# Writing Linear Equations 

 0ESSENTIAL QUESTION
How can you use linear equations to solve real-world problems?

LESSON 5.1
Writing Linear
Equations from Situations and Graphs COMMON
CORE 8.F. 4

LESSON 5.2
Writing Linear Equations from a Table
COMMON
CORE 8.F. 4

LESSON 5.3
Linear Relationships and Bivariate Data
8.SP.1, 8.SP.2,
8.SP. 3

Real-World Video
Linear equations can be used to describe many situations related to shopping. If a store advertised four books for $\$ 32.00$, you could write and solve a linear equation to find the price of each book.

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## Are

Complete these exercises to review skills you will need for this module.

## Write Fractions as Decimals

EXAMPLE $\quad \frac{0.5}{0.8}=$ ?

| Multiply the numerator and the denominator | $\frac{0.5 \times 10}{0.8 \times 10}=\frac{5}{8}$ |
| :--- | :---: |
| by a power of 10 so that the denominator is a | 0.625 |
| whole number. | $8 \longdiv { 5 0 0 0 }$ |
|  | $\frac{-48}{20}$ |
| Write the fraction as a division problem. | $\frac{-16}{40}$ |
| Write a decimal point and zeros in the dividend. | $\frac{-40}{0}$ |
| Place a decimal point in the quotient. |  |
| Divide as with whole numbers. |  |

Write each fraction as a decimal.

1. $\frac{3}{8}$
2. $\frac{0.3}{0.4}$
3. $\frac{0.13}{0.2}$
4. $\frac{0.39}{0.75}$

## Inverse Operations

| EXAMPLE | $\begin{aligned} 5 n & =20 \\ \frac{5 n}{5} & =\frac{20}{5} \\ n & =4 \end{aligned}$ | $n$ is multiplied by 5 . <br> To solve the equation, use the inverse operation, division. |
| :---: | :---: | :---: |
|  | $k+7=9$ | $7 \mathrm{is} \mathrm{added} \mathrm{to} \mathrm{k}$. |
|  | $k+7-7=9-7$ | To solve the equation, use the inverse operation, subtraction. |
|  | $k=2$ |  |

Solve each equation using the inverse operation.
5. $7 p=28$
6. $h-13=5$ $\qquad$
7. $\frac{y}{3}=-6$ $\qquad$ 8. $b+9=21$
9. $c-8=-8$ $\qquad$ 10. $3 n=-12$ $\qquad$
11. $-16=m+7$ $\qquad$ 12. $\frac{t}{-5}=-5$

## Reading Start-Up

## Visualize Vocabulary

## Use the $\checkmark$ words to complete the diagram. You can put more than one word in each bubble.



## Understand Vocabulary

## Complete the sentences using the preview words.

1. A set of data that is made up of two paired variables is $\qquad$ .
2. When the rate of change varies from point to point, the relationship is a $\qquad$ .

## Active Reading

Tri-Fold Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label the columns "What I Know," "What I Need to Know," and "What I Learned." Complete the first two columns before you read. After studying the module, complete the third column.


MODULE 5

# Unpocking the Stondords 

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

## 8.F. 4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship ... Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## Key Vocabulary

rate of change (tasa de cambio)
A ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

## What It Means to You

You will learn how to write an equation based on a situation that models a linear relationship.

## UNPACKING EXAMPLE 8.F.4

In 2006 the fare for a taxicab was an initial charge of $\$ 2.50$ plus $\$ 0.30$ per mile. Write an equation in slope-intercept form that can be used to calculate the total fare.

The constant charge is $\$ 2.50$.
The rate of change is $\$ 0.30$ per mile.
The input variable, $x$, is the number of miles driven.
So $0.3 x$ is the cost for the miles driven.
The equation for the total fare, $y$, is as follows:
$y=0.3 x+2.5$

## 8.SP. 3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

## Key Vocabulary

bivariate data (datos bivariados)
A set of data that is made up of two paired variables.


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## What It Means to You

You will see how to use a linear relationship between sets of data to make predictions.

## UNPACKING EXAMPLE 8.SP. 3

The graph shows the temperatures in degrees Celsius inside the earth at certain depths in kilometers. Use the graph to write an equation and find the temperature at a depth of 12 km .

