Discrete, Computational and Algebraic Topology

November 10–14, 2014, U Copenhagen, Denmark

Abstracts

Karim Adiprasito (Einstein Institute for Mathematics, Hebrew U, Israel)

Some combinatorial and geometric aspects of simplicial convex polytopes

The purpose of the talk will be to demonstrate a surprising connection between three rather different subjects:

- from graph theory: chordality, a basic property that imposes a condition on the shortest induced cycles of a graph, and stands in relation to matchings and colorings;
- from toric algebraic geometry: the Hard Lefschetz Theorem for simplicial polytopes, a cornerstone of the theory of convex polytopes that enabled Stanley, Billera and Lee to give a complete characterization of face numbers of simplicial convex polytopes;
- from approximation theory: the overall "shape" of a polytope, measured for instance by the distance to a smooth convex body it approximates.

Robert Adler (Electrical Engineering, Technion – Israel Institute of Technology)

Pondering Persistence and Extolling Euler

The theory and practice of topological data analysis over the past decade has been dominated by the notion and application of persistent homology. Given a representation of the persistent homology of a filtered structure, it is easy to compute the Euler characteristic curve (ECC) which gives the value of the Euler characteristic of the structure at each point of the filtration.

Although ECCs contains less information than, for example, persistence diagrams, they are typically much easier to compute numerically, using techniques not much different to those used by Euler almost three centuries ago. From a practical point of view, ECCs often seem to contain most of the useful information that is necessary for data analysis and, from a more theoretical point of view, it has turned out to be much easier to study ECCs for random structures than persistent homology or even Betti numbers at some point of a filtration.

I shall describe the relation between persistent homology and ECCs in a number of scenarios, in particular emphasising the rich theory of the ECC in situations of underlying randomness.

Paul Bendich (Duke U, USA)

Multi-scale Looping, Bending, and Branching Analysis of Brain Artery Trees

The statistical analysis of a population of brain artery trees is considered. New representations of these tree structured data objects are developed, using ideas from topological data analysis. Specifically, a number of representations of each brain tree, using persistence diagrams that quantify branching and looping at multiple scales, are considered. Novel approaches to the statistical analysis, through various summaries of the persistence diagrams, lead to improved correlations with covariates such as age and sex, relative to earlier analyses of this data set. Strikingly, these correlations remain strong even after controlling for more obvious geometric differences in the set of trees.

This is joint work with Alex Pieloch, J.S. Marron, Ezra Miller, and Sean Skwerer, and the dataset was obtained from CASILab at UNC-Chapel Hill.

Anders Björner (KTH Stockholm, Sweden)

Some thoughts on topological methods for complexity

The talk will discuss some challenges to topological combinatorics posed by complexity theory.

Pavle V. M. Blagojević (U Bonn/FU Berlin, Germany; SANU Belgrade, Serbia)
joint with Florian Frick, Albert Haase, and Günter M. Ziegler

Topology of the Grünbaum hyperplane mass partition problem

Let $\mathcal{M} = \{\mu_1, \ldots, \mu_j\}$ be a collection of *masses*, i.e., finite Borel measures on $\mathbb{R}^d$ such that every affine hyperplane has measure zero, and let $\mathcal{H} = \{H_1, \ldots, H_k\}$ be an arrangement of k hyperplanes in $\mathbb{R}^d$. Then $\mathcal{H}$ *equiparts* the collection of masses $\mathcal{M}$ if it cuts each of the j masses into 2^k orthants of equal measure, that is, if

$$\mu_\ell(\mathcal{O}_g^{\mathcal{H}}) = \tfrac{1}{2^k}\mu_\ell(\mathbb{R}^d)$$

for $1 \leq \ell \leq j$ and $g \in (\mathbb{Z}/2)^k$, where $\mathcal{O}_g^{\mathcal{H}}$ will be our notation for the orthants determined by $\mathcal{H}$. In his paper from 1960 Grünbaum suggested the following general measure partition problem:

Problem. *Determine the minimal dimension $d = \Delta(j, k)$ such that for every collection $\mathcal{M}$ of j masses in $\mathbb{R}^d$ there exists an arrangement of k hyperplanes $\mathcal{H}$ that equiparts $\mathcal{M}$.*

The Grünbaum hyperplane mass partition problem was extensively studied over more than a half century by many authors. It was a playground for testing the strengths of various different topological methods. The absence of complete answers and presence of number of claims with incomplete or even invalid proofs makes this problem into one of most challenging problems of Topological Combinatorics.

