Workshop on Traffic Assignment with Equilibrium Methods

Presented by:

David Boyce and Michael Florian

Northwestern University and University of Montreal

Sponsored by:

Subcommittee on Network Equilibrium Modeling
Transportation Network Modeling Committee

January 9, 2005, 8:30 am – 12:00 pm

Contents of second part

► Variable demand equilibrium assignment

► Multi-modal models : equilibration

► Some large scale applications

► Stochastic user equilibrium assignment
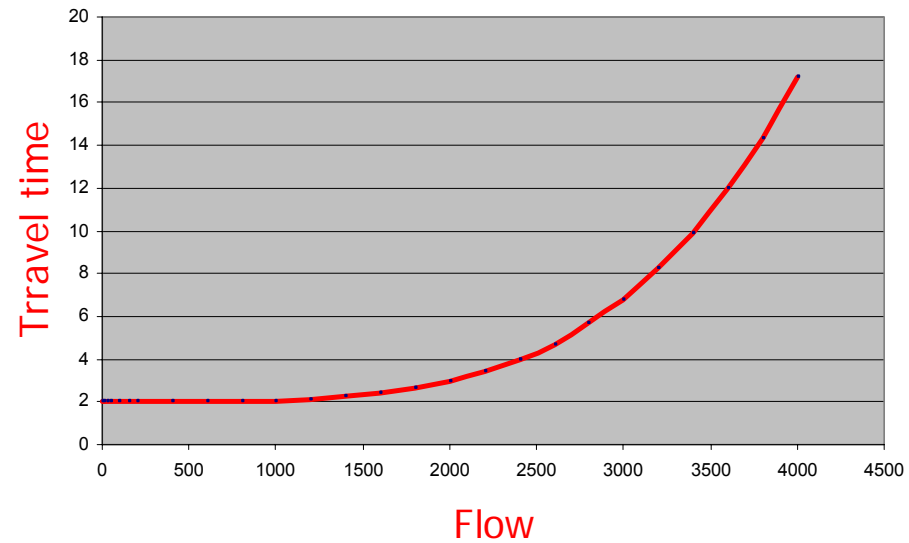
Variable demand equilibrium assignment

► The demand for travel is assumed to be given by a direct demand function which has the property that as travel time (cost) increases the demand decreases

► The network equilibrium is established when all used routes are of equal time (cost) and the travel time leads to a demand which yields the equilibrium link flows

One Link – One Origin-Destination Pair
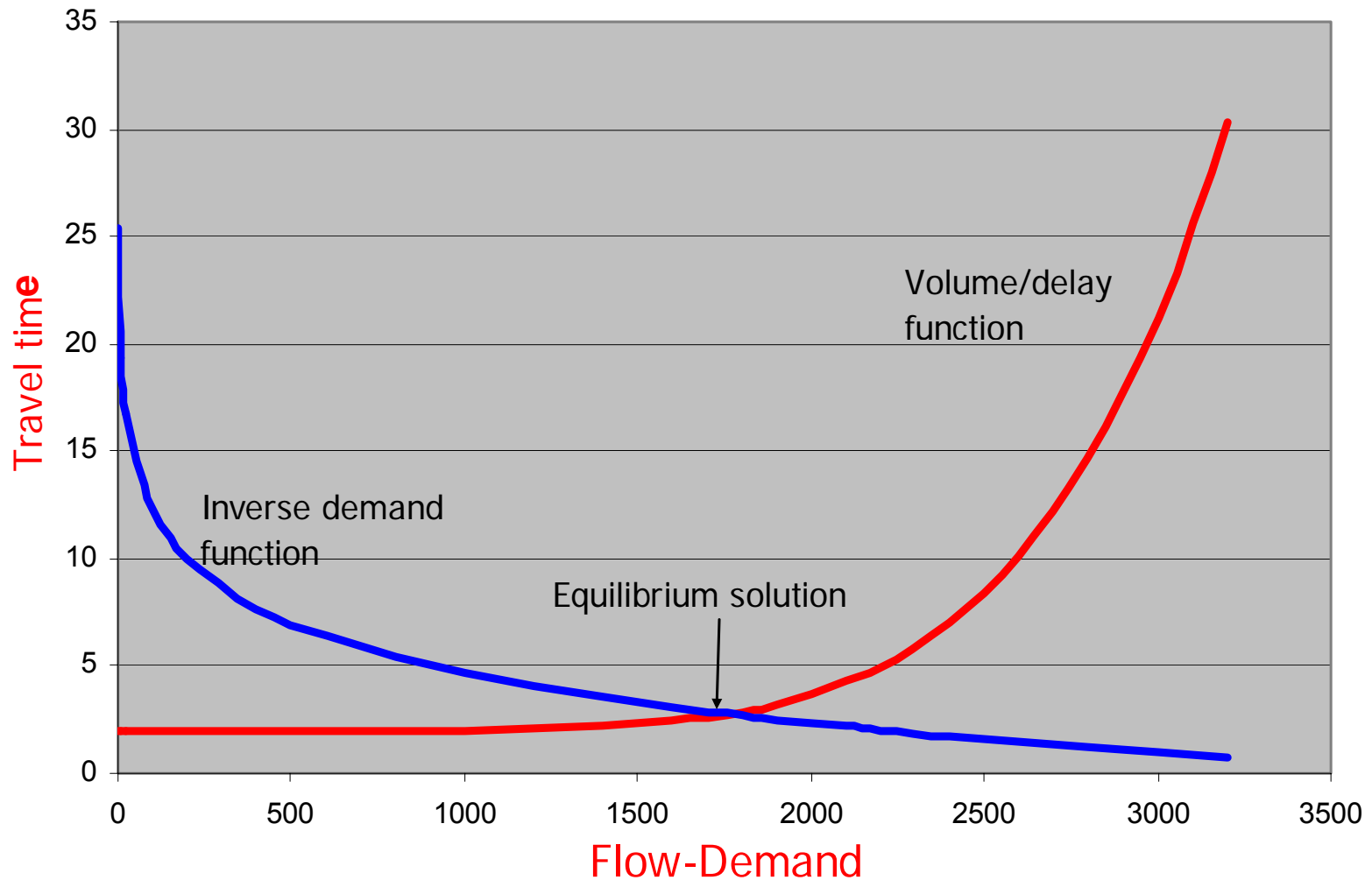
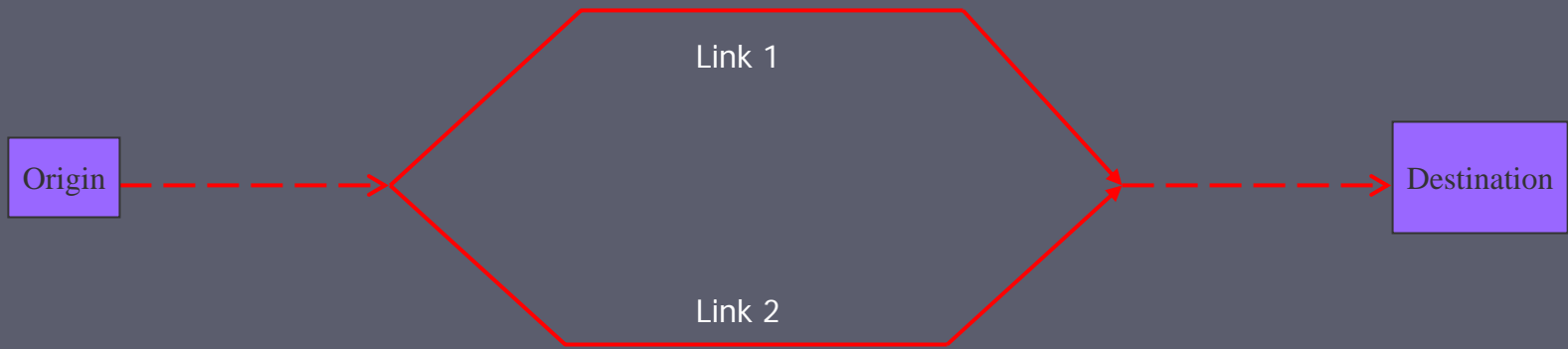**Demand function**
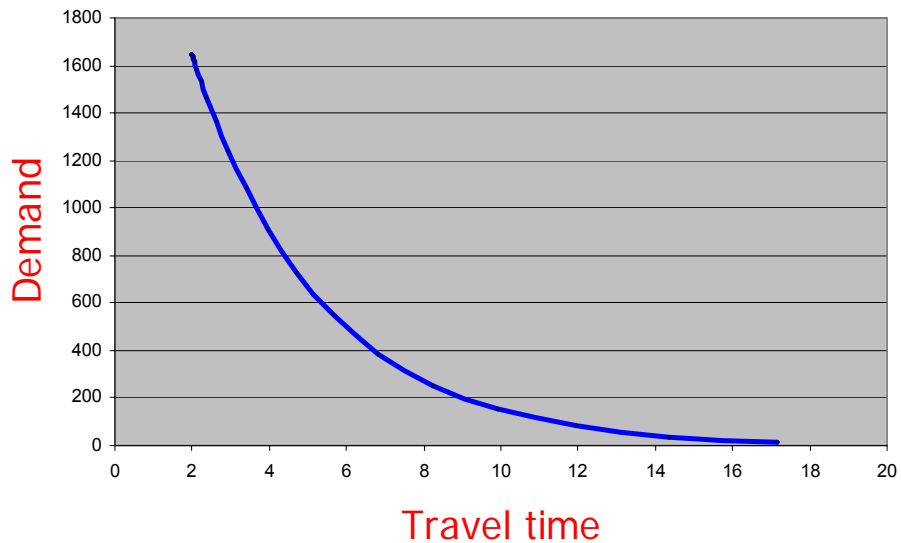
Volume/delay function

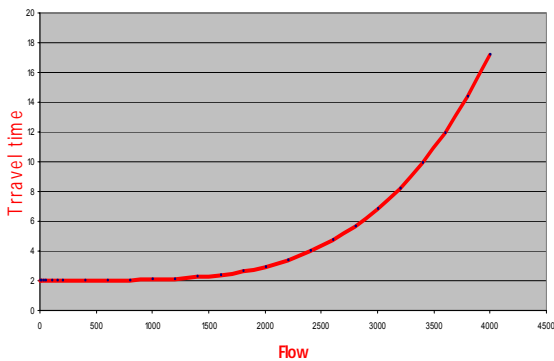# Demand-performance equilibrium
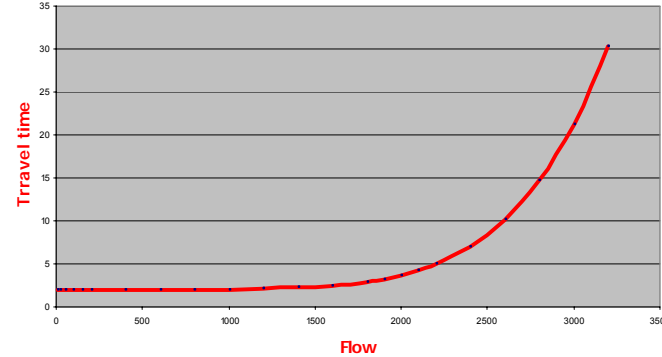

Volume/delay function

A two link variable demand example

► A volume delay function is associated with each link and a demand function is associated with the origin-destination pair
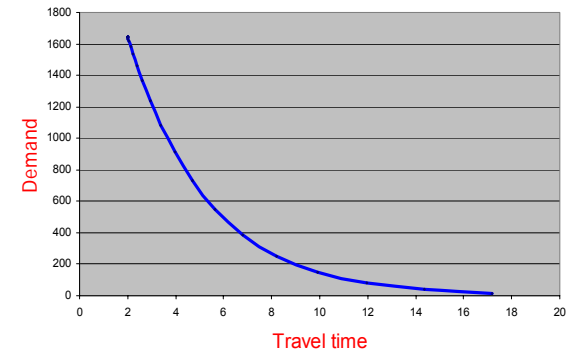
Two link – graphical solution

Graphical Equilibrium Solution

Any horizontal line is an equilibrium solution

Travel time

Flow
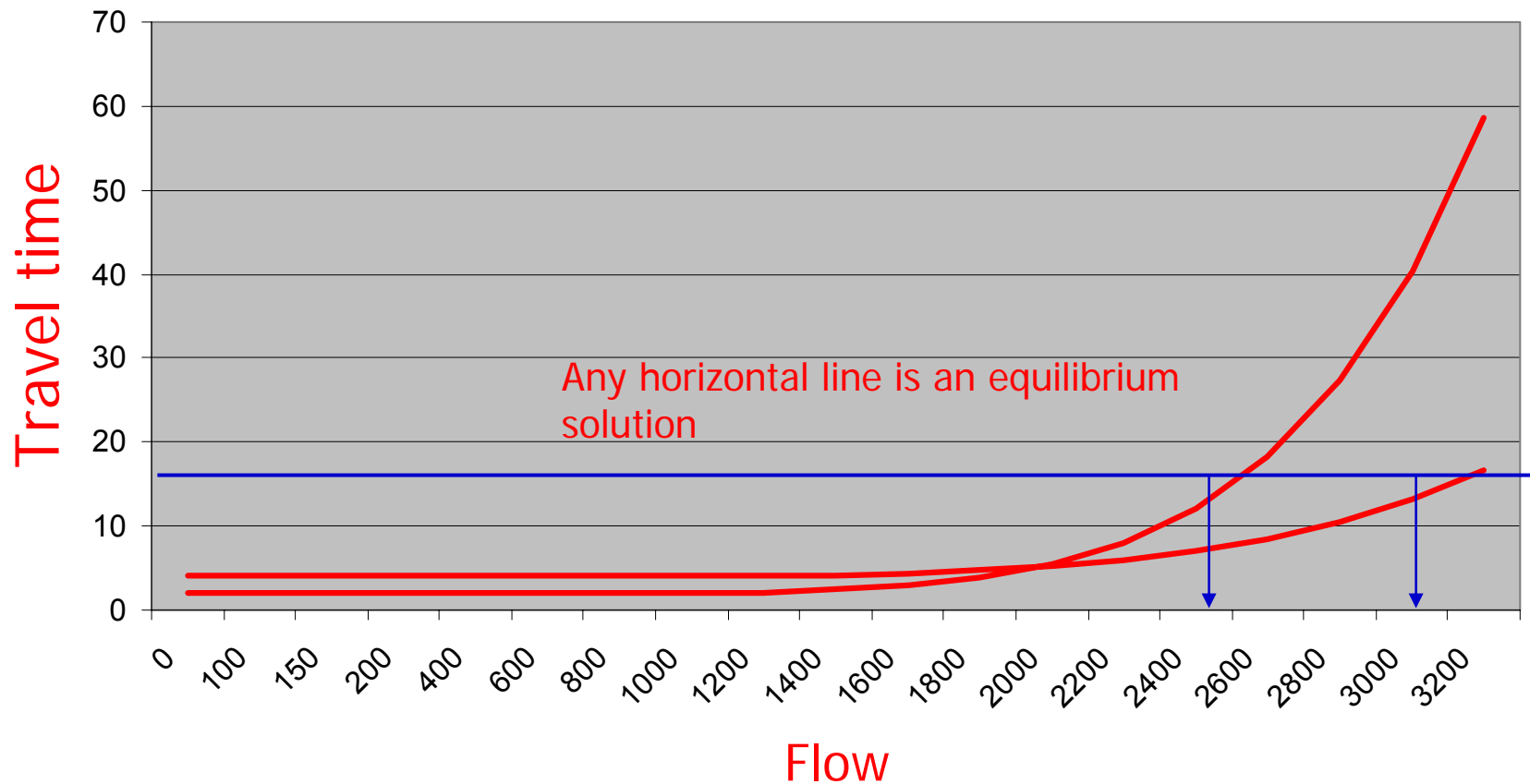
The two-link equilibrium solution

► The travel time on both links is the same

► The demand on each link is that which corresponds to the travel time of the two paths

► One computes the aggregate of the two volume/delay functions. It is the lower free flow time volume/delay function until it reaches the free flow time of the higher travel time function. Then for each equilibrium travel time

► One adds the corresponding flow on the two links.

► Then the intersection of the inverse demand function with the aggregate impedance curve is the equilibrium solution for the link that gives the minimum travel time

Two-link equilibrium solution

Graphical Equilibrium Solution

Two-link equilibrium solution

Graphical Equilibrium Solution

Travel time vs. Flow-Demand

Composite impedance function
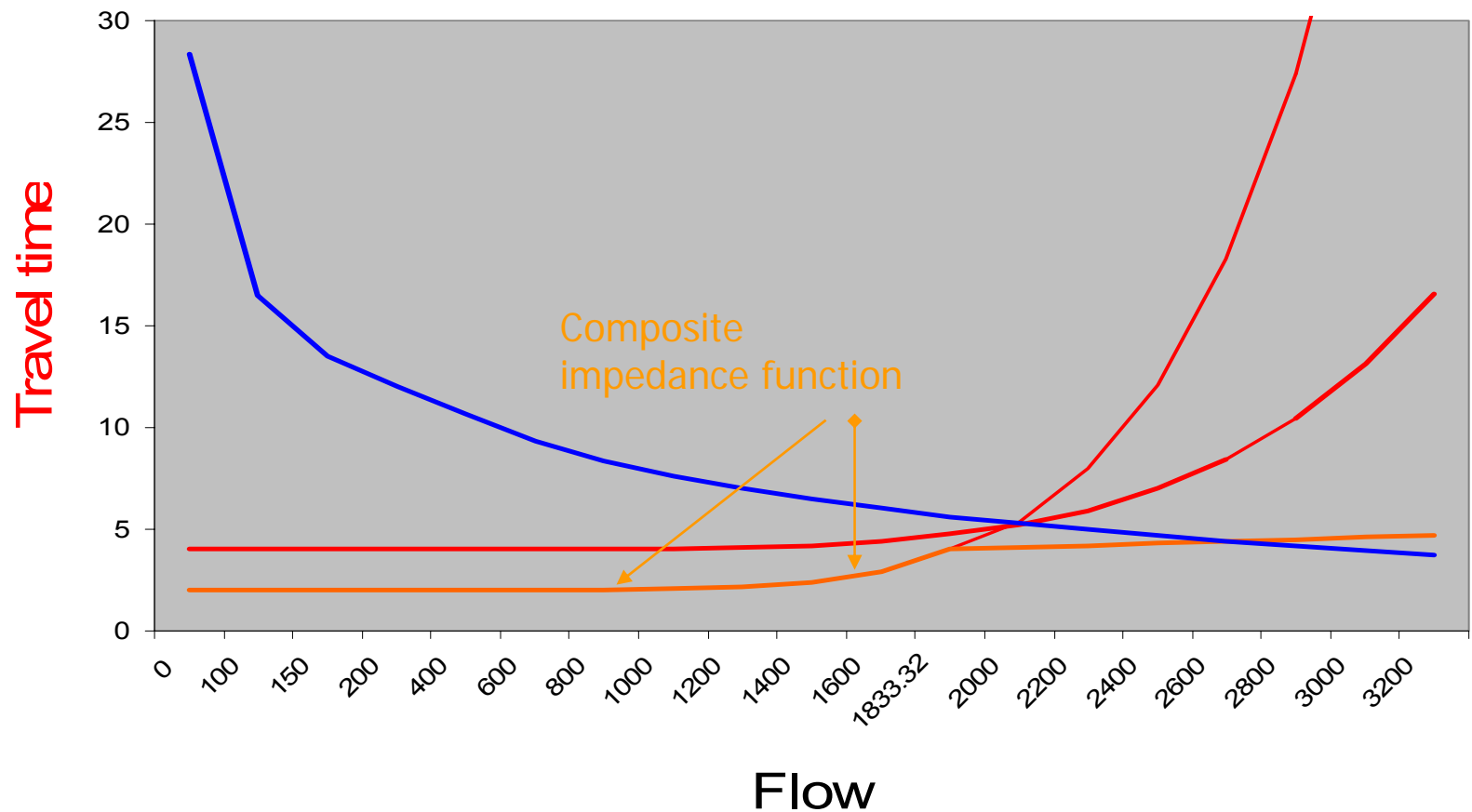
Equilibrium solution
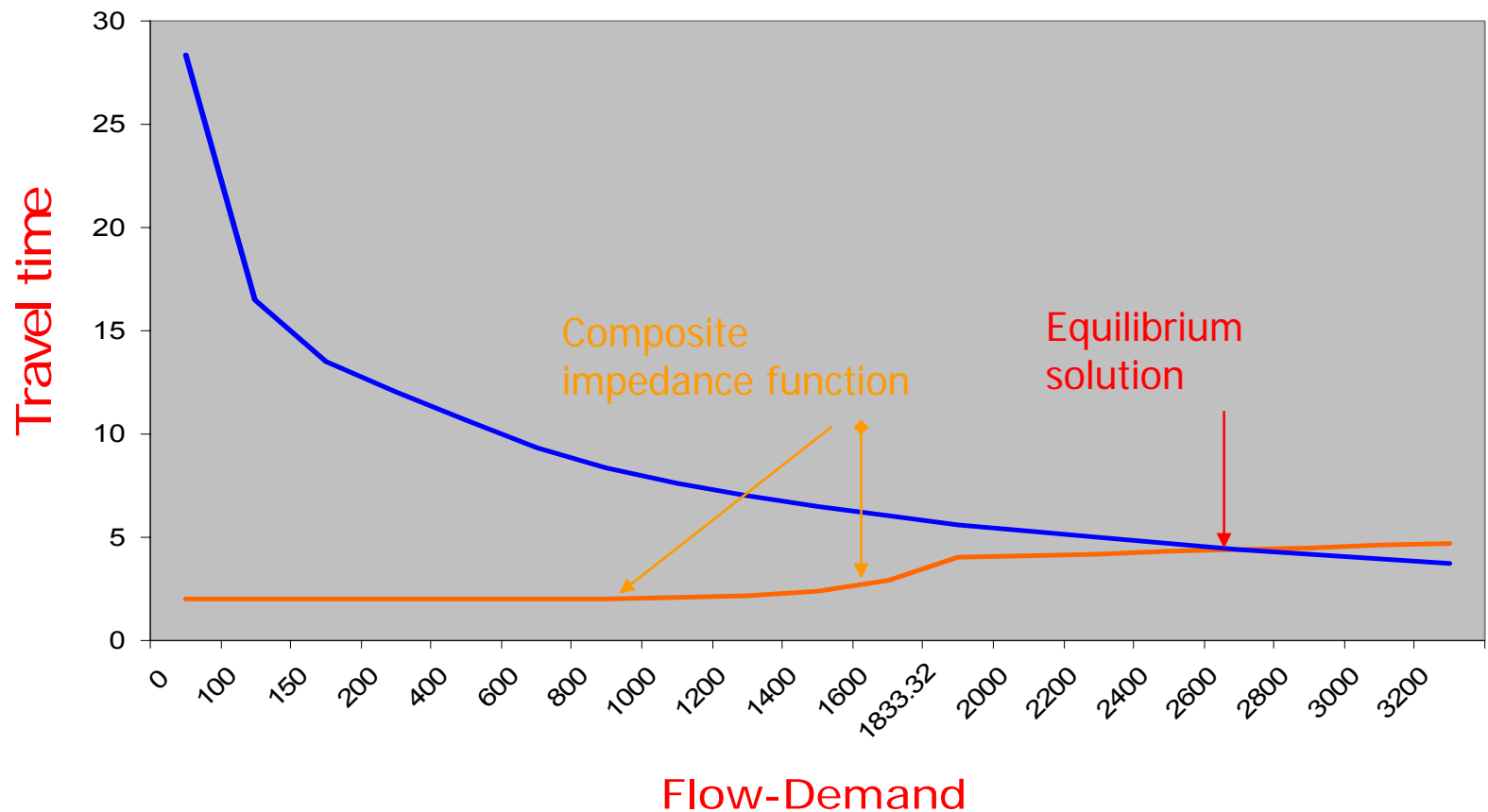
Variable demand network equilibrium

► This is a well solved problem

► Numerous algorithms have been proposed to solve this problem

► The applications are few since the demand is usually not specified by a direct demand function

► Has been used for combined mode choice assignment equilibration

- Variable demand equilibrium assignment

- Multi-modal models : equilibration

- Some large scale applications

- Stochastic user equilibrium assignment

The demand is usually represented by a sequence of models that determine:

► The production and attraction of trips

► The destination choice

► The mode choice

► Classical demand models are aggregated by zone

► More modern demand modeling practice is based on disaggregate behavior, tours and activities

General comments on demand procedures

► All the approaches to demand modeling have the property that, as trip time increases for a mode the demand for that mode decreases, if all other factors influencing demand are kept constant.

► Hence, often the variable demand equilibration is not carried with a well defined mathematical model that can be analyzed rigorously.

The need for averaging or "feedback"

► In order to equilibrate the demand and network performance in complex models one needs to use an averaging or "feedback" method that would ensure the consistency of the modeling suite used.

► That is, ensure that the travel times that correspond to the link flows are indeed those that yield the demand that is assigned to the network.

Transportation Planning Models with "Feedback"

Production Attraction

↓

Trip Distribution

↓

Mode Choice

↓

Trip Assignment

↓

Impedance

Feedback Mechanism
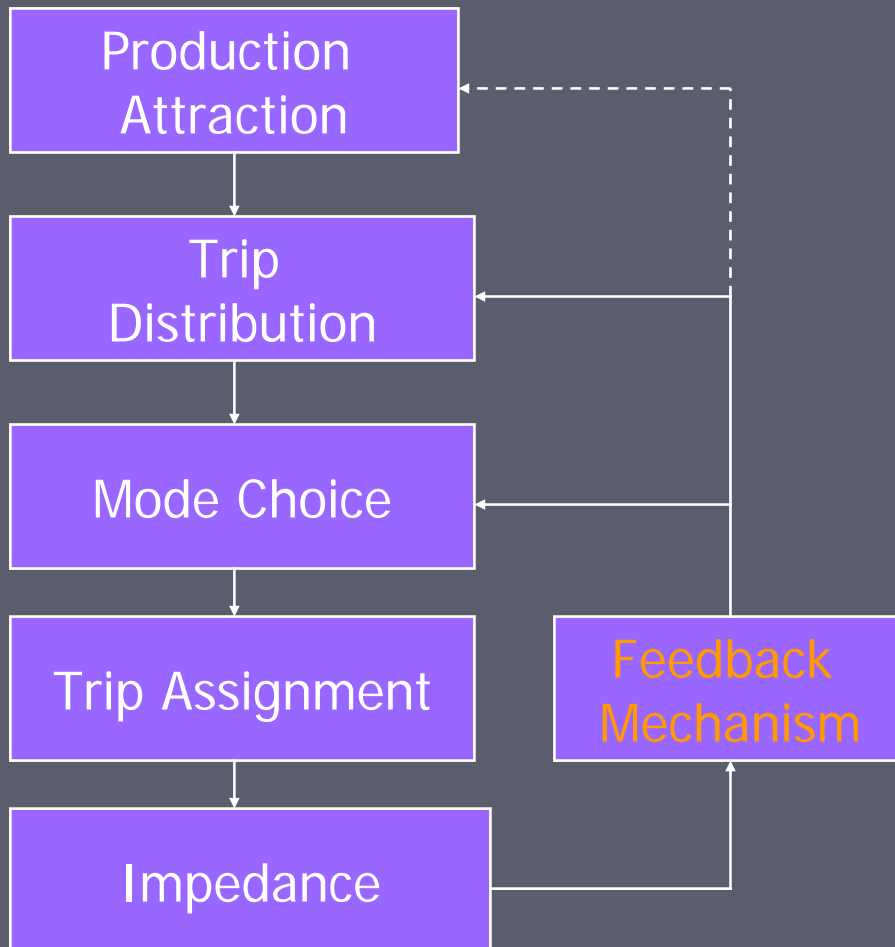
- The "Feedback Mechanism" may take different forms depending on the knowledge that one has of the underlying model.

- If it is an optimization model the step sizes are computed according to a descent algorithm

- Otherwise one carries out "ad-hoc" feedback by some averaging scheme.

Combined models that have well understood mathematical formulations

► Combined trip distribution-assignment

► Combined trip distribution-mode choice-assignment

► Can be solved by methods which have well known convergence properties since they can be formulated as optimization models with the appropriate constraints

► Key point : starting from a feasible solution the method finds an auxiliary solution by generating a new solution and a step size to combine with the current solution by using the objective function of the problem

► New solution = Current solution *(1-step size)+
Auxiliary Solution * step size

Complex models of the type that can not be formulated as optimization models

► Solved by using "ad-hoc" equilibration methods that are inspired from optimization models but do not have the same reassuring properties

► The Method of Successive Averages (MSA) is one of the more popular ones that is used in attempting to equilibrate complex models.

► The averaging scheme uses step sizes which are related to the number of iterations carried out but are not based on an objective function; for instance if k is the iteration number

► New solution = Current solution $*(1-1/k)+$
                                                    Auxiliary Solution $*1/k$

Various convergence measures

The measures of gap that can be used in "ad-hoc" equilibration procedures can be related to link flow changes or changes in total travel time, VHT or VMT.

► Relative changes in link flows:

  SUM (|current flows-previous flows|/SUM (current flows)

► Relative changes in total travel costs:

SUM (travel costs*(current flows – prevs. flows)/SUM (travel costs*current flows)

  This convergence measure is intuitively related to the notion of a relative or normalized gap.

Various convergence measures

► Some practitioners just use change in VMT (Vehicle Miles Traveled) from one iteration to the next

SUM (travel dist.*current flows – travel dist.*prevs. flows)

► Another convergence measure may be the changes in VHT (Vehicle hours traveled)

SUM (travel time*current flows – travel time*prevs. flows)

which is related to the normalized travel cost measure.

Various convergence measures

► These convergence measures are satisfactory since they all measure how close to a consistent solution are the resulting demands and flows.

► In the absence of a well formulated optimization model one has to resort to these types of convergence measures; the results are satisfactory as will be shown with some examples from practice.

► One also assumes that the equilibrium solution is unique which may or may not be the case

- ► Variable demand equilibrium assignment

- ► Multi-modal models : equilibration

- ► Some large scale applications

- ► Stochastic user equilibrium assignment

Examples of Equilibration:
Some examples derived from practice where "feedback" was used

Metro Portland, OR
Winnipeg, Manitoba
Santiago, Chile
SCAG, California

(None of these are current models)

The Metro Portland Model Study on Equilibration

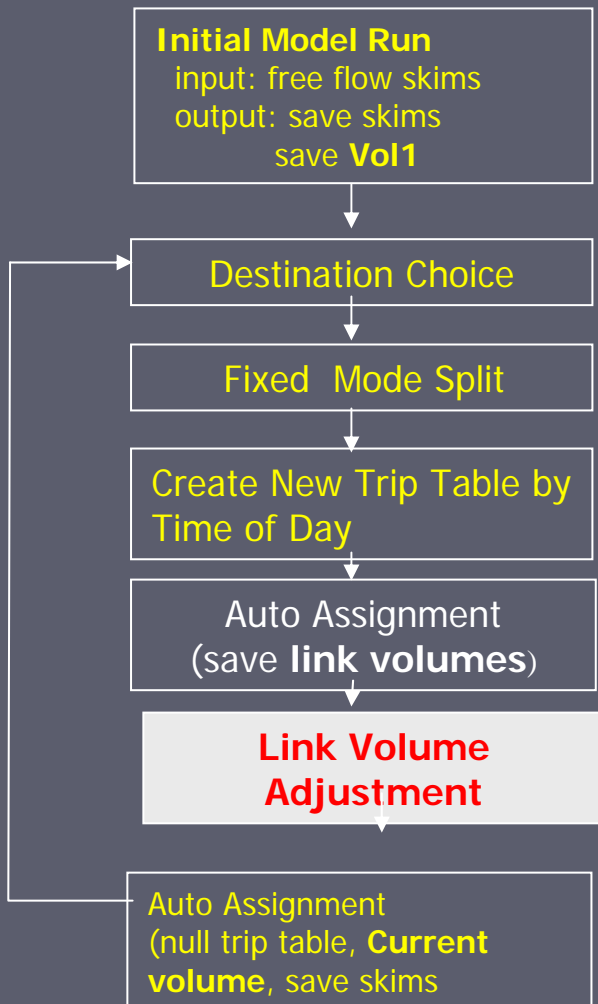► This part of the presentation is based on work done by Dick Walker, Cindy Pederson and Scott Higgins

► The paper is entitled " Equilibrating the Input and Output Impedances in the Demand Modeling Process" November 1995

► The paper is publicly available

The Metro Portland Model Study on Equilibration

► The dimensions of the application are:

1,260 zones

8,034 nodes

22,546 links

7 trip purposes

2 time periods

Method of Successive Averages (MSA) Variant 1

Model Application

Initial Model Run
input: free flow skims
output: save skims
save **Vol1**

↓

Destination Choice

↓

Fixed Mode Split

↓

Create New Trip Table by Time of Day

↓

Auto Assignment
(save **link volumes**)

↓

Link Volume Adjustment

↓

Auto Assignment
(null trip table, **Current volume**, save skims

Link Weighting Technique at Each Model Iteration

1st Iteration
volume = Vol2
$AVG1 = \left(\frac{1}{2}\right) Vol1 + \left(\frac{1}{2}\right) Vol2$ = Averaged volume

2nd Iteration
volume = Vol3
$AVG2 = \left(\frac{2}{3}\right) AVG1 + \left(\frac{1}{3}\right) Vol3$ = Averaged volume

3rd Iteration
volume = Vol4
$AVG3 = \left(\frac{3}{4}\right) AVG2 + \left(\frac{1}{4}\right) Vol4$ = Averaged volume

4th Iteration
volume = Vol5
$AVG4 = \left(\frac{4}{5}\right) AVG3 + \left(\frac{1}{5}\right) Vol5$ = Averaged volume

5th Iteration
volume = Vol6
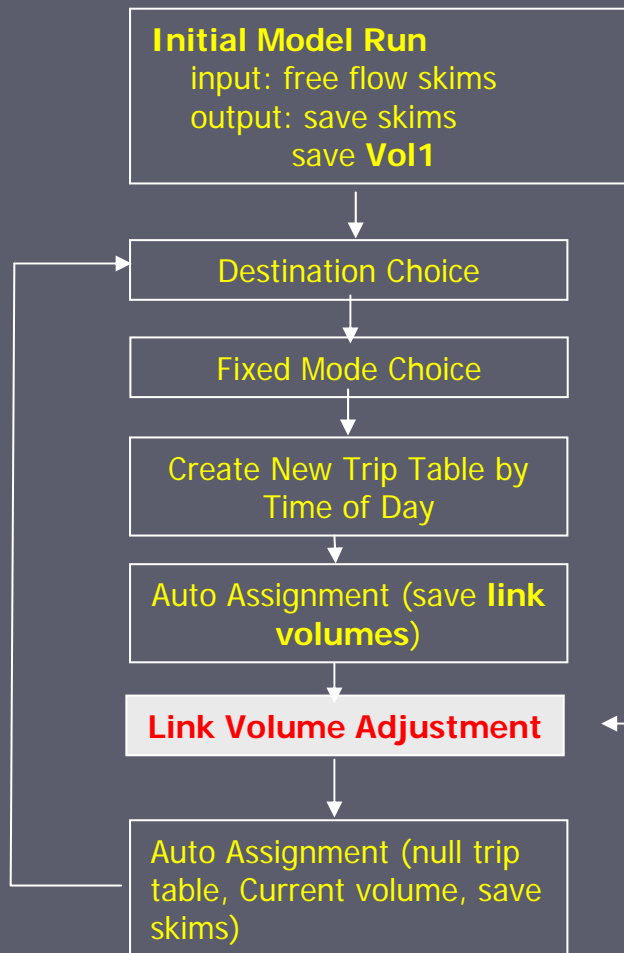$AVG5 = \left(\frac{5}{6}\right) AVG4 + \left(\frac{1}{6}\right) Vol6$ = Averaged volume

and so on ...

The implied step size

► The step size used is in this variant of MSA is simply 1/iteration number

► That is, the last volume obtained is given a decreasing weight as the number of iterations increase

► This is the most conventional form of the MSA method

Method of Successive Averages (MSA) Variant 2

Model Application

Initial Model Run
  input: free flow skims
  output: save skims
    save **Vol1**

↓

Destination Choice

↓

Fixed Mode Choice

↓

Create New Trip Table by Time of Day

↓

Auto Assignment (save **link volumes**)

↓

Link Volume Adjustment

↓

Auto Assignment (null trip table, Current volume, save skims)

Link Averaging Technique at Each Model Iteration

1st Iteration
volume = Vol2
AVG1 $= \dfrac{Vol1 + Vol2}{2} =$ Averaged volume

2nd Iteration
volume = Vol3
AVG2 $= \dfrac{AVG1 + Vol3}{2} =$ Averaged volume

3rd Iteration
volume = Vol4
AVG3 $= \dfrac{AVG2 + Vol4}{2} =$ Averaged volume

4th Iteration
volume = Vol5
AVG4 $= \dfrac{AVG3 + Vol5}{2} =$ Averaged volume

5th Iteration
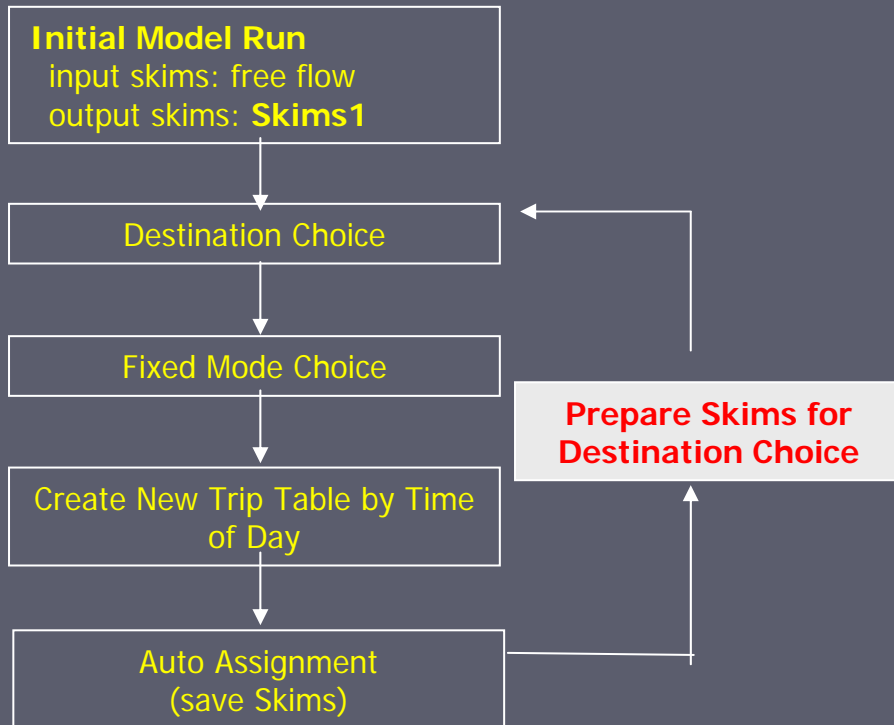volume = Vol6
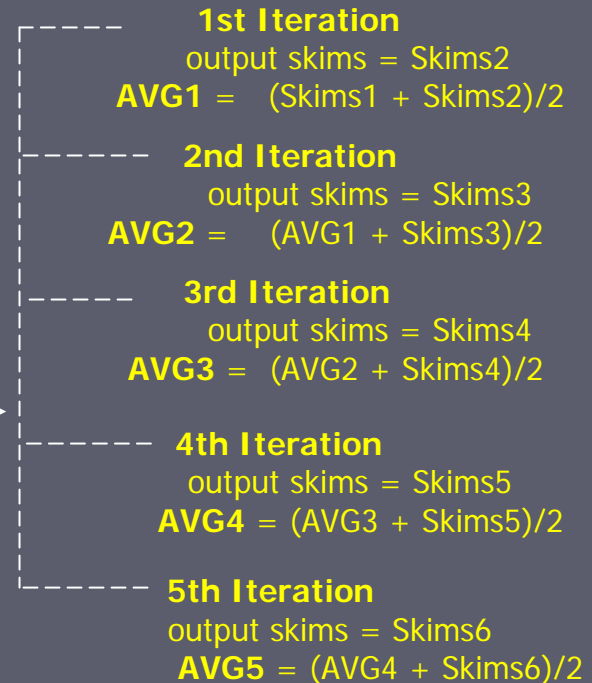**AVG5** $= \dfrac{AVG4 + Vol6}{2} =$ Averaged volume

and so on...

The implied step size

► In this variant the step size is always ½.

► That is, the last volume is always given more weight. Previous iteration volumes are given implicit weights of ½**number of iterations

► This is a less common version of the MSA method

Travel Costs Averaging Method

Model Application

Skim Preparation Technique at Each Model Iteration

Initial Model Run
input skims: free flow
output skims: **Skims1**

Destination Choice

Fixed Mode Choice

Create New Trip Table by Time of Day

Auto Assignment
(save Skims)

Prepare Skims for Destination Choice

1st Iteration
output skims = Skims2
AVG1 =   (Skims1 + Skims2)/2

2nd Iteration
output skims = Skims3
AVG2 =   (AVG1 + Skims3)/2

3rd Iteration
output skims = Skims4
AVG3 = (AVG2 + Skims4)/2

4th Iteration
output skims = Skims5
AVG4 = (AVG3 + Skims5)/2

5th Iteration
output skims = Skims6
AVG5 = (AVG4 + Skims6)/2
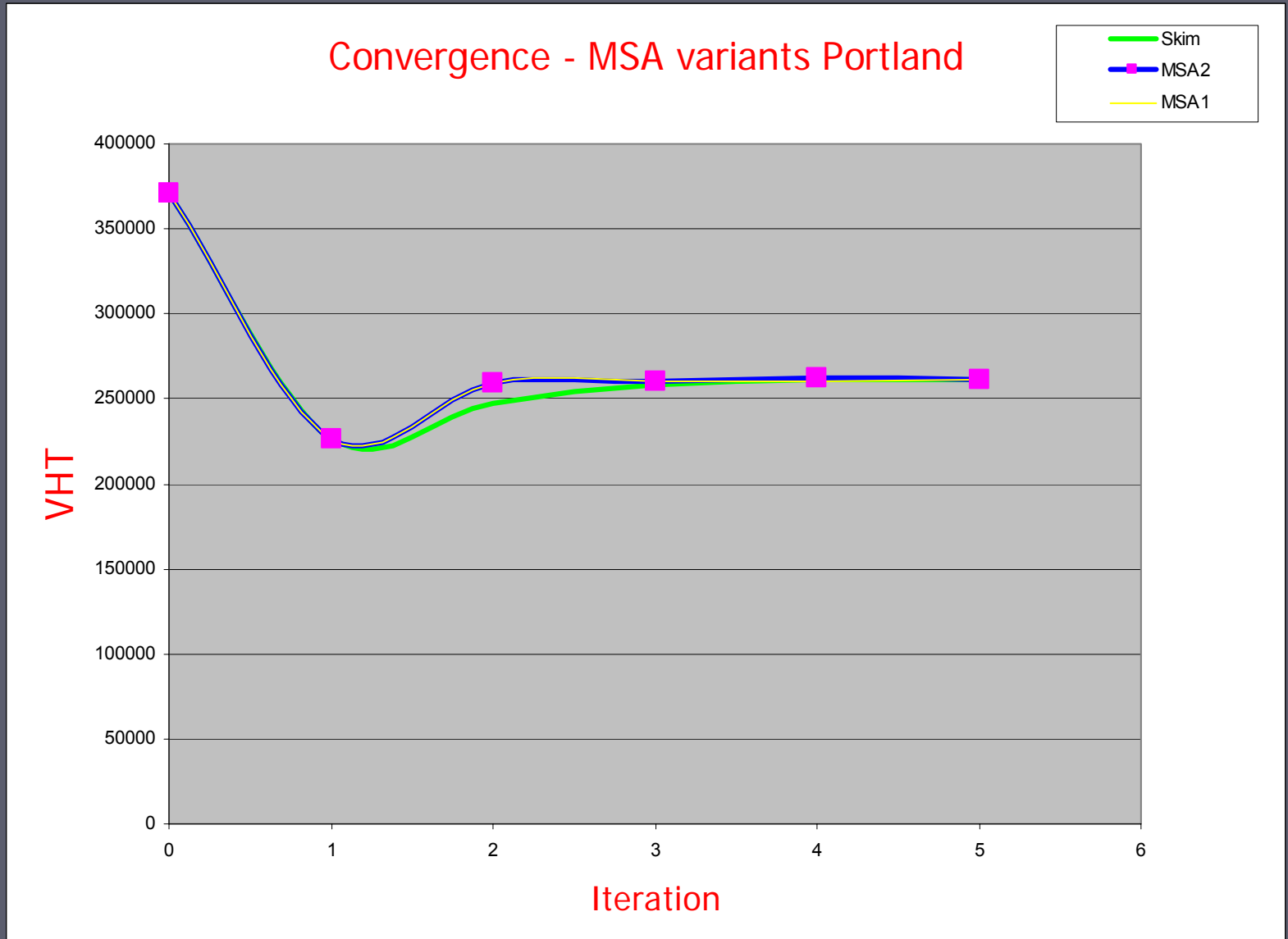
and so on ....

Impedance averaging

► This variant of MSA is less common but in some circumstances it may be useful

► The results obtained resemble those obtained with link volume averaging but require more iterations

► The next two slides show the convergence results for VHT and VMT. Link flow differences were not given

Convergence of VHT

Convergence - MSA variants Portland

Convergence of VMT

Convergence MSA variants Portland

Legend: SKIM, MSA2, MSA3

Y-axis: VMT(1000s vehicle per hour)
X-axis: Iteration
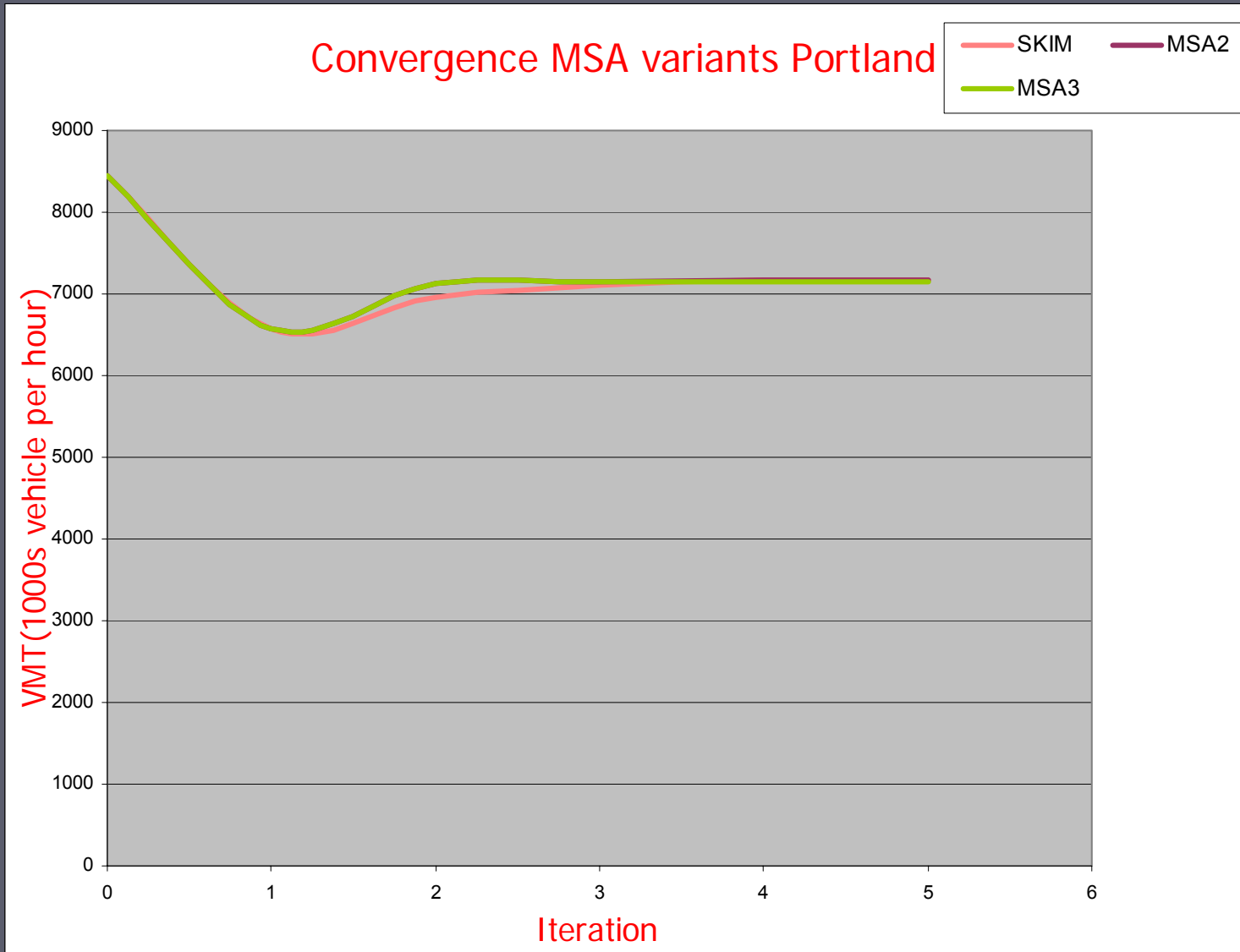
The Winnipeg, Canada Model

► This application is based on a rather old data base. The purpose here is to demonstrate the the equilibration mechanism.

► The model is a rather conventional four step model where the equilibration involves the trip distribution, the mode choice and the assignment on both road and transit networks.

► The problem size is modest: 154 zones, 1,100 nodes, 3000 links, 67 transit lines, 2 modes, 1 trip purpose.

The Winnipeg base network

Bare network

The Winnipeg, Canada Model

Generation Attraction

Trip Distribution

Mode Choice

Trip Assignment: road and transit

Conventional MSA

Impedances
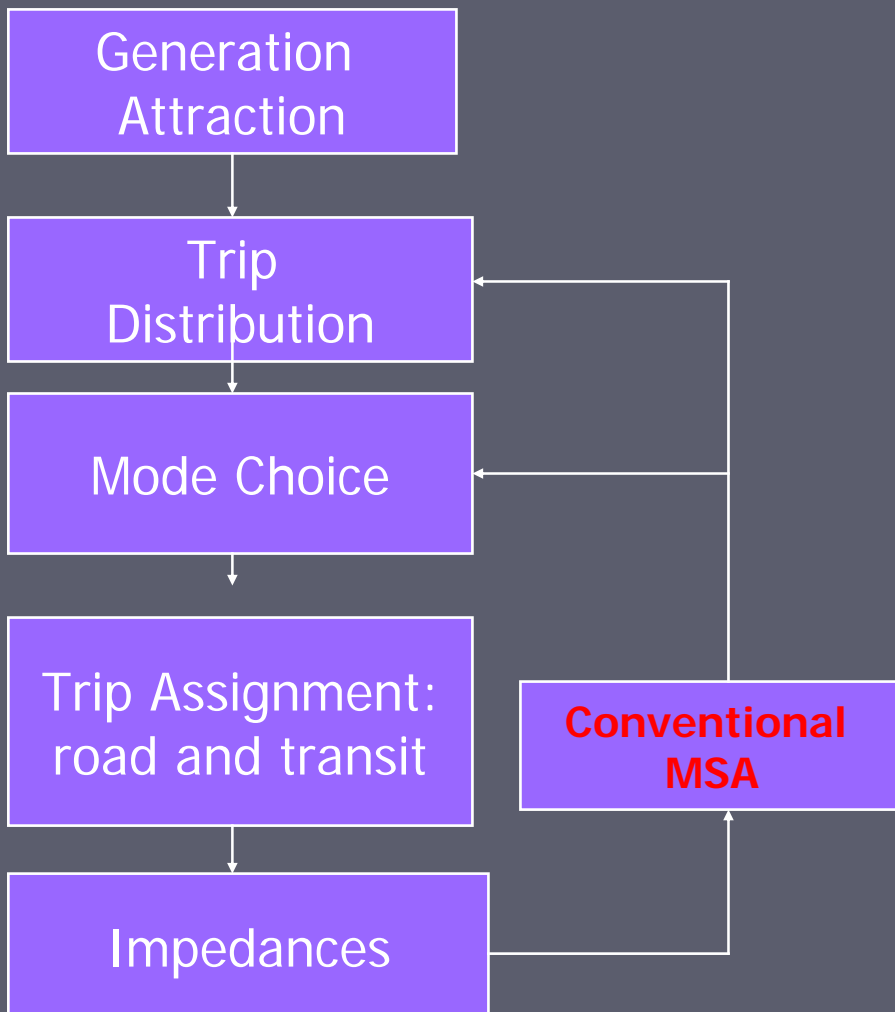
- The "Feedback Mechanism" that was used here is the conventional MSA method

- The trip distribution model is a three dimensional model

- The mode choice is a binomial logit model

- There are only two modes : bus and car

The Winnipeg, Canada Model

► The next two slides give the convergence results for the link difference and normalized gap measures for two variants of the trip distribution model: two-dimensional and three-dimensional models.

Winnipeg Convergence Results

WPG Model Convergence with 2-D Balancing

Santiago, Chile - Base network (2001)

Base Network of Santiago, Chile
(Year 2001)

base network
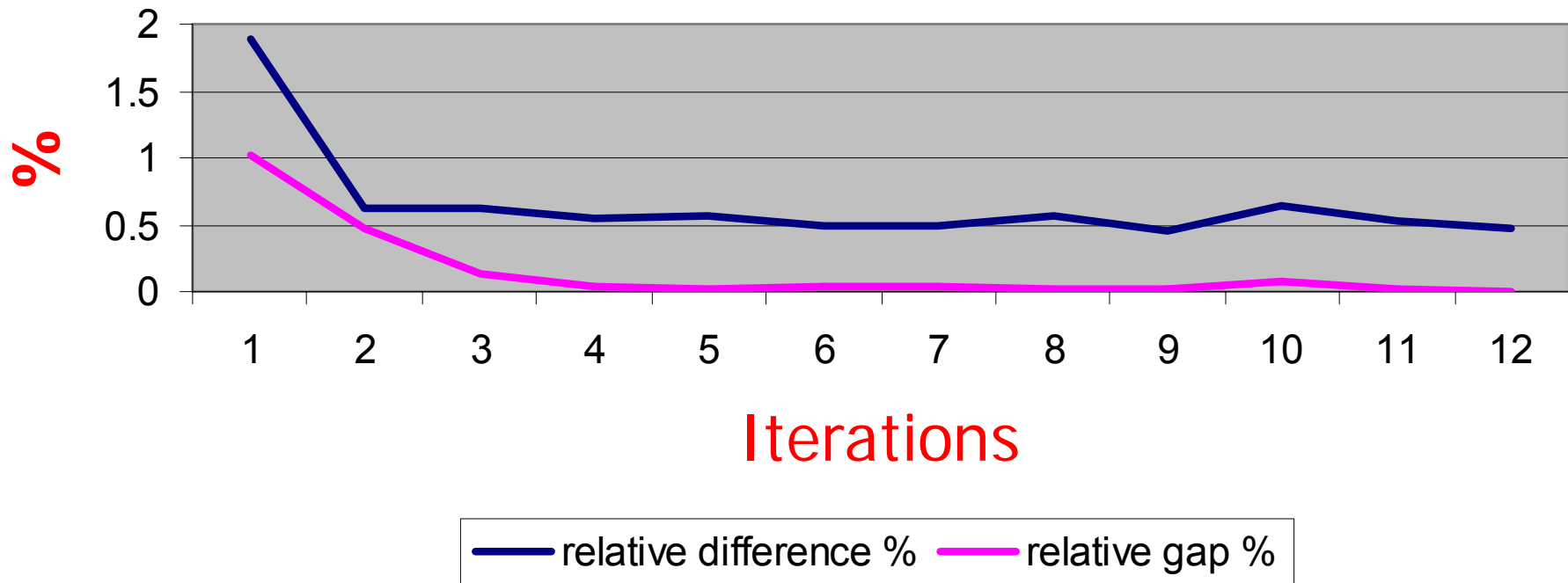connector
metro
parking zone
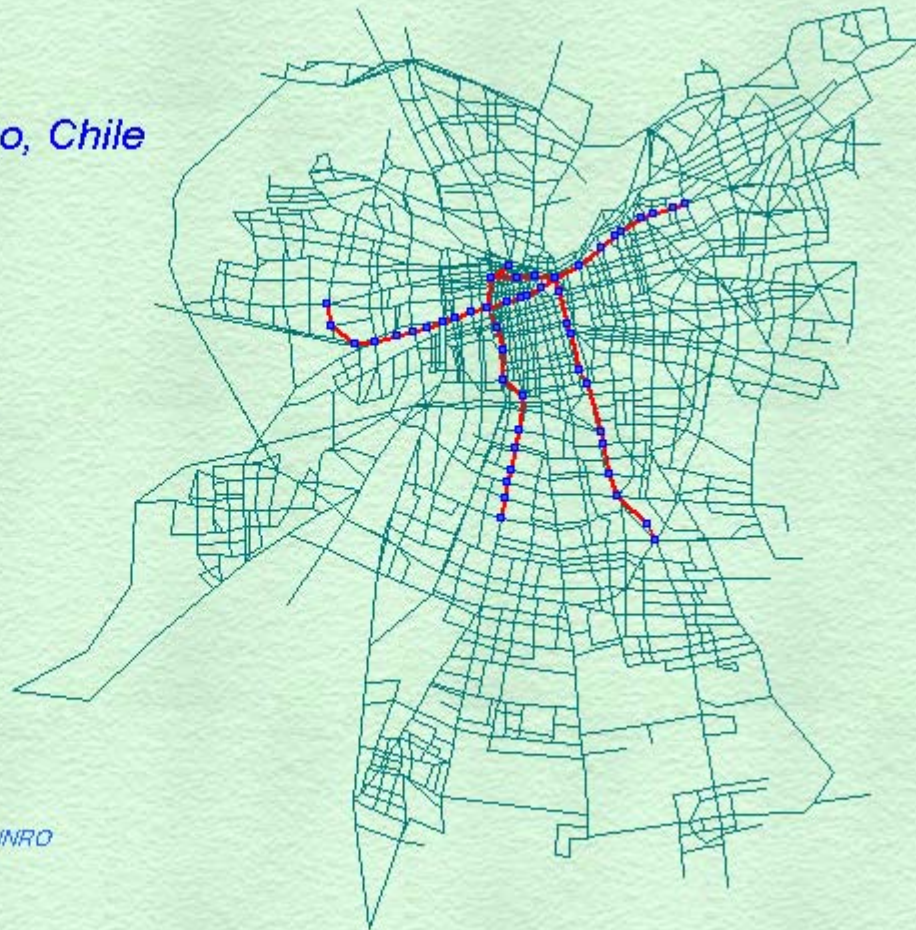
Plot generated by Enif 2002-01-23 10:30:20 at INRO

INRO

Santiago, Chile Strategic Planning Model
- developed by Fernandez and DeCea (ESTRAUS)

- Base network

  - 409 centroids including 49 parking locations

  - 1808 nodes, 11,331 directional links

  - 1116 transit lines and 52468 line segments

  - 11 modes, including 4 combined modes

    (bus-metro, txc-metro, auto-metro and auto passenger-metro)

- The demand

  - subdivided into 13 socio-economic classes

  - 3 trip purposes (work, study, other)

  - driving license holders can access to 11 modes

  - no license holders can access to 9 modes

Mode Choice Model

The mode choice function is a simple multinomial logit function :

```
                              ┌──────────────┐
                              │     root     │
                              └──────────────┘
```

| walk | auto | auto pass. | taxi | taxi coll. | bus | metro | bus metro | txc metro | auto metro | auto pass. metro |

Solution Procedure

Start and Initialize

Trip Distribution and Mode Choice

Auto Assignment

MSA Auto Volume

Auto Skims

Convergence?

end

Transit Auto equivalent flow

Congested time

Multiple Transit Assignments

Standard Transit Assignment for buses

Transit Assignment with adjusted headway for metro

Auto Impedance

Bus Impedance

Metro MSA Impedance

Park-and-Ride Model for auto-metro and bus-metro

All Impedances

Santiago, Chile Strategic Planning Model

- this is a different algorithm than the one used by Fernandez and DeCea

► The next slide shows the convergence of the MSA algorithm that uses link flow averaging for the car network and travel time averaging for the transit network.

► The convergence of both the car demand and link flows are given for two variants: uncongested transit assignment and eqiuilibrium transit assignment.

Convergence of equilibration

Normalized Gap vs. Iterations
congested vs. non-congested metro assignment with metro capacity
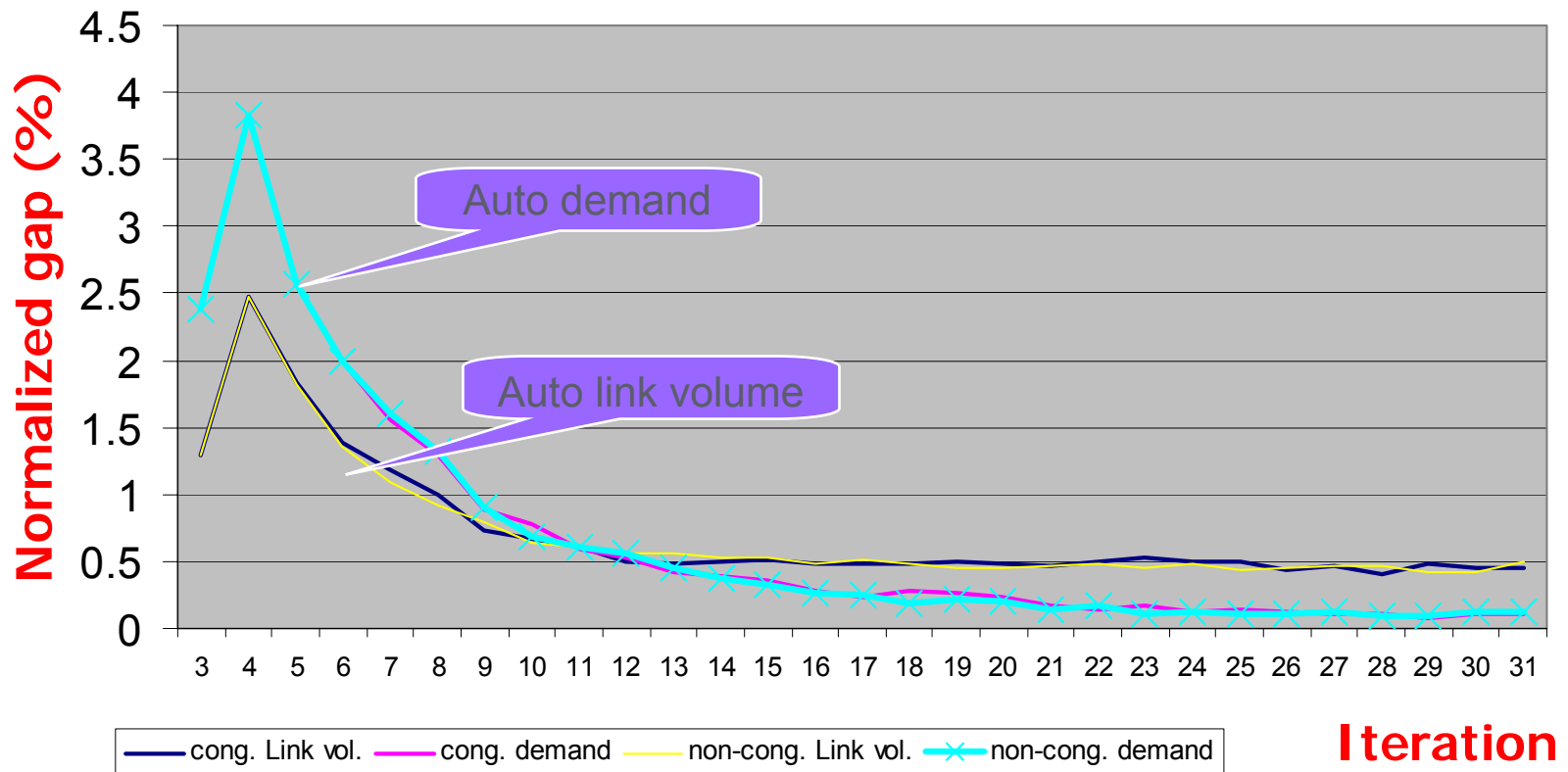
The SCAG California Model

► This is a very large model of the aggregate variety

► The model dimensions are impressive:
15 modes
4 time periods
13 trip purposes – trip distribution models
5 mode choice models

The SCAG California Model (1997)
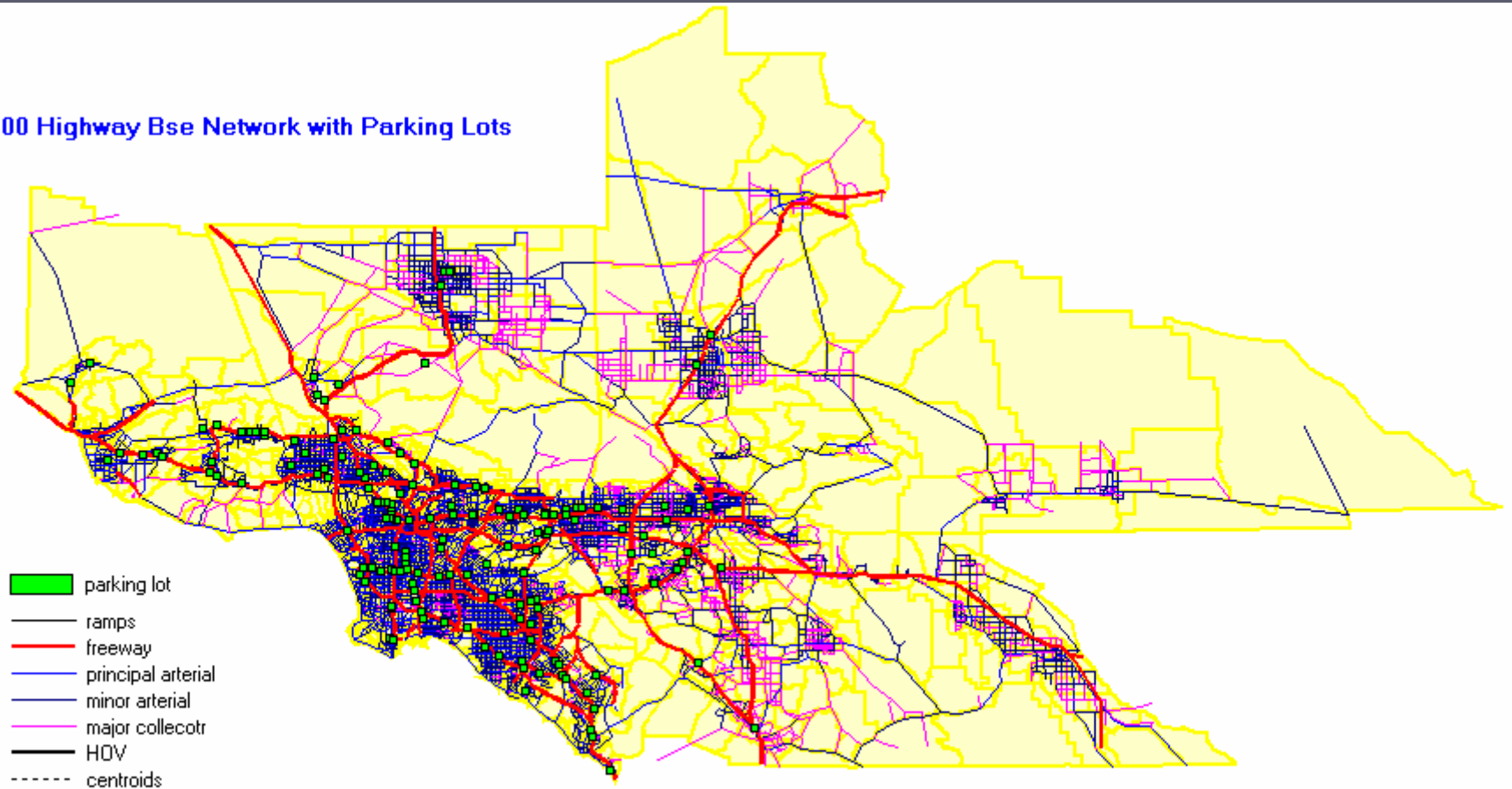
► There are 3325 zones (3217 SCAG region and externals; 108 zones for parking lots)

► 26290 Nodes and 108897 directional links

► 672 transit lines for 21 operators and 60926 transit line segments
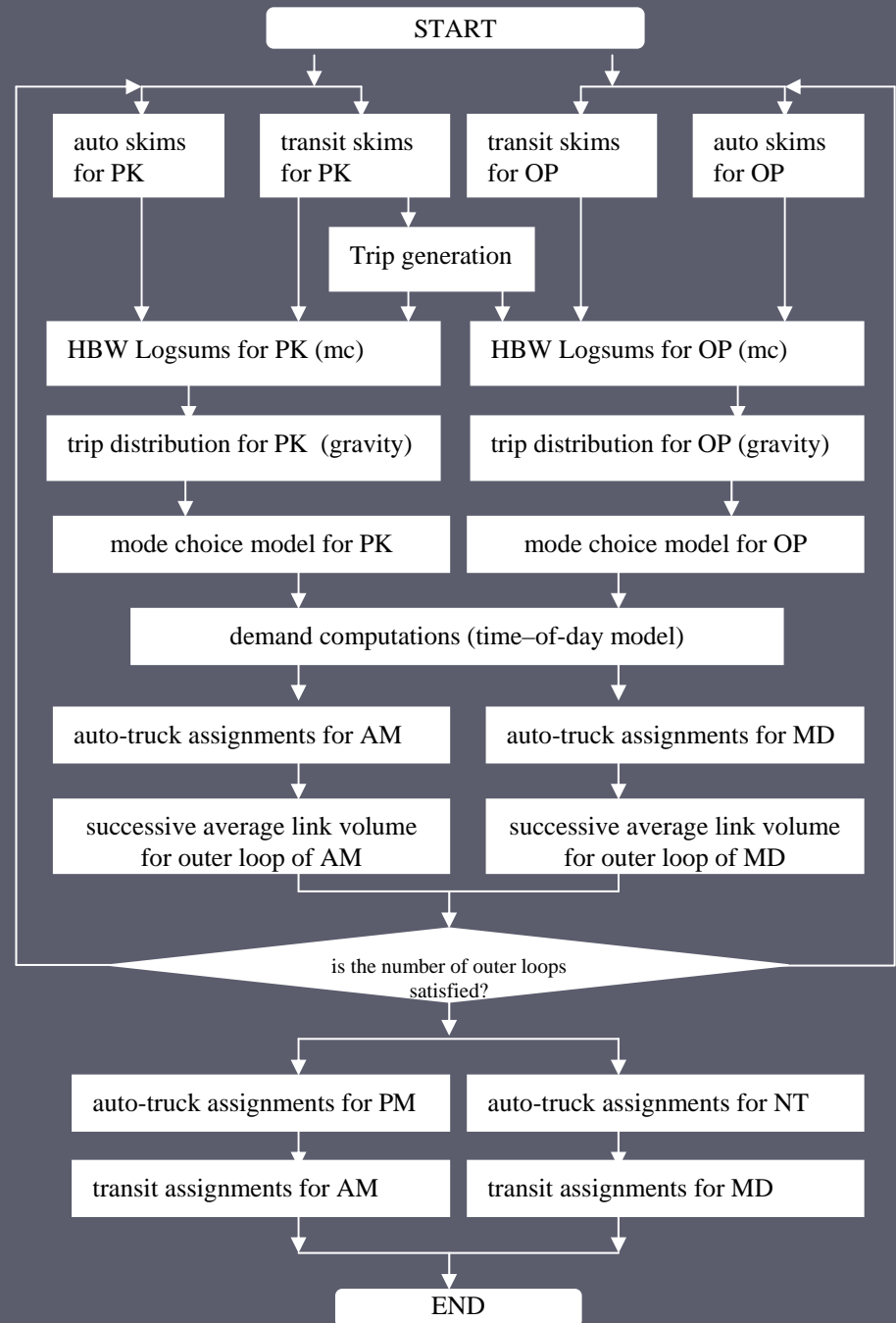
The SCAG Highway Network
with parking lots

2000 Highway Bse Network with Parking Lots

parking lot
ramps
freeway
principal arterial
minor arterial
major collecotr
HOV
centroids

THE SCAG MODEL COMPUTATIONS FLOW CHART:

- Run times on AIX RS/6000:

  – Auto skim for one mode: 9 min
  – Transit skims for one mode: 1-2 hr (10 matrices per mode)
  – Trip distribution for one mode: 3 min
  – Transit assignment for one mode: 9 min
  – Auto assignment for 6 classes :1 iteration: 18 min

```
                            START

 auto skims    transit skims   transit skims    auto skims
  for PK         for PK          for OP           for OP

                  Trip generation

 HBW Logsums for PK (mc)        HBW Logsums for OP (mc)

 trip distribution for PK (gravity)   trip distribution for OP (gravity)

 mode choice model for PK       mode choice model for OP

        demand computations (time–of–day model)

 auto-truck assignments for AM     auto-truck assignments for MD

 successive average link volume    successive average link volume
  for outer loop of AM              for outer loop of MD

            is the number of outer loops
                  satisfied?

 auto-truck assignments for PM     auto-truck assignments for NT

 transit assignments for AM        transit assignments for MD

                    END
```
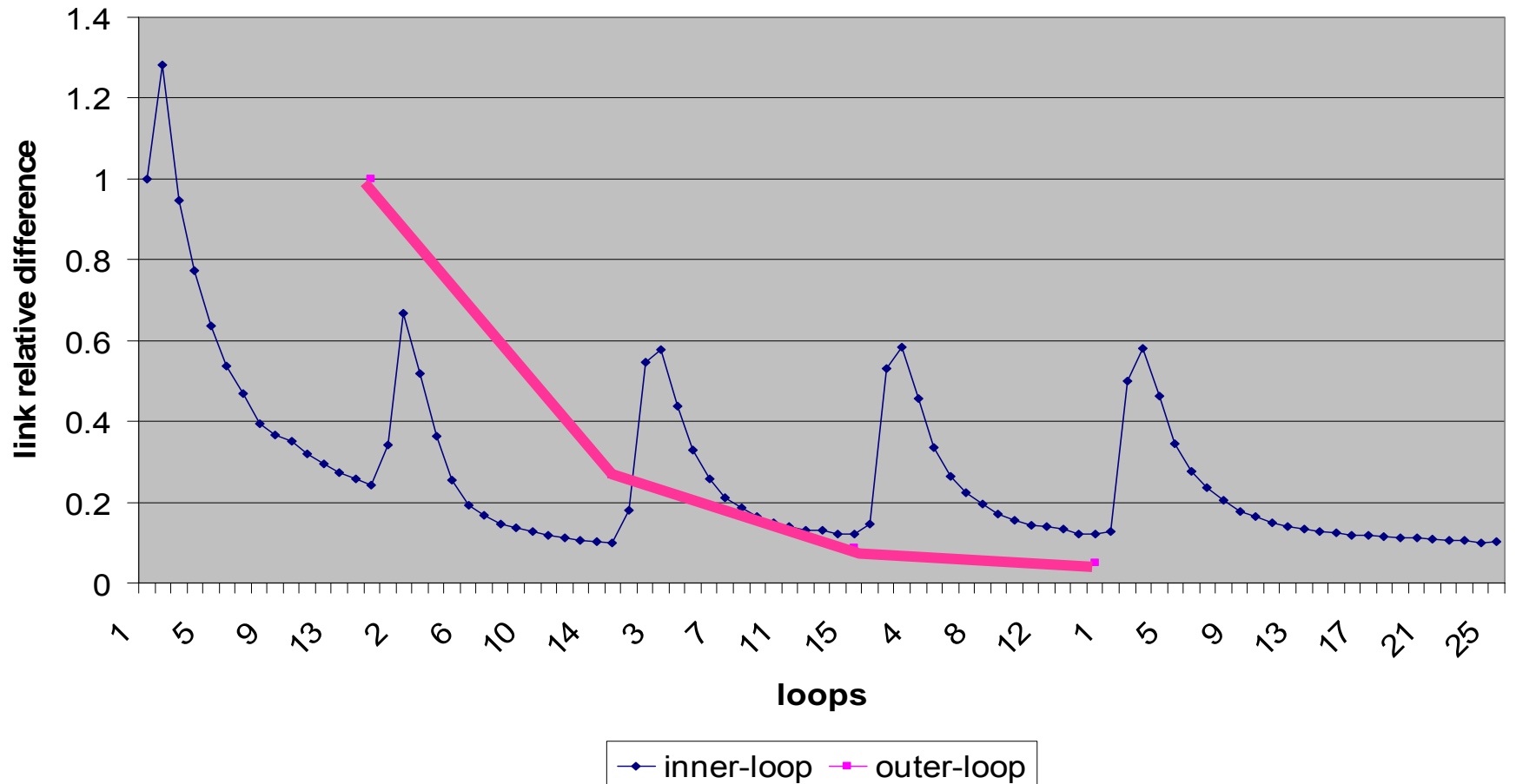
The SCAG California Model (1997)

► The next slide shows the convergence of both inner iterations of the car assignment (in blue) and the outer iterations (in red). Only 5 iterations were carried out.

SCAG MSA Convergence results

SCAG Model Convergence (AM peak)

Comments on the SCAG results

▶ Each inner loop is so time consuming that the number of iterations are limited by the total computational time.

▶ Depending on which initial solution is chosen there may be 2%-5% difference in the resulting VMT values.

▶ Convergence is probably very approximate.

Conclusions on "ad-hoc" equilibration

► The "ad-hoc" equilibration methods do converge empirically.

► If the network is congested one may have to carry out at least 15 iterations and the results may still exhibit 2%-5% variation.

► This raises issues about the necessity of very fine solutions in the inner loop.

► A lot is left to the judgment of the analyst who carries out the study.

► Variable demand equilibrium assignment

► Multi-modal models : equilibration

► Some large scale applications

► Stochastic user equilibrium assignment

Stochastic user equilibrium

For each origin-destination pair of zones, all used routes have equal <u>perceived</u> travel times, and no unused route has a lower <u>perceived</u> travel time.

► The models that are used are based on various distribution for the perception errors: logit, probit, uniform.

The computational methods used

► The logit path choice combined with the user optimal principle may be formulated as an optimization problem (In the case of constant travel times STOCH by Dial deserves special mention).

► Hence it may be solved by using a descent algorithm with well defined convergence properties.

► The probit path choice requires a solution by Monte Carlo simulation and the use of the MSA method. In this case a convergence proof exists.

Uses of Stochastic assignment

► In relatively un-congested conditions.

► When congestion dominates the stochastic and user equilibrium yield similar results.

► In mildly congested conditions the results may reflect better the paths chosen.

► A nice property of stochastic assignment is that the path flows are unique, which is not the case with deterministic equilibrium assignment.

Stochastic assignment - summary

► The use of stochastic assignment enriches the methods used for determining path choice on lightly congested networks.

► There is no hard and fast rule which may be used to determine when stochastic assignment should be used. It is up to the analyst that carries out the study.