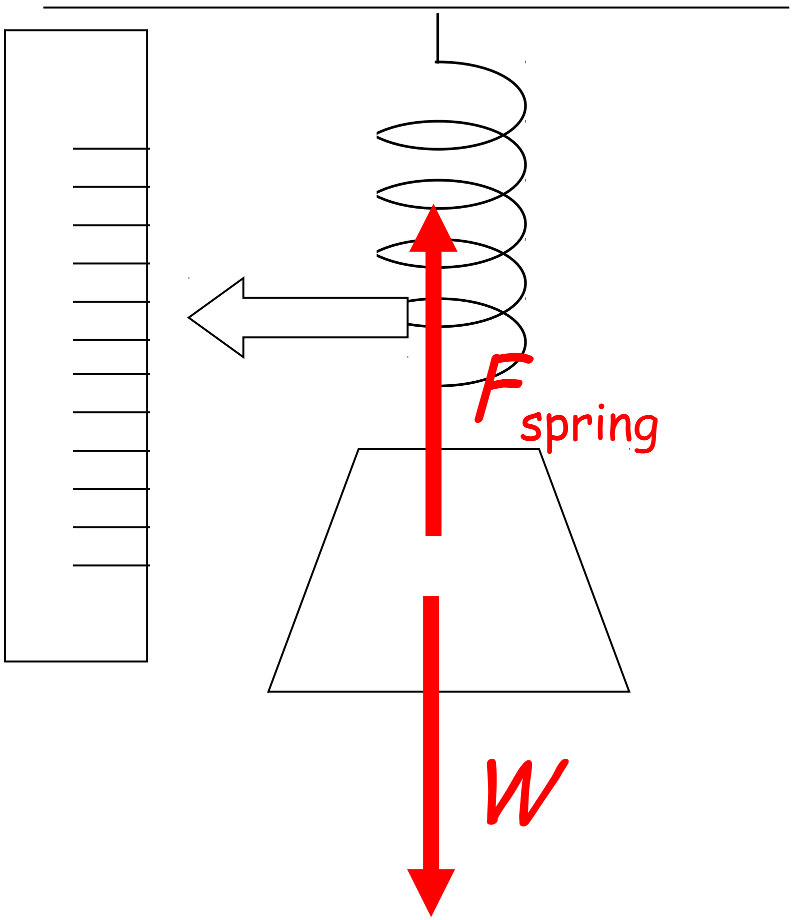
k is called the spring constant, and needs to be measured for each spring.

Work done by the force of a spring when moving an object attached to it is:

$$W_s(\text{from } 0 \text{ to } x_f) = \overline{F}_s x_f = \frac{1}{2} k x_f^2$$



Hooke's law is the basis of traditional scales.



$$F_{spring} - W = 0$$
$$k\Delta x - mg = 0$$
$$\Rightarrow m = \frac{k}{g\Delta x}$$

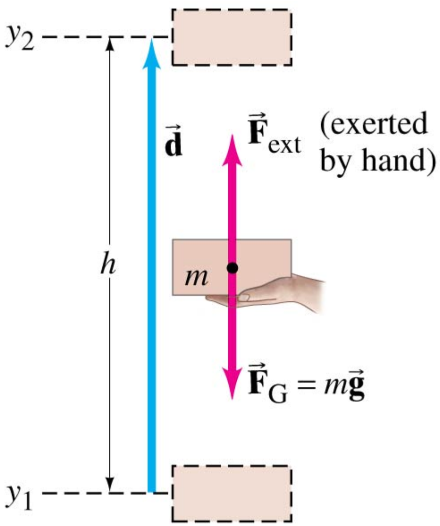
DEMO: Springs and weights

Potential Energy of gravitational force

An object can have potential energy by virtue of its surroundings.

Familiar examples of potential energy:

- A compressed or streched spring
- A stretched elastic band
- An object at some height above the ground



In raising a mass m to a height h, the work done by the external force is

$$W_{ext} = F_{ext}\, d\cos 0^\circ = mgh = mg(y_2 - y_1)$$

We therefore define the gravitational potential energy:

$$PE_G = mgy$$

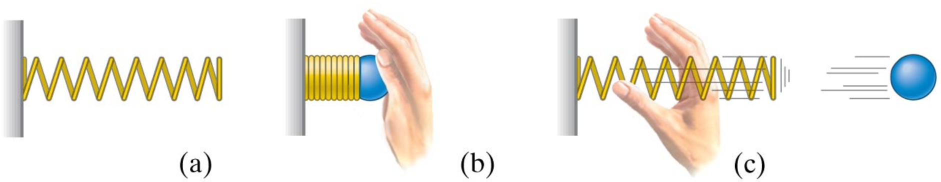
If $PE_G = mgy$, where do we measure y from?

It turns out not to matter, as long as we are consistent about where we choose $y = 0$. Only changes in potential energy can be measured.

Potential Energy of spring

This potential energy can become kinetic energy if the object is dropped.