In this talk we review known results on the Grünbaum hyperplane mass partition problem, present new proofs for known results, explain gaps in proofs of number of claims and finally establish new values of $\Delta(j, k)$.

Peter Bubenik (Cleveland State U, USA)

Statistical Topological Data Analysis

Topological Data Analysis (TDA) can detect multiscale, nonlinear features not readily accessible using more traditional methods. The standard TDA summary, the barcode (or persistence diagram), has three great features: it is a complete discrete invariant, it is computable, and it is stable. However, it is not directly amenable to standard techniques from statistics and machine learning. To bridge the gap between TDA and statistics and machine learning, one can convert the barcode into a vector. I will present one way of doing this, the persistence landscape, that does not lose any information. I will discuss its statistical and computational properties and give some biological applications.

Gunnar Carlsson (Stanford U, USA)

The algebraic geometry of persistence barcodes

Persistence barcodes have been shown to be useful in understanding various kinds of complex and high dimensional data sets. It turns out that the set of these barcodes carries various kinds of structures, including that of a metric space and that of an "infinite algebraic variety" and further that these structures are useful in applying the methodology to databases whose entries are geometric objects (such as databases of compounds) and in generalizing it to multidimensional versions of persistence. I will discuss the variety structures with applications.

Michael Farber (Queen Mary, U London, UK)

Large random spaces and groups

I will discuss several models producing random simplicial complexes. We are interested in geometric and topological properties of random simplicial complexes which are satisfied with probability tending to 1 as the number of vertices of the complex tends to infinity. In the talk I will explain why for random simplicial complexes the Whitehead and Eilenberg–Ganea Conjectures hold. I shall also describe torsion and the cohomological dimension of fundamental groups of random simplicial complexes.

Alexander Gaifullin (Steklov Mathematical Institute, Russia)

Volumes of polyhedra and collapses of simplicial complexes

It is well-known that the square of the area of a triangle is a polynomial in the squares of the lengths of its sides. This is Heron's formula. Similarly, there is a formula, which is now called the Cayley–Menger formula, that expresses the square of the volume of a simplex as a polynomial in the squares of its edge lengths. An amazing generalization of this formula was obtained by Sabitov in 1996. He proved that the volume of an arbitrary simplicial polyhedron in the three-dimensional Euclidean space is a root of a monic polynomial whose coefficients are polynomials in the squares of the edge lengths of the polyhedron. Sabitov used a rather difficult algebraic technique of extracting variables by means of resultants. Later an alternative prove was given by Connelly, Sabitov, and Walz in which this technique was replaced by the usage of places of fields. The proof became more involved but much easier to understand. As a direct corollary of his theorem, Sabitov obtained that the volume of an arbitrary flexible polyhedron in the three-dimensional Euclidean space is constant during the flexion.

Attempts to generalize Sabitov's theorem to higher dimensions faced difficulties of both algebraic and combinatorial nature. The difficulties of combinatorial nature are caused by the fact that triangulations of spheres of dimensions higher than two cannot be simplified monotonically by bistellar moves or any other local moves. In the talk we shall present a proof of a higher dimensional analogue of Sabitov's theorem that has been recently obtained by the speaker. The main point in this proof is the fact that certain special simplicial complexes associated to places of fields of rational functions in the coordinates of the vertices of the polyhedron admit collapses to subcomplexes of sufficiently small dimensions.

Michael Joswig (TU Berlin, Germany)

Heuristics for Sphere Recognition

The problem of determining whether a given (finite abstract) simplicial complex is homeomorphic to a sphere is undecidable. Still, the task naturally appears in a number of practical applications and can often be solved, even for huge instances, with the use of appropriate heuristics. We report on the current status of suitable techniques and their limitations. We also present implementations in polymake and relevant test examples.

Joint work with Frank H. Lutz and Mimi Tsuruga.

Matthew Kahle (Ohio State U, USA)

The length of the longest bar in random persistent homology

In recent years, a number of papers have studied topological features of "random geometric complexes", particularly various facts about their expected homology. One of the motivations for this is establishing a probabilistic null hypothesis for topological data analysis. In practice, however, one usually computes persistent homology over a range of parameter, rather than homology alone. Detailed results for persistent homology of random geometric complexes have been harder to come by.

I will present new work which quantifies the length of the longest bar in persistent homology, up to a constant factor. This is an important step toward quantifying the statistical significance of topological signals in data. This is joint work with Omer Bobrowski and Primož Škraba.

Roman Karasev (MIPT, Russia)

Covering dimension using toric varieties

We consider some classical theorems that can be described as "quantitative covering dimension" theorems, like the Lebesgue theorem about the cube and the Knaster–Kuratowski–Mazurkiewicz theorem about the simplex. Though it is not very hard to prove these theorems by almost elementary means, we explore another way to handle them using the correspondence between convex polytopes and toric varieties through the moment map.

This provides a unified point of view on such results and allows to generalize them to some extent. In particular, it gives an answer to a question of Dömötör Pálvölgyi on `http://mathoverflow.net/questions/105471/is-this-stronger-knaster-kuratowski-mazurkiewicz-lemma-true`.

Claudia Landi (UniMORE, Italy)

A stable combinatorial distance for Reeb graphs of surfaces

Reeb graphs are combinatorial signatures that capture shape properties from the perspective of a chosen function. One of the most important questions is whether Reeb graphs are robust against function perturbations that may occur because of noise and approximation errors in the data acquisition process. In this work we tackle the problem of stability by providing an editing distance between Reeb graphs of orientable surfaces in terms of the cost necessary to transform one graph into another by edit operations. Our main result is that the editing distance between two Reeb graphs is upper bounded by the extent of the difference of the associated functions, measured by the maximum norm. This yields the stability property under function perturbations.

Jesper M. Møller (U Copenhagen, Denmark)

Chromatic numbers of manifolds

Neža Mramor Kosta (U Ljubljana, Slovenia)

Discrete Morse functions on infinite complexes

A discrete Morse function on a finite cell complex divides the cells of complex into, roughly, the critical and the regular cells, where data on the critical cells determines the homology and the homotopy type of the cell complex. In the infinite case, new features appear. In addition to critical cells, infinite rays of the discrete vector field also contain information on the homology and the shape of the complex. In this talk we will discuss the differences between discrete Morse theory on finite and infinite complexes and some of the new phenomena that appear in the infinite case.

Marian Mrozek (Jagiellonian U, Poland)

Morse–Forman–Conley theory for combinatorial multivector fields

In late 90' R. Forman defined a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. He also studied combinatorial vector fields without acyclicity assumption, studied its chain recurrent set and proved Morse inequalities in this setting.

In this talk we consider a generalized concept of combinatorial multivector field and present an extension of the Morse–Forman theory towards the Conley index theory.

This is research in progress.

Vidit Nanda (U Penn, USA)

Discrete Morse Theory for Local Systems

Forman's discrete Morse theory is by now the standard method by which large (co)homology computations over a fixed coefficient ring R are rendered tractable. Slightly more general – and hence more challenging – is the problem of computing cohomology of local systems over regular CW complexes. Here each cell is assigned its own R-module and the boundary operators are replaced by aggregates of compatible R-linear maps. This talk will focus on how one adapts the discrete Morse theoretic framework and algorithms to this generalized setting. As a simple consequence, we also produce algorithms for distributed computation of ordinary cellular cohomology with R-coefficients.

This is joint work with Justin Curry and Robert Ghrist.

Martin Raussen (Aalborg U, Denmark)

Combinatorial and topological models for spaces of schedules

Higher Dimensional Automata are topological models for concurrent computation in the form of cubical complexes. A schedule gives rise to a directed path (d-path), and d-homotopies (preserving the directions) of such d-paths leave the results of computations invariant.

I shall describe and discuss several models for the homotopy type of the space of traces (schedules up to reparametrization) for particularly simple HDAs: as a prodsimplicial complex – with products of simplices as building blocks – and as a configuration space living in a product of simplices. In favourable cases, these models allow calculations of homology groups and other topological invariants of the trace spaces.