The initial temperature is $20^{\circ} \mathrm{C}$. It increases at a rate of $10^{\circ} \mathrm{C} / \mathrm{km}$.

The equation is $t=10 d+20$. At a depth of 12 km , the temperature is $140^{\circ} \mathrm{C}$.

Temperature Inside Earth


# LESSON Equations from Situations and Graphs 

How do you write an equation to model a linear relationship given a graph or a description?

## EXPLORE ACTIVITY <br> neald

 COMMONCORE
8.F. 4

## Writing an Equation in Slope-Intercept Form

Greta makes clay mugs and bowls as gifts at the Crafty Studio. She pays a membership fee of $\$ 15$ a month and an equipment fee of $\$ 3.00$ an hour to use the potter's wheel, table, and kiln. Write an equation in the form $\boldsymbol{y}=\boldsymbol{m x}+\boldsymbol{b}$ that Greta can use to calculate her monthly costs.

A What is the input variable, $x$, for this situation?


What is the output variable, $y$, for this situation?
$\qquad$

## Math Talk

Mathematical Practices
B During April, Greta does not use the equipment at all. What will be her number of hours ( $x$ ) for April? $\qquad$
What change could the studio make that would make a difference to the $y$-intercept of the
What will be her cost ( $y$ ) for April? $\qquad$
What will be the $y$-intercept, $b$, in the equation? $\qquad$ equation?

C Greta spends 8 hours in May for a cost of $\$ 15+8(\$ 3)=$ $\qquad$ .

In June, she spends 11 hours for a cost of $\qquad$ .

From May to June, the change in $x$-values is $\qquad$ .

From May to June, the change in $y$-values is $\qquad$ .

What will be the slope, $m$, in the equation? $\qquad$
D Use the values for $m$ and $b$ to write an equation for Greta's costs in the form $y=m x+b$ : $\qquad$

## Writing an Equation from a Graph

You can use information presented in a graph to write an equation in slope-intercept form.

## EXAMPLE 1 (Brald

A video club charges a one-time membership fee plus a rental fee for each DVD borrowed. Use the graph to write an equation in slope-intercept form to represent the amount spent, $y$, on $x$ DVD rentals.

STEP 1 Choose two points on the graph, $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, to find the slope.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
m & =\frac{18-8}{8-0} \\
m & =\frac{10}{8}=1.25
\end{aligned}
$$

Find the change in $y$-values

## Math Talk

Mathematical Practices
If the graph of an equation is a line that goes through the origin, what is the value

STEP 2 Read the $y$-intercept from the graph.
The $y$-intercept is 8 .
STEP 3 Use your slope and $y$-intercept values to write an equation in slope-intercept form.
$y=m x+b \quad$ Slope-intercept form

$$
y=1.25 x+8 \quad \text { Substitute } 1.25 \text { for } m \text { and } 8 \text { for } y \text {. }
$$

## Reflect

1. What does the value of the slope represent in this context?
2. Describe the meaning of the $y$-intercept.

## YOUR TURN

3. The cash register subtracts $\$ 2.50$ from a $\$ 25$ Coffee Café gift card for every medium coffee the customer buys. Use the graph to write an equation in slope-intercept form to represent this situation.

Amount on Gift Card
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## Writing an Equation from a Description

You can use information from a description of a linear relationship to find the slope and $y$-intercept and to write an equation.

## EXAMPLE 2

Red


West Main St. Office Rentals 曹
Offices for rent at convenient locations.
Monthly Rates:
600 square feet for $\$ 750$ 900 square feet for $\mathbf{\$ 1 1 5 0}$

The output variable is the monthly rent.
STEP 2 Write the information given in the problem as ordered pairs.
The rent for 600 square feet of floor space is $\$ 750$ : $(600,750)$
The rent for 900 square feet of floor space is $\$ 1150$ : $(900,1150)$
STEP 3 Find the slope.

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{1150-750}{900-600}=\frac{400}{300}=\frac{4}{3}
$$

STEP 4 Find the $y$-intercept. Use the slope and one of the ordered pairs.

$$
\begin{array}{ll}
y=m x+b & \text { Slope-intercept form } \\
750=\frac{4}{3} \cdot 600+b & \text { Substitute for } y, m \text {, and } x . \\
750=800+b & \text { Multiply. } \\
-50=b & \text { Subtract } 800 \text { from both sides. }
\end{array}
$$

STEP 5 Substitute the slope and $y$-intercept.

$$
\begin{array}{ll}
y=m x+b & \text { Slope-intercept form } \\
y=\frac{4}{3} x-50 & \text { Substitute } \frac{4}{3} \text { for } m \text { and }-50 \text { for } b .
\end{array}
$$

## Reflect

4. Without graphing, tell whether the graph of this equation rises or falls from left to right. What does the sign of the slope mean in this context?

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5. Hari's weekly allowance varies depending on the number of chores he does. He received $\$ 16$ in allowance the week he did 12 chores, and $\$ 14$ in allowance the week he did 8 chores. Write an equation for his allowance in slope-intercept form.

## Guided Practice

1. Li is making beaded necklaces. For each necklace, she uses 27 spacers, plus 5 beads per inch of necklace length. Write an equation to find how many beads Li needs for each necklace. (Explore Activity)
a. input variable: $\qquad$
b. output variable: $\qquad$
c. equation:
2. Kate is planning a trip to the beach. She estimates her average speed to graph her expected progress on the trip. Write an equation in slope-intercept form that represents the situation. (Example 1)

Choose two points on the graph to find the slope.
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=$ $\qquad$


Driving time (h)

Read the $y$-intercept from the graph: $b=$ $\qquad$
Use your slope and $y$-intercept values to write an equation in slope-intercept form.
3. At $59^{\circ} \mathrm{F}$, crickets chirp at a rate of 76 times per minute, and at $65^{\circ}$ F, they chirp 100 times per minute. Write an equation in slope-intercept form that represents the situation. (Example 2)

Input variable: $\qquad$ Output variable: $\qquad$
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=$ $\qquad$ Use the slope and one of the ordered
pairs in $y=m x+b$ to find $b$. $\qquad$ $=$ $\qquad$
$\qquad$ $+b ;$ $\qquad$ $=b$

Write an equation in slope-intercept form. $\qquad$

## ESSENTIAL QUESTION CHECK-IN

4. Explain what $m$ and $b$ in the equation $y=m x+b$ tell you about the graph of the line with that equation.

### 5.1 Independent Practice

8.F. 4

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5. A dragonfly can beat its wings 30 times per second. Write an equation in slope-intercept form that shows the relationship between flying time in seconds and the number of times the dragonfly beats its wings.
6. A balloon is released from the top of a platform that is 50 meters tall. The balloon rises at the rate of 4 meters per second. Write an equation in slope-intercept form that tells the height of the balloon above the ground after a given number of seconds.

## The graph shows a scuba diver's ascent over time.

7. Use the graph to find the slope of the line. Tell what the slope means in this context.
$\qquad$
$\qquad$
8. Identify the $y$-intercept. Tell what the $y$-intercept means in this context.

Scuba Diver's Ascent

9. Write an equation in slope-intercept form that represents the diver's depth over time.
10. The formula for converting Celsius temperatures to Fahrenheit temperatures is a linear equation. Water freezes at $0^{\circ} \mathrm{C}$, or $32^{\circ} \mathrm{F}$, and it boils at $100^{\circ} \mathrm{C}$, or $212^{\circ} \mathrm{F}$. Find the slope and $y$-intercept for a graph that gives degrees Celsius on the horizontal axis and degrees Fahrenheit on the vertical axis. Then write an equation in slope-intercept form that converts degrees Celsius into degrees Fahrenheit.
$\qquad$
11. The cost of renting a sailboat at a lake is $\$ 20$ per hour plus $\$ 12$ for lifejackets. Write an equation in slope-intercept form that can be used to calculate the total amount you would pay for using this sailboat.


## The graph shows the activity in a savings account.

12. What was the amount of the initial deposit that started this savings account?
$\qquad$

13. Find the slope and $y$-intercept of the graphed line.

$\qquad$
14. Write an equation in slope-intercept form for the activity in this savings account.
$\qquad$
15. Explain the meaning of the slope in this graph.
$\qquad$
$\qquad$
ค.0.T.
FOCUS ON HIGHER ORDER THINKING
16. Communicate Mathematical Ideas Explain how you decide which part of a problem will be represented by the variable $x$, and which part will be represented by the variable $y$ in a graph of the situation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
17. Represent Real-World Problems Describe what would be true about the rate of change in a situation that could not be represented by a graphed line and an equation in the form $y=m x+b$.
$\qquad$
$\qquad$
$\qquad$
18. Draw Conclusions Must $m$, in the equation $y=m x+b$, always be a positive number? Explain.

# LESSON <br> 5.2 <br> Writing Linear Equations from a Table 

8.F. 4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value... . Interpret the rate of change and initial value. . . .

How do you write an equation to model a linear relationship given a table?

## Graphing from a Table to Write an Equation

You can use information from a table to draw a graph of a linear relationship and to write an equation for the graphed line.


## EXAMPLE 1

Red
orld COMMON CORE

The table shows the temperature of a fish tank during an experiment. Graph the data, and find the slope and $y$-intercept from the graph. Then write the equation for the graph in slope-intercept form.

| Time (h) | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature ( ${ }^{\circ}$ F) | 82 | 80 | 78 | 76 | 74 | 72 |

STEP 1 Graph the ordered pairs from the table (time, temperature).

STEP 2 Draw a line through the points.
STEP 3 Choose two points on the graph to find the slope: for example, choose $(0,82)$ and $(1,80)$.
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad$ Use the slope formula.

$m=\frac{80-82}{1-0} \quad$ Substitute $(0,82)$ for $\left(x_{1}, y_{1}\right)$ and $(1,80)$ for $\left(x_{2}, y_{2}\right)$.
$m=\frac{-2}{1}=-2$
Simplify.
STEP 4 Read the $y$-intercept from the graph.
$b=82$
STEP 5 Use these slope and $y$-intercept values to write an equation in slope-intercept form.
$y=m x+b$

- $y=-2 x+82$

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1. The table shows the volume of water released by Hoover Dam over a certain period of time. Graph the data, and find the slope and $y$-intercept from the graph. Then write the equation for the graph in slope-intercept form.


| Water Released from Hoover Dam |  |
| :---: | :---: |
| Time $(\mathbf{s})$ | Volume of water $\left(\mathbf{m}^{\mathbf{3}}\right)$ |
| 5 | 75,000 |
| 10 | 150,000 |
| 15 | 225,000 |
| 20 | 300,000 |



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## EXAMPLE 2



Elizabeth's cell phone plan lets her choose how many minutes are included each month. The table shows the plan's monthly cost $y$ for a given number of included minutes $x$. Write an equation in slope-intercept form to represent the situation.

| Minutes included, $\boldsymbol{x}$ | 100 | 200 | 300 | 400 | 500 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cost of plan $(\boldsymbol{\$}), \boldsymbol{y}$ | 14 | 20 | 26 | 32 | 38 |

STEP 1 Notice that the change in cost is the same for each increase of 100 minutes. So, the relationship is linear. Choose any two ordered pairs from the table to find the slope.

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{(20-14)}{(200-100)}=\frac{6}{100}=0.06
$$

STEP 2 Find the $y$-intercept. Use the slope and any point from the table.

$$
\begin{aligned}
y & =m x+b & & \text { Slope-intercept form } \\
14 & =0.06 \cdot 100+b & & \text { Substitute for } y, m \text {, and } x . \\
14 & =6+b & & \text { Multiply. } \\
8 & =b & & \text { Subtract } 6 \text { from both sides. }
\end{aligned}
$$

STEP 3 Substitute the slope and $y$-intercept.

$$
\begin{array}{ll}
y=m x+b & \text { Slope-intercept form } \\
y=0.06 x+8 & \text { Substitute } 0.06 \text { for } m \text { and } 8 \text { for } b .
\end{array}
$$

## Reflect

2. What is the base price for the cell phone plan, regardless of how many minutes are included? What is the cost per minute? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
3. What If? Elizabeth's cell phone company changes the cost of her plan as shown below. Write an equation in slope-intercept form to represent the situation. How did the plan change?

| Minutes included, $\boldsymbol{x}$ | 100 | 200 | 300 | 400 | 500 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Cost of plan (\$), $\boldsymbol{y}$ | 30 | 35 | 40 | 45 | 50 |

$\qquad$

## Math Tralk

Mathematical Practices
Explain the meaning of the slope and $y$-intercept of

## YOUR TURN

4. A salesperson receives a weekly salary plus a commission for each computer sold. The table shows the total pay, $p$, and the number of computers sold, $n$. Write an equation in slope-intercept form to represent this situation.

| Number of <br> computers sold, $\boldsymbol{n}$ | 4 | 6 | 8 | 10 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Total pay (\$), $\boldsymbol{p}$ | 550 | 700 | 850 | 1000 | 1150 |

$\qquad$
5. To rent a van, a moving company charges $\$ 40.00$ plus $\$ 0.50$ per mile. The table shows the total cost, $c$, and the number of miles driven, $d$. Write an equation in slope-intercept form to represent this situation.

| Number of miles <br> driven, $\boldsymbol{d}$ | 10 | 20 | 30 | 40 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Total cost (\$), $\boldsymbol{c}$ | 45 | 50 | 55 | 60 | 65 |

## Guided Practice

1. Jaime purchased a $\$ 20$ bus pass. Each time he rides the bus, a certain amount is deducted from the pass. The table shows the amount, $y$, left on his pass after $x$ rides. Graph the data, and find the slope and $y$-intercept from the graph or from the table. Then write the equation for the graph in slope-intercept form. (Example 1)

| Number of rides, $\boldsymbol{x}$ | 0 | 4 | 8 | 12 | 16 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Amount left on pass (\$), $\boldsymbol{y}$ | 20 | 15 | 10 | 5 | 0 |



The table shows the temperature $(y)$ at different altitudes $(x)$.
This is a linear relationship. (Example 2)

| Altitude (ft), $\boldsymbol{x}$ | 0 | 2,000 | 4,000 | 6,000 | 8,000 | 10,000 | 12,000 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature ( $\left.{ }^{\circ} \mathrm{F}\right), \boldsymbol{y}$ | 59 | 51 | 43 | 35 | 27 | 19 | 11 |

2. Find the slope for this relationship.
$\qquad$
3. Write an equation in slope-intercept form that represents this relationship.
$\qquad$

## ? ESSENTIAL QUESTION CHECK-IN

6. Describe how you can use the information in a table showing a linear relationship to find the slope and $y$-intercept for the equation.

### 5.2 Independent Practice


7. The table shows the costs of a large cheese pizza with toppings at a local pizzeria. Graph the data, and find the slope and $y$-intercept from the graph. Then write the equation for the graph in slope-intercept form.

| Number of toppings, $\boldsymbol{t}$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Total cost $\mathbf{( \$ )}, \boldsymbol{C}$ | 8 | 10 | 12 | 14 | 16 | 18 |

Cost of Large Pizza

9. A friend gave Ms. Morris a gift card for a local car wash. The table shows the linear relationship of how the value left on the

| Number of car washes, $\boldsymbol{x}$ | 0 | 8 | 12 |
| :--- | ---: | ---: | ---: |
| Amount left on card (\$), $\boldsymbol{y}$ | 30 | 18 | 12 | card relates to the number of car washes.

a. Write an equation that shows the number of dollars left on the card.
b. Explain the meaning of the negative slope in this situation.
$\qquad$
$\qquad$
c. What is the maximum value of $x$ that makes sense in this context? Explain.

The tables show linear relationships between $x$ and $y$. Write an equation in slope-intercept form for each relationship.
10.

| $x$ | -2 | -1 | 0 | 2 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | -1 | 0 | 1 | 3 |

11. 

| $x$ | -4 | 1 | 0 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 14 | 4 | 6 | -6 |

12. Finance Desiree starts a savings account with $\$ 125.00$. Every month, she deposits $\$ 53.50$.
a. Complete the table to model the situation.

| Month, $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Amount in <br> Savings (\$), $y$ |  |  |  |  |  |

b. Write an equation in slope-intercept form that shows how much money Desiree has in her savings account after $x$ months.
c. Use the equation to find how much money Desiree will have in savings after 11 months.
13. Monty documented the amount of rain his farm received on a monthly basis, as shown in the table.

| Month, $\boldsymbol{x}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Rainfall (in.), $\boldsymbol{y}$ | 5 | 3 | 4.5 | 1 | 7 |

a. Is the relationship linear? Why or why not?
$\qquad$
$\qquad$
b. Can an equation be written to describe the amount of rain? Explain.
14. Analyze Relationships If you have a table that shows a linear relationship, when can you read the value for $b$, in $y=m x+b$, directly from the table without drawing a graph or doing any calculations? Explain.
$\qquad$
$\qquad$
15. What If? Jaíme graphed linear data given in the form (cost, number). The $y$-intercept was 0 . Jayla graphed the same data given in the form

# LEsson Linear Relationships and Bivariate Data 

8.SP. 1

Construct and interpret scatter plots for bivariate measurement data.. Describe patterns such as... linear association, and nonlinear association. Also 8.SP.2, 8.SP. 3

## Finding the Equation of a Linear Relationship

You can use the points on a graph of a linear relationship to write an equation for the relationship. The equation of a linear relationship is $y=m x+b$, where $m$ is the rate of change, or slope, and $b$ is the value of $y$ when $x$ is 0 .


Math On the Spot

## EXAMPLE 1

Red
world
Common
CORE
8.SP. 2

A handrail runs alongside a stairway. As the horizontal distance from the bottom of the stairway changes, the height of the handrail changes. Show that the relationship is linear, and then find the equation for the relationship.

STEP 1 Show that the relationship is linear.


Horizontal distance (ft)

## Math Talk

Mathematical Practices
What does the slope of the equation represent in this situation? What does the $y$-intercept represent?

STEP 2 Write the equation of the linear relationship.

Choose two points Choose a point and use the slope to to find the slope.
$(5,7)$ and $(25,23)$

$$
\begin{aligned}
m & =\frac{23-7}{25-5} \\
& =\frac{16}{20} \\
& =0.8
\end{aligned}
$$

substitute values for $x, y$, and $m$.

$$
\begin{aligned}
y & =m x+b \\
7 & =0.8(5)+b \\
7 & =4+b \\
3 & =b
\end{aligned}
$$

The equation of the linear relationship is $y=0.8 x+3$.

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## YOUR TURN

Find the equation of each linear relationship.
1.

2.

| Hours $(\boldsymbol{x})$ | Number of <br> units $(\boldsymbol{y})$ |
| :---: | :---: |
| 2 | 480 |
| 15 | 3,600 |
| 24 | 5,760 |
| 30 | 7,200 |
| 48 | 11,520 |
| 55 | 13,200 |



Math On the Spot

## Making Predictions

You can use an equation of a linear relationship to predict a value between data points that you already know.

## EXAMPLE 2 (3cal

 common COREThe graph shows the cost for taxi rides of different distances. Predict the cost of a taxi ride that covers a distance of 6.5 miles.

STEP 1 Write the equation of the linear relationship.

$$
\begin{array}{rlrl}
(2,7) \text { and }(6,15) & & \begin{array}{l}
\text { Select two points. } \\
m
\end{array}=\frac{15-7}{6-2} & \\
& =\frac{8}{4} & & \begin{array}{l}
\text { Calculate the rate } \\
\text { of change. }
\end{array} \\
& =2 & & \text { Simplify. } \\
y & =m x+b & & \\
15 & =2(6)+b & & \begin{array}{l}
\text { Fill in values for } \\
15
\end{array} \\
=12+b, y, \text { and } m . \\
3 & =b & & \text { Simplify. } \\
& & \text { Solve for } b .
\end{array}
$$



The equation of the linear relationship is $y=2 x+3$.
You can check your equation using another point on the graph. Try $(8,19)$. Substituting gives $19=2(8)+3$. The right side simplifies to 19 , so $19=19 . \checkmark$

STEP 2 Use your equation from Step 1 to predict the cost of a 6.5-mile taxi ride.