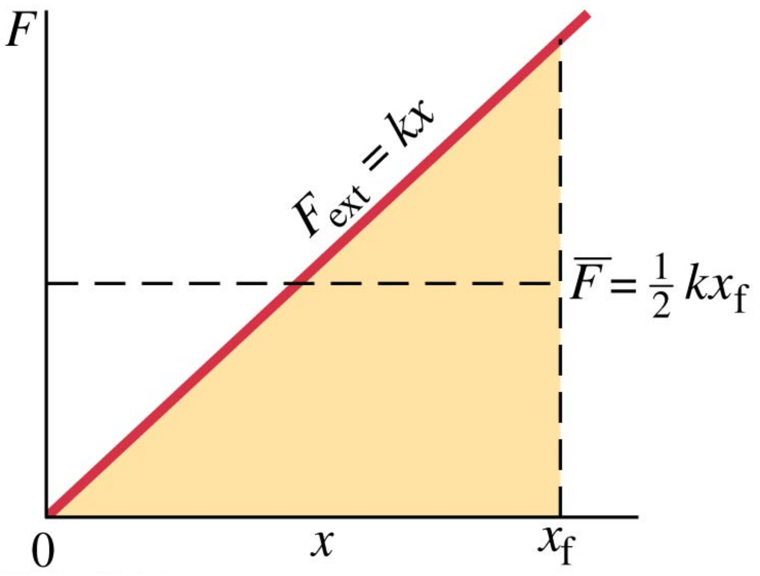
Potential energy is a property of a system as a whole, not just of the object (because it depends on external forces).



Potential energy can also be stored in a spring when it is compressed; the figure below shows potential energy yielding kinetic energy.

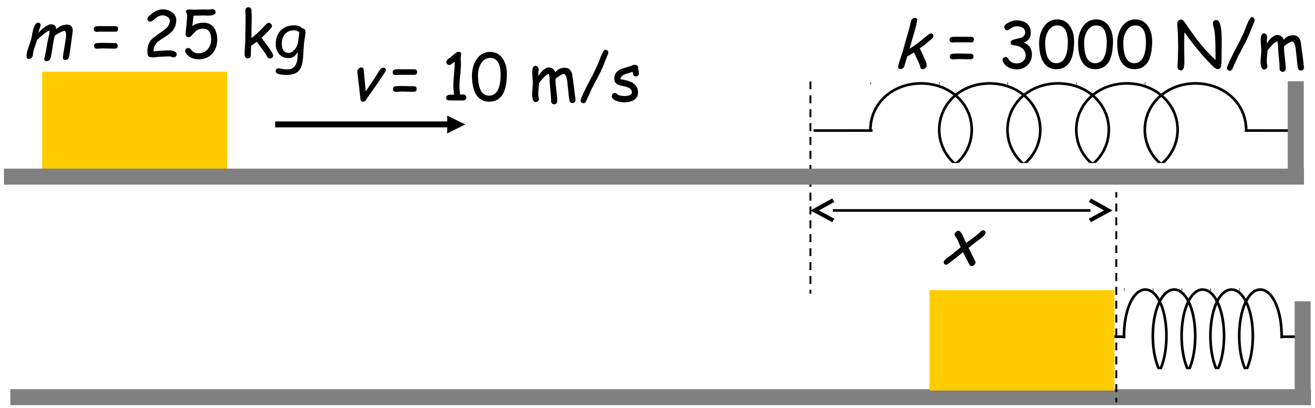
$$PE_{spring} = \overline{F}\, x_f = \frac{1}{2} k x_f^2$$

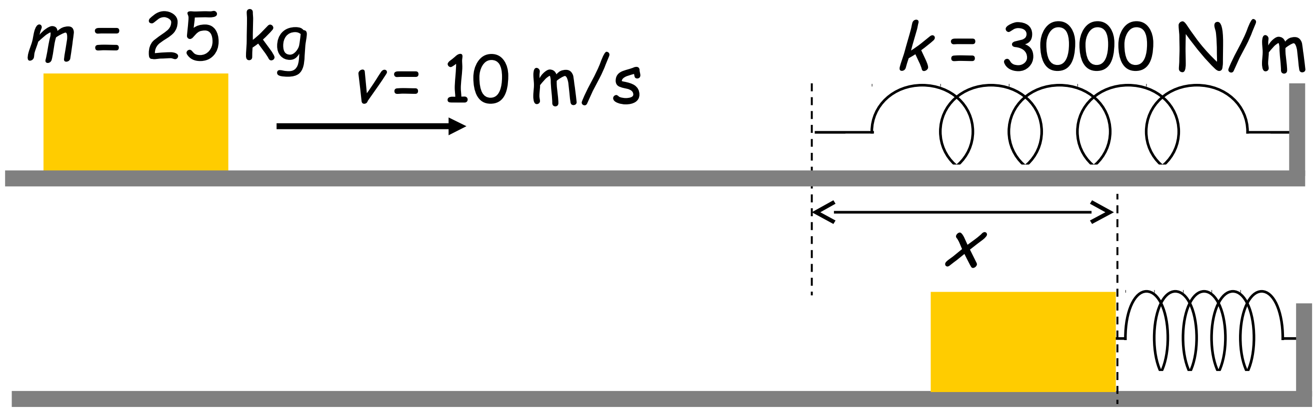
$= $ area of a triangle



Example: Box and spring

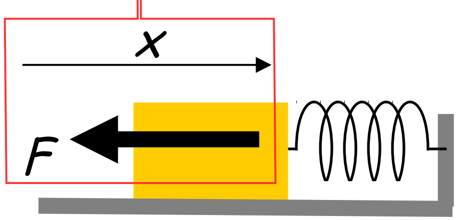
A box of mass m = 25 kg slides on a horizontal frictionless surface with an initial speed v_0 = 10 m/s. How far will it compress the spring before coming to rest if k = 3000 N/m?





Use the work-kinetic energy theorem: $W = \Delta KE$

$$W = -\frac{1}{2}kx^2$$



$$\Delta KE = 0 - \frac{1}{2}mv^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(25\text{ kg})(10\text{ m/s})^2}{3000\text{ N/m}}} = 0.91\text{ m}$$

Answer E