WORKED EXAMPLE 3.1 Calculating a Frequency from a Wavelength

The light blue glow given off by mercury streetlamps has a wavelength of 436 nm. What is its frequency in hertz?

Strategy

We are given a wavelength and need to find the corresponding frequency. Wavelength and frequency are inversely related by the equation $\lambda\nu = c$, which can be solved for ν. Don't forget to convert from nanometers to meters.

Solution

$$\nu = \frac{c}{\lambda} = \frac{\left(3.00 \times 10^{8}\,\frac{\cancel{\text{m}}}{\text{s}}\right)}{(436\,\cancel{\text{nm}})\left(\frac{1\,\cancel{\text{m}}}{10^{9}\,\cancel{\text{nm}}}\right)}$$

$$= 6.88 \times 10^{14}\ \text{s}^{-1} = 6.88 \times 10^{14}\ \text{Hz}$$

The frequency of the light is $6.88 \times 10^{14}\ \text{s}^{-1}$, or 6.88×10^{14} Hz.

Problem 3.1 What is the frequency of a gamma ray with $\lambda = 3.56 \times 10^{-11}$ m? Of a radar wave with $\lambda = 10.3$ cm?

Problem 3.2 What is the wavelength in meters of an FM radio wave with frequency $\nu = 102.5$ MHz? Of a medical X ray with $\nu = 9.55 \times 10^{17}$ Hz?

Key Concept Problem 3.3 Two electromagnetic waves are represented to the left.
(a) Which wave has the higher frequency?
(b) Which wave represents a more intense beam of light?
(c) Which wave represents blue light, and which represents red light?

WORKED EXAMPLE 3.2 Using the Balmer–Rydberg Equation

What are the two longest-wavelength lines in nanometers in the series of the hydrogen spectrum when $m = 1$ and $n > 1$?

Strategy

The wavelength λ is greatest when n is smallest; that is, when $n = 2$ and $n = 3$.

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \qquad \text{where } m = 1$$

Solution

Solving the equation first for $n = 2$ gives

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = (1.097 \times 10^{-2}\ \text{nm}^{-1})\left(1 - \frac{1}{4}\right) = 8.228 \times 10^{-3}\ \text{nm}^{-1}$$

or

$$\lambda = \frac{1}{8.228 \times 10^{-3}\ \text{nm}^{-1}} = 121.5\ \text{nm}$$

Solving the equation next for $n = 3$ gives

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = (1.097 \times 10^{-2}\ \text{nm}^{-1})\left(1 - \frac{1}{9}\right) = 9.751 \times 10^{-3}\ \text{nm}^{-1}$$

or

$$\lambda = \frac{1}{9.751 \times 10^{-3}\ \text{nm}^{-1}} = 102.6\ \text{nm}$$

The two longest-wavelength lines are at 121.5 nm and 102.6 nm.

WORKED EXAMPLE 3.3 Using the Balmer–Rydberg Equation

What is the shortest-wavelength line in nanometers in the series of the hydrogen spectrum when $m = 1$ and $n > 1$?

Strategy

The shortest-wavelength line occurs when n is infinitely large so that $1/n^2$ is zero. That is, if $n = \infty$, then $1/n^2 = 0$.

$$\text{Volume} = \frac{454\ \cancel{g}\ \text{gold}}{19.31\ \cancel{g}/\text{cm}^3} = 23.5\ \text{cm}^3\ \text{gold}$$

Solution

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = (1.097 \times 10^{-2}\ \text{nm}^{-1})(1 - 0) = 1.097 \times 10^{-2}\ \text{nm}^{-1}$$

or

$$\lambda = \frac{1}{1.097 \times 10^{-2}\ \text{nm}^{-1}} = 91.16\ \text{nm}$$

The shortest-wavelength line is at 91.16 nm.

Problem 3.4 The Balmer equation can be extended beyond the visible portion of the electromagnetic spectrum to include lines in the ultraviolet. What is the wavelength in nanometers of ultraviolet light in the Balmer series corresponding to a value of $n = 7$?

Problem 3.5 What is the longest-wavelength line in nanometers in the infrared series for hydrogen where $m = 3$?

Problem 3.6 What is the shortest-wavelength line in nanometers in the infrared series for hydrogen where $m = 3$?