## Reflect

3. What If? Suppose a regulation changes the cost of the taxi ride to $\$ 1.80$ per mile, plus a fee of $\$ 4.30$. How does the price of the 6.5 mile ride compare to the original price?
$\qquad$
$\qquad$
4. How can you use a graph of a linear relationship to predict an unknown value of $y$ for a given value of $x$ within the region of the graph?
$\qquad$
$\qquad$
$\qquad$
5. How can you use a table of linear data to predict a value?
$\qquad$
$\qquad$
$\qquad$

## YOUR TURN

Paulina's income from a job that pays her a fixed amount per hour is shown in the graph. Use the graph to find the predicted value.
6. Income earned for working 2 hours
$\qquad$
7. Income earned for working 3.25 hours
$\qquad$
8. Total income earned for working for five 8-hour days all at the standard rate $\qquad$


## Contrasting Linear and Nonlinear Data

Bivariate data is a set of data that is made up of two paired variables. If the relationship between the variables is linear, then the rate of change (slope) is constant. If the graph shows a nonlinear relationship, then the rate of change varies between pairs of points.

Andrew has two options in which to invest $\mathbf{\$ 2 0 0}$. Option A earns simple interest of 5\%, while Option B earns interest of 5\% compounded annually. The table shows the amount of the investment for both options over 20 years. Graph the data and describe the differences between the two graphs.

|  | Option A | Option B |
| :---: | :---: | :---: |
| Year, $\boldsymbol{x}$ | Total (\$) | Total (\$) |
| 0 | 200.00 | 200.00 |
| 5 | 250.00 | 255.26 |
| 10 | 300.00 | 325.78 |
| 15 | 350.00 | 415.79 |
| 20 | 400.00 | 530.66 |

STEP 1 Graph the data from the table for Options $A$ and $B$ on the same coordinate grid.

STEP 2 Find the rate of change between pairs of
 points for Option A and classify the relationship.

| Option A | Rate of Change |
| :---: | :---: |
| $(0,200)$ and $(5,250)$ | $m=\frac{250-200}{5-0}=$ |
| $(5,250)$ and $(10,300)$ |  |
| $(10,300)$ and $(15,350)$ |  |

The rate of change between the data values is $\qquad$ so the graph of Option A shows a $\qquad$ relationship.

STEP 3 Find the rate of change between pairs of points for Option B and classify the relationship.

| Option B | Rate of Change |
| :---: | :---: |
| $(0,200)$ <br> and <br> $(5,255.26)$ | $m=\frac{252.26-200}{5-0} \approx$ |
| $(5,255.26)$ <br> and <br> $(10,325.78)$ |  |
| $(10,325.78)$ <br> and <br> $(15,415.79)$ |  |

The rate of change between the data values is $\qquad$ ,
so the graph of Option $B$ shows a $\qquad$ relationship.

## Reflect

9. Why are the graphs drawn as lines or curves and not discrete points?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
10. Can you determine by viewing the graph if the data have a linear or nonlinear relationship? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
11. Draw Conclusions Find the differences in the account balances to the nearest dollar at 5 year intervals for Option B. How does the length of time that money is in an account affect the advantage that compound interest has over simple interest?
$\qquad$
$\qquad$
$\qquad$

## Guided Practice

Use the following graphs to find the equation of the linear relationship. (Example 1)
1.

3. The graph shows the relationship between the number of hours a kayak is rented and the total cost of the rental. Write an equation of the relationship. Then use the equation to predict the cost of a rental that lasts 5.5 hours. (Example 2)
2.



Does each of the following graphs represent a linear relationship?
Why or why not? (Explore Activity)

$\qquad$ $\underline{ }$

## ESSENTIAL QUESTION CHECK-IN


$\qquad$
$\qquad$

### 5.3 Independent Practice

## Does each of the following tables represent a linear relationship?

 Why or why not?7. 

| Number <br> of boxes | Weight (kg) |
| :---: | :---: |
| 3 | 15 |
| 9 | 45 |
| 21 | 105 |

8. 

| Day | Height (cm) |
| :---: | :---: |
| 5 | 30 |
| 8 | 76.8 |
| 14 | 235.2 |

Explain whether or not you think each relationship is linear.
9. the cost of equal-priced DVDs and the number purchased
10. the height of a person and the person's age
$\qquad$
11. the area of a square quilt and its side length
$\qquad$
$\qquad$
12. the number of miles to the next service station and the number of kilometers
$\qquad$
$\qquad$
13. Multistep The Mars Rover travels 0.75 feet in 6 seconds. Add the point to the graph. Then determine whether the relationship between distance and time is linear, and if so, predict the distance that the Mars Rover would travel in 1 minute.

14. Make a Conjecture Zefram analyzed a linear relationship, found that the slope-intercept equation was $y=3.5 x+16$, and made a prediction for the value of $y$ for a given value of $x$. He realized that he made an error calculating the $y$-intercept and that it was actually 12 . Can he just subtract 4 from his prediction if he knows that the slope is correct? Explain.
$\qquad$
$\qquad$
7.0.1. focus on hicher order thinking
15. Communicate Mathematical Ideas The table shows a linear relationship. How can you predict the value of $y$ when $x=6$ without finding the equation of the relationship?

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 4 | 38 |
| 8 | 76 |
| 12 | 114 |

$\qquad$
16. Critique Reasoning Louis says that if the differences between the values of $x$ are constant between all the points on a graph, then the relationship is linear. Do you agree? Explain.
$\qquad$
$\qquad$
$\qquad$
17. Make a Conjecture Suppose you know the slope of a linear relationship and one of the points that its graph passes through. How could you predict another point that falls on the graph of the line?
$\qquad$
$\qquad$
$\qquad$
18. Explain the Error Thomas used $(7,17.5)$ and $(18,45)$ from a graph to find the equation of a linear relationship as shown. What was his mistake?

$$
\begin{aligned}
m & =\frac{45-7}{18-17.5}=\frac{38}{0.5}=79 \\
y & =79 x+b \\
45 & =79 \cdot 18+b \\
45 & =1422+b, \text { so } b=-1377
\end{aligned} \text { The equation is } y=79 x-1377 .
$$

## Ready to Go On?

### 5.1 Writing Linear Equations from Situations and Graphs

Write the equation of each line in slope-intercept form.

2.


### 5.2 Writing Linear Equations from a Table

Write the equation of each linear relationship in slope-intercept form.
3.

| $x$ | 0 | 100 | 200 | 300 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 1.5 | 36.5 | 71.5 | 106.5 |

4. 

| $x$ | 25 | 35 | 45 | 55 |
| :--- | :--- | :--- | :--- | :--- |
| $y$ | 94 | 88 | 82 | 76 |

### 5.3 Linear Relationships and Bivariate Data

Write the equation of the line that connects each set of data points.
5.

6.


## ESSENTIAL QUESTION

7. Write a real-world situation that can be represented by a linear relationship.

## Selected Response

1. An hourglass is turned over with the top part filled with sand. After 3 minutes, there are 855 mL of sand in the top half. After 10 minutes, there are 750 mL of sand in the top half. Which equation represents this situation?
(A) $y=285 x$
(B) $y=-10.5 x+900$
(C) $y=-15 x+900$
(D) $y=75 x$
2. Which graph shows a linear relationship?
(A)

(B)

(C)

(D)

3. What are the slope and $y$-intercept of the relationship shown in the table?

| $\boldsymbol{x}$ | 10,000 | 20,000 | 30,000 |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{y}$ | 2,500 | 3,000 | 3,500 |

(A) slope $=0.05, y$-intercept $=1,500$
(B) slope $=0.5, y$-intercept $=1,500$
(C) slope $=0.05, y$-intercept $=2,000$
(D) slope $=0.5, y$-intercept $=2,000$
4. Which is the sum of $3.15 \times 10^{7}+$ $9.3 \times 10^{6} ?$ Write your answer in scientific notation.
(A) $4.08 \times 10^{7}$
(B) $4.08 \times 10^{6}$
(C) $0.408 \times 10^{8}$
(D) $40.8 \times 10^{6}$

## Mini-Task

5. Franklin's faucet was leaking, so he put a bucket underneath to catch the water. After a while, Franklin started keeping track of how much water was in the bucket. His data is in the table below.

| Hours | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: |
| Quarts | 5 | 6.5 | 8 | 9.5 |

a. Is the relationship linear or nonlinear?
b. Write the equation for the relationship.
$\qquad$
c. Predict how much water will be in the bucket after 14 hours if Franklin doesn't stop the leak.