Joint work with Lisbeth Fajstrup (Aalborg) and Krzysztof Ziemiański (Warsaw).

Francisco Santos (U Cantabria, Spain)

Many triangulated odd-dimensional spheres

It is known that the $(2k-1)$-sphere has at most $2^{O(n^k \log n)}$ combinatorially distinct triangulations with n vertices, for every $k \geq 2$. In this talk I show how to construct at least $2^{\Omega(n^k)}$ such triangulations, improving on the previous constructions which gave $2^{\Omega(n^{k-1})}$ in the general case (Kalai) and $2^{\Omega(n^{5/4})}$ for $k = 2$ (Pfeifle–Ziegler).

We also construct $2^{\Omega\left(n^{k-1+\frac{1}{k}}\right)}$ geodesic (a.k.a. star-convex) n-vertex triangualtions of the $(2k-1)$-sphere. As a step for this (in the case $k = 2$) we construct n-vertex 4-polytopes containing $\Omega(n^{3/2})$ facets that are not simplices, or with $\Omega(n^{3/2})$ edges of degree three.

This is joint work with E. Nevo and S. Wilson, `http://arxiv.org/abs/1408.3501`.

Primož Škraba (Jožef Stefan Institute, Slovenia)

Persistence of Different Shapes

In this talk I will talk about recent work with M. Vejdemo-Johansson and J. Pita Costa on a generalized notion of persistence based on bars of different shapes. The general idea is that given a shape which captures information we are looking for, there is a general procedure on computing something persistence-like. For example in standard persistence we assign the time a simplex appears in a filtration, so the shape can be thought of as an open interval. For zig-zag persistence, since we can remove simplices, the corresponding shape is an interval. The talk will be heavily motivated by examples and the resulting algorithms rather than formalisms.

John M. Sullivan (TU Berlin, Germany)

The Ribbonlength of Knot Diagrams

The ropelength problem asks to minimize the length of a knotted space curve such that a unit tube around the curve remains embedded. A two-dimensional analog has a much more combinatorial flavor: we require a unit-width ribbon around a knot diagram to be immersed with consistent crossing information. Attempting to characterize critical points for ribbonlength leads us to new results about the medial axis of an immersed disk in the plane, including a certain topological stability for thin disks. This is joint work with Elizabeth Denne and Nancy Wrinkle.

Uli Wagner (IST Austria, Austria)

Eliminating Multiple Intersections and Tverberg Points

Generalizing classical results about embeddings (maps without double points), we study conditions under which a finite k-dimensional simplicial complex K can be mapped into d-dimensional Euclidean space without triple, quadruple, or higher-multiplicity intersection points.

In particular, we focus on the setting of topological Tverberg-type problems, namely on the question whether K admits a map into d-space that has no r-fold Tverberg point, i.e., no image point with r preimages in pairwise disjoint simplices of K.

We present higher-multiplicity analogues of the Whitney trick and, more generally, of the Haefliger–Weber theorem, which guarantee that under suitable codimension restrictions, a well-known necessary condition for the existence of maps without r-fold Tverberg points is also sufficient.

More specifically, if $d \geq \frac{r+1}{r}k + 3$, then K admits a map into d-space without r-fold Tverberg point if and only if the r-fold deleted product of K admits an equivariant map into a sphere of dimension $d(r-1)-1$ (with respect to a suitable action of the symmetric group).

Nathalie Wahl (U Copenhagen, Denmark)

Combinatorial problems arising in Topology

I'll present combinatorial problems arising in string topology and homological stability for groups. In string topology, one defines operations on the chains of the free loop space of a manifold parametrized by certain graph complexes whose homology are compactifications of the moduli space of Riemann surfaces. One would like to compute the homology of these chain complexes. Some first computations of these homology groups where achieved by Egas, but many questions remain open. In homological stability, one constructs families of simplicial complexes associated to families of groups. One would like to know when such complexes are highly connected, i.e., when their reduced homology vanishes in a range. High connectivity is known to hold in many examples, with different proofs in each case, and there is no known example where we know that high connectivity does not hold.