WORKED EXAMPLE 3.4 Calculating the Energy of a Photon from its Frequency

What is the energy in joules of radar waves with $\nu = 3.35 \times 10^8$ Hz? What is the energy in kilojoules per mole?

Strategy

The energy of a photon with frequency ν can be calculated with the equation $E = h\nu$. To find the energy per mole of photons, the energy of one photon must be multiplied by Avogadro's number (Section 2.6).

Solution

$$E = h\nu = (6.626 \times 10^{-34}\ \mathrm{J} \cdot \cancel{\mathrm{s}})(3.35 \times 10^{8}\ \cancel{\mathrm{s}^{-1}}) = 2.22 \times 10^{-25}\ \mathrm{J}$$

$$\left(2.22 \times 10^{-25} \frac{\mathrm{J}}{\cancel{\mathrm{photon}}}\right)\left(6.022 \times 10^{23} \frac{\cancel{\mathrm{photon}}}{\mathrm{mol}}\right) = 0.134\ \mathrm{J/mol}$$

$$= 1.34 \times 10^{-4}\ \mathrm{kJ/mol}$$

Problem 3.7 What is the energy in kilojoules per mole of photons corresponding to the shortest-wavelength line in the series of the hydrogen spectrum when $m = 1$ and $n > 1$? (Worked Example 3.3)?

Problem 3.8 The biological effects of a given dose of electromagnetic energy generally become more serious as the energy of the radiation increases: Infrared radiation has a pleasant warming effect; ultraviolet radiation causes tanning and burning; and X rays can cause considerable tissue damage. What energies in kilojoules per mole are associated with the following wavelengths: infrared radiation with $\lambda = 1.55 \times 10^{-6}$ m, ultraviolet light with $\lambda = 250$ nm, and X rays with $\lambda = 5.49$ nm?

WORKED EXAMPLE 3.5 Using Quantum Numbers to Identify an Orbital

Identify the shell and subshell of an orbital with the quantum numbers $n = 3$, $l = 1$, $m_l = 1$.

Strategy

The principal quantum number n gives the shell number, and the angular-momentum quantum number l gives the subshell designation. The magnetic quantum number m_l is related to the spatial orientation of the orbital.

Solution

A value of $n = 3$ indicates that the orbital is in the third shell, and a value of $l = 1$ indicates that the orbital is of the p type. Thus, the orbital has the designation $3p$.

WORKED EXAMPLE 3.6 Assigning Quantum Numbers to an Orbital

Give the possible combinations of quantum numbers for a 4*p* orbital.

Strategy

The designation 4*p* indicates that the orbital has a principal quantum number $n = 4$ and an angular-momentum quantum number $l = 1$. The magnetic quantum number *ml* can have any of the three values −1, 0, or +1.

Solution

The allowable combinations are

$$n = 4,\ l = 1,\ m_l = -1 \qquad n = 4,\ l = 1,\ m_l = 0 \qquad n = 4,\ l = 1,\ m_l = +1$$

Problem 3.10 Extend Table 3.1 to show allowed combinations of quantum numbers when $n = 5$. How many orbitals are in the fifth shell?

TABLE 3.1 Allowed Combinations of Quantum Numbers n, l, and m_l for the First Four Shells

n	l	m_l	Orbital Notation	Number of Orbitals in Subshell	Number of Orbitals in Shell
1	0	0	1*s*	1	1
2	0	0	2*s*	1	4
	1	−1, 0, +1	2*p*	3	
3	0	0	3*s*	1	9
	1	−1, 0, +1	3*p*	3	
	2	−2, −1, 0, +1, +2	3*d*	5	
4	0	0	4*s*	1	16
	1	−1, 0, +1	4*p*	3	
	2	−2, −1, 0, +1, +2	4*d*	5	
	3	−3, −2, −1, 0, +1, +2, +3	4*f*	7	

WORKED EXAMPLE 3.6 Assigning Quantum Numbers to an Orbital

Continued

Problem 3.11 Give orbital notations for electrons in orbitals with the following quantum numbers:
(a) $n = 2, l = 1, m_l = 1$ **(b)** $n = 4, l = 3, m_l = -2$ **(c)** $n = 3, l = 2, m_l = 1$

Problem 3.12 Give the possible combinations of quantum numbers for the following orbitals:
(a) A 3*s* orbital **(b)** A 2*p* orbital **(c)** A 4*d* orbital

WORKED EXAMPLE 3.7 Calculating the Energy Difference between Two Orbitals

What is the energy difference in kilojoules per mole between the first and second shells of the hydrogen atom if the lowest-energy emission in the spectral series with $m = 1$ and $n = 2$ occurs at $\lambda = 121.5$ nm?

Strategy

The lowest-energy emission line in the spectral series with $m = 1$ and $n = 2$ corresponds to the emission of light as an electron falls from the second shell to the first shell, with the energy of that light equal to the energy difference between shells. Knowing the wavelength of the light, we can calculate the energy of one photon using the equation, $E = hc/\lambda$, and then multiply by Avogadro's number to find the answer in joules (or kilojoules) per mole:

Solution

$$E = \frac{hcN_A}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \cancel{\text{s}})\left(3.00 \times 10^{8}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(10^{9}\ \frac{\cancel{\text{nm}}}{\cancel{\text{m}}}\right)(6.022 \times 10^{23}\ \text{mol}^{-1})}{121.5\ \cancel{\text{nm}}}$$

$$= 9.85 \times 10^{5}\ \text{J/mol} = 985\ \text{kJ/mol}$$

The energy difference between the first and second shells of the hydrogen atom is 985 kJ/mol.

Problem 3.15 Calculate in kilojoules per mole the energy necessary to completely remove an electron from the first shell of a hydrogen atom ($R\infty = 1.097 \times 10^{-2}\ \text{nm}^{-1}$).

WORKED EXAMPLE 3.8 Assigning a Ground-State Electron Configuration to an Atom

Give the ground-state electron configuration of arsenic, $Z = 33$, and draw an orbital-filling diagram, indicating the electrons as up or down arrows.

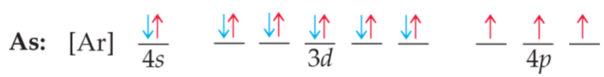
Strategy

Think of the periodic table as having *s*, *p*, *d*, and *f* blocks of elements, as shown in Figure 3.17. Start with hydrogen at the upper left, and fill orbitals until 33 electrons have been added. Remember that only two electrons can go into an orbital and that each one of a set of degenerate orbitals must be half filled before any one can be completely filled.

Solution

$$\textbf{As:}\ 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^3 \text{ or } [\text{Ar}]\ 4s^2\ 3d^{10}\ 4p^3$$

An orbital-filling diagram indicates the electrons in each orbital as arrows. Note that the three 4*p* electrons all have the same spin:



WORKED KEY CONCEPT EXAMPLE 3.9 Identifying an Atom from its Ground-State Electron Configuration

Identify the atom with the following ground-state electron configuration:

$$[\mathrm{Kr}]\ \underline{\downarrow\uparrow}\quad \underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}\quad \underline{}\ \underline{}\ \underline{}$$

Strategy

One way to do this problem is to identify the electron configuration and decide which atom has that configuration. Alternatively, you can just count the electrons, thereby finding the atomic number of the atom.

Solution

The atom whose ground-state electron configuration is depicted is in the fifth row because it follows krypton. It has the configuration $5s^2\ 4d^5$, which identifies it as technetium. Alternatively, it has $36 + 7 = 43$ electrons and is the element with Z = 43.

Problem 3.17 Give expected ground-state electron configurations for the following atoms, and draw orbital-filling diagrams for parts **(a)–(c)**.

(a) Ti (Z = 22)
(b) Zn (Z = 30)
(c) Sn (Z = 50)
(d) Pb (Z = 82)

WORKED KEY CONCEPT EXAMPLE 3.9 Identifying an Atom from its Ground-State Electron Configuration

Continued

Problem 3.18 As we'll see in the next chapter, an *ion* is a charged species formed by the gain or loss of one or more electrons from the corresponding neutral atom. What do you think is a likely ground-state electron configuration for the sodium ion, Na^{+}, formed by loss of an electron from a neutral sodium atom? What is a likely ground-state electron configuration for the chloride ion, Cl^{-}, formed by adding an electron to a neutral chlorine atom?

Key Concept Problem 3.19 Identify the atom with the following ground-state electron configuration: