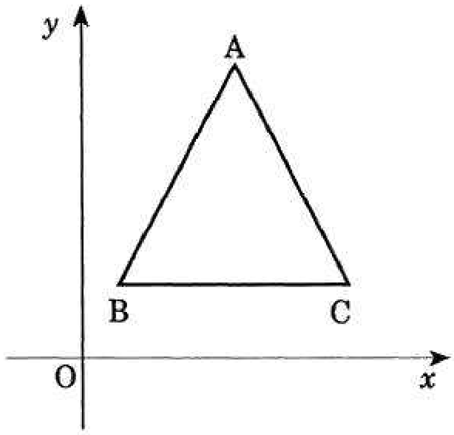
GCC Straight Line

[SQA] 1. A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).

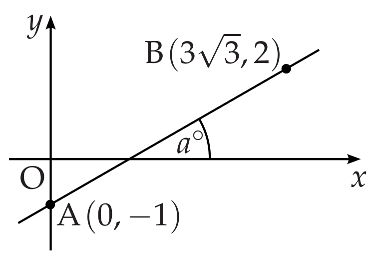


(*a*) Show that the triangle is isosceles. **(2)**

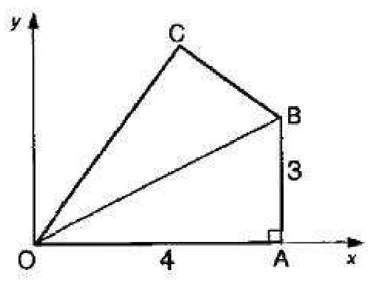
(*b*) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. **(7)**

(ii) Hence show that H lies one quarter of the way up DA. **(1)**

[SQA] 2. Find the size of the angle $a°$ that the line joining the points $A(0, -1)$ and $B(3\sqrt{3}, 2)$ makes with the positive direction of the x-axis. **3**



[SQA] 3. The diagram shows a kite OABC.
A is the point (4,0) and B is the point (4,3).
Calculate the gradient of OC correct to two decimal places. **3**



[SQA] 4. Find the equation of the line through the point $(3, -5)$ which is parallel to the line with equation $3x + 2y - 5 = 0$. **2**

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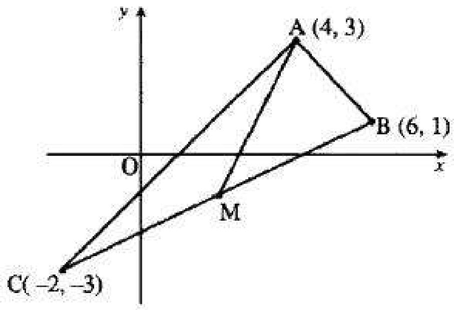
All others © Higher Still Notes

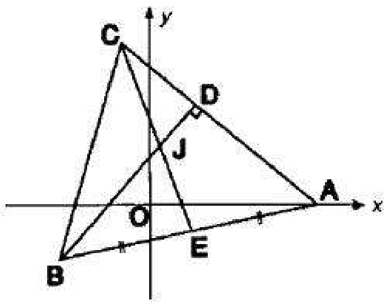
[SQA] 5. A triangle ABC has vertices A(4, 3), B(6, 1) and C(−2, −3) as shown in the diagram. Find the equation of AM, the median from A. 3



[SQA] 6. In the diagram A is the point (7,0), B is (−3,−2) and C(−1,8). The median CE and the altitude BD intersect at J.

(a) Find the equations of CE and BD. 6

(b) Find the co-ordinates of J. 2



[SQA] 7. Triangle PQR has vertex P on the x-axis, as shown in the diagram.

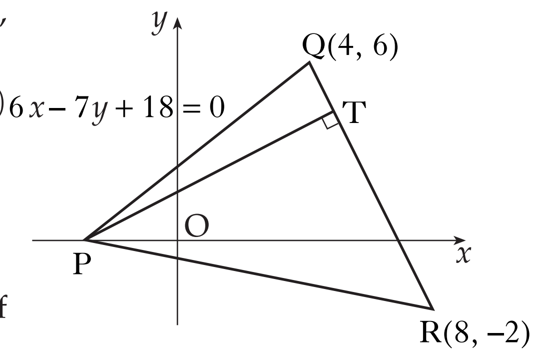
Q and R are the points $(4, 6)$ and $(8, -2)$ respectively.

The equation of PQ is $6x - 7y + 18 = 0$.

(a) State the coordinates of P. **1**

(b) Find the equation of the altitude of the triangle from P. **3**

(c) The altitude from P meets the line QR at T. Find the coordinates of T. **4**



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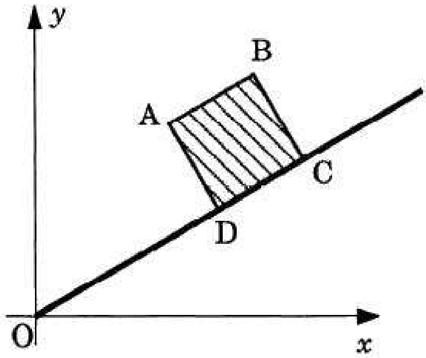
[SQA] 8. ABCD is a square. A is the point with coordinates (3,4) and ODC has equation $y = \frac{1}{2}x$.



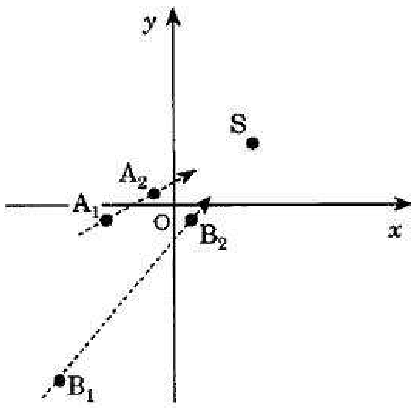
(*a*) Find the equation of the line AD. **(3)**

(*b*) Find the coordinates of D. **(3)**

(*c*) Find the area of the square ABCD. **(2)**

[SQA] 9. A Royal Navy submarine exercising in the Firth of Clyde is stationary on the seabed below a point S on the surface. S is the point (5, 4) as shown.
A radar operator observes the frigate 'Achilles' sailing in a straight line, passing through the points A_1 (–4, –1) and A_2 (–1, 1).
Similarly the frigate 'Belligerent' is observed sailing in a straight line, passing through the points B_1 (–7, –11) and B_2 (1, –1).

If both frigates continue to sail in straight lines, will either or both frigates pass directly over the submarine? 5



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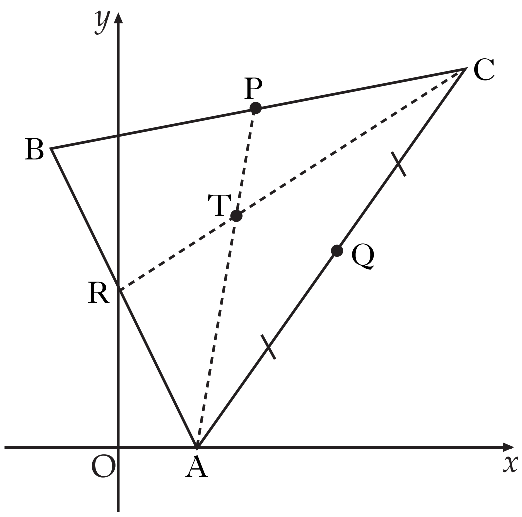
10. Triangle ABC has vertices $A(4,0)$, $B(4,16)$ and $C(18,20)$, as shown in the diagram opposite.

Medians AP and CR intersect at the point $T(6,12)$.

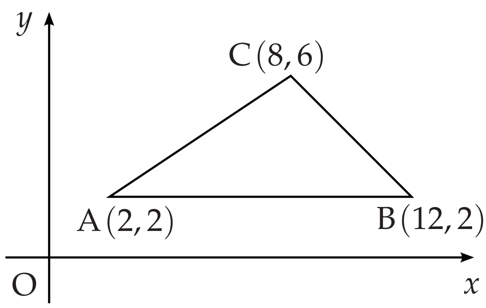


(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

(*c*) Find the ratio in which T divides BQ. **2**

[SQA] 11. Triangle ABC has vertices $A(2,2)$, $B(12,2)$ and $C(8,6)$.



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

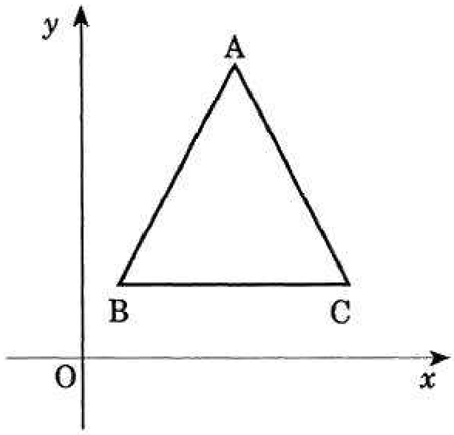
(*d*) Hence find the equation of the circle passing through A, B and C. **2**

[END OF QUESTIONS]

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

GCC Straight Line

[SQA] 1. A triangle ABC has vertices A(4, 8), B(1, 2) and C(7, 2).



(*a*) Show that the triangle is isosceles. **(2)**

(*b*) (i) The altitudes AD and BE intersect at H, where D and E lie on BC and CA respectively. Find the coordinates of H. **(7)**

(ii) Hence show that H lies one quarter of the way up DA. **(1)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(*a*)	2	C	CN	G1	proof	1995 P2 Q1
(*b*)	8	C	CN	G8, G7, G1	(i) $H(4, \frac{7}{2})$, (ii) proof	

(a) •1 Calculate the length of the sides

•2 $AB = AC = \sqrt{3^2 + 6^2}$

(b) •3 knows to find equ. of an altitude

•4 $m_{AC} = -2$

•5 $m_{BE} = \frac{1}{2}$

•6 $y - 2 = \frac{1}{2}(x - 1)$

•7 $x = 4$ stated or implied

•8 knows how to find intersection

•9 $H = \left(4, \frac{7}{2}\right)$

•10 $DA = 6$ and $DH = 1\frac{1}{2}$

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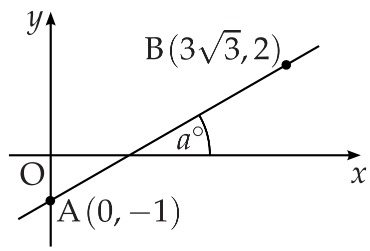
All others © Higher Still Notes

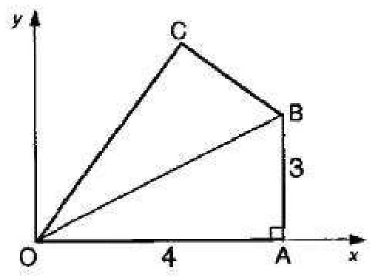
[SQA] 2. Find the size of the angle $a°$ that the line joining the points $A(0,-1)$ and $B(3\sqrt{3},2)$ makes with the positive direction of the x-axis. **3**



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	NC	G2	30	2000 P1 Q3

•1 ss: know how to find gradient or equ.	•1 $\frac{2-(-1)}{3\sqrt{3}-0}$
•2 pd: process	•2 $\tan a$ = gradient *stated or implied by* •3
•3 ic: interpret exact value	•3 $a = 30$

[SQA] 3. The diagram shows a kite OABC. A is the point (4,0) and B is the point (4,3). Calculate the gradient of OC correct to two decimal places. **3**



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CR	G2		1992 P1 Q13

•1 *strat:* i.e. try to evaluate $C\hat{O}A$

•2 $A\hat{O}B = 36.9°$

•3 $\tan 73.7° = 3.428$

•4 $\times \cos x$

[SQA] 4. Find the equation of the line through the point $(3,-5)$ which is parallel to the line with equation $3x + 2y - 5 = 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	2	C	CN	G3, G2		1991 P1 Q1

•1 $m = -\frac{3}{2}$ stated or implied by •2

•2 $y-(-5) = -\frac{3}{2}(x-3)$

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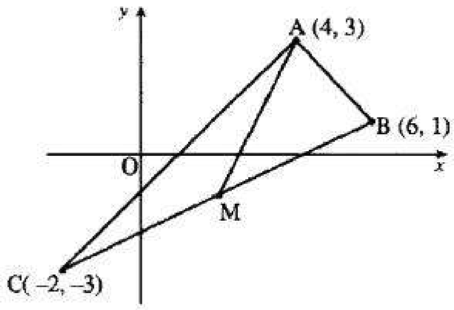
[SQA] 5. A triangle ABC has vertices A(4, 3), B(6, 1) and C(−2, −3) as shown in the diagram. Find the equation of AM, the median from A. 3



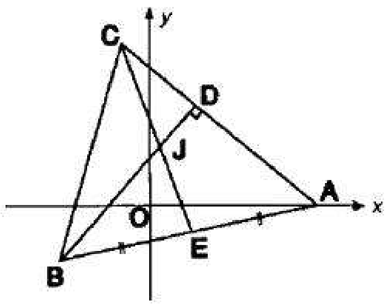
Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	CN	G3, G3		1998 P1 Q1

•[1] $M = (2, -1)$
•[2] $m_{AM} = 2$
•[3] $y - (-1) = 2(x - 2)$

[SQA] 6. In the diagram A is the point (7,0), B is (−3,− 2) and C(−1,8). The median CE and the altitude BD intersect at J.

(a) Find the equations of CE and BD. 6

(b) Find the co-ordinates of J. 2



Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	6	C	NC	G3, G5, G8		1992 P1 Q2
(b)	2	C	NC	G8		

•[1] $E = (2, -1)$
•[2] $m_{CE} = -3$
•[3] $y - (-1) = -3(x - 2)$ or $y - 8 = -3(x - (-1))$
•[4] $m_{AC} = -1$
•[5] $m_{BD} = -1$
•[6] $y - (-2) = 1(x - (-3))$
•[7] strat: attempt to solve simultaneously
•[8] $J = (1, 2)$

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All others © Higher Still Notes

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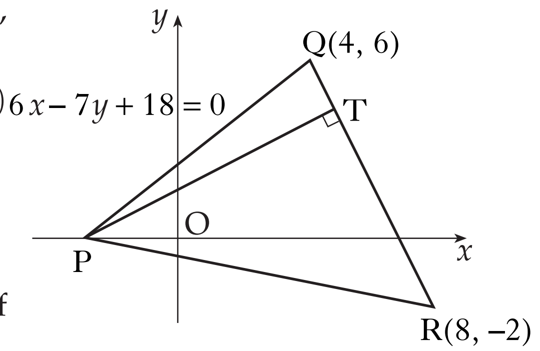
Q and R are the points $(4, 6)$ and $(8, -2)$ respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(a) State the coordinates of P. **1**

(b) Find the equation of the altitude of the triangle from P. **3**

(c) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	1	C	CN	G4	$P(-3, 0)$	2009 P1 Q21
(b)	3	C	CN	G7	$y = \frac{1}{2}(x+3)$	
(c)	4	C	CN	G8	$T(5, 4)$	

•[1] ic: interpret x-intercept	•[1] $P = (-3, 0)$
•[2] pd: find gradient (of QR)	•[2] $m_{QR} = -2$
•[3] ss: know and use $m_1 m_2 = -1$	•[3] $m_{\text{alt.}} = \frac{1}{2}$
•[4] ic: state equ. of altitude	•[4] $y - 0 = \frac{1}{2}(x+3)$
•[5] ic: state equ. of line (QR)	•[5] $y + 2 = -2(x - 8)$
•[6] ss: prepare to solve sim. equ.	•[6] $x - 2y = -3$ and $2x + y = 14$
•[7] pd: solve for x	•[7] $x = 5$
•[8] pd: solve for y	•[8] $y = 4$

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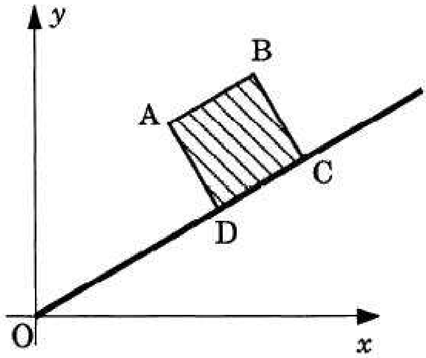

(a) Find the equation of the line AD. **(3)**

(b) Find the coordinates of D. **(3)**

(c) Find the area of the square ABCD. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	3	C	CN	G5, G3		1994 P2 Q2
(b)	2	C	CN	G1		
(c)	2	C	CN	G1		

(a) •[1] using $m_1 m_2 = -1$

•[2] $m_{AD} = -2$

•[3] $y - 4 = -2(x - 3)$

(b) •[4] strategy for sim. equations

•[5] $2x + y = 10$ or equiv

•[6] (4,2)

(c) •[7] strategy : find length of AD

•[8] 5

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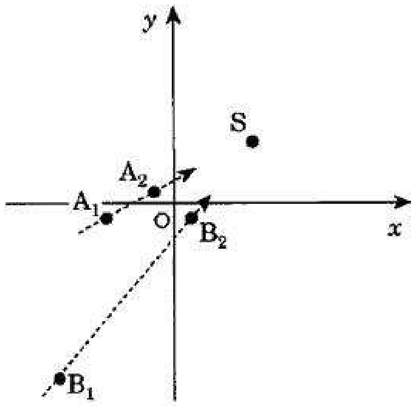

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	5	C	CN	G8		1995 P1 Q6

•1	strat: *compare* gradients	•1	strat: st lines and substitution
•2	$m_{A_1A_2} = \frac{2}{3}$	•2	A_1A_2: $y+1 = \frac{2}{3}(x+4)$ or equivalent
•3	$m_{A_2S} = \frac{1}{2}$ *or* $m_{A_1S} = \frac{5}{9}$ so not heading for S	•3	$4+1 \neq \frac{2}{3}(5+4)$ so not heading for S
•4	$m_{B_1B_2} = \frac{5}{4}$	•4	B_1B_2: $y+11 = \frac{5}{4}(x+7)$ or equivalent
•5	$m_{B_2S} = \frac{5}{4}$ *or* $m_{B_1S} = \frac{5}{4}$ so heading for S	•5	$4+11 = \frac{5}{4}(5+7)$ so heading for S

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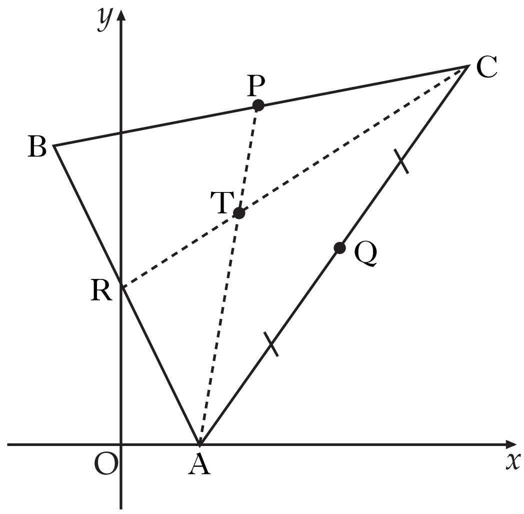
10. Triangle ABC has vertices A(4,0), B(4,16) and C(18,20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6,12).



(*a*) Find the equation of median BQ. **3**

(*b*) Verify that T lies on BQ. **1**

(*c*) Find the ratio in which T divides BQ. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CN	G7	$y-16=-\frac{2}{5}(x-(-4))$	2010 P1 Q21
(*b*)	1	C	CN	A6	proof	
(*c*)	2	C	CN	G24	2 : 1	

•[1] ss: know and find midpoint of AC	•[1] $(11,10)$
•[2] pd: calculate gradient of BQ	•[2] $-\frac{6}{15}$ or equiv
•[3] ic: state equation	•[3] $y-16=-\frac{2}{5}(x-(-4))$ or $y-10=-\frac{2}{5}(x-11)$
•[4] ic: substitute in for T and complete	•[4] $2(6)+5(12)=12+60=72$
•[5] ss: valid method for finding the ratio	•[5] e.g. vector approach $\overrightarrow{BT}=\begin{pmatrix}10\\-4\end{pmatrix}, \overrightarrow{TQ}=\begin{pmatrix}5\\-2\end{pmatrix}$
•[6] ic: complete to simplified ratio	•[6] 2 : 1

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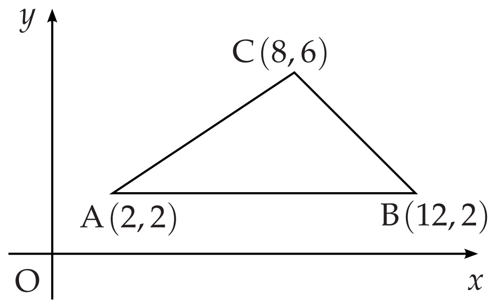
[SQA] 11. Triangle ABC has vertices A $(2,2)$, B $(12,2)$ and C $(8,6)$.



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	1	C	CN	G3, G7	$x=7$	2001 P2 Q7
(*b*)	4	C	CN	G7	$3x+2y=23$	
(*c*)	1	C	CN	G8	$(7,1)$	
(*d*)	2	A/B	CN	G8, G9, G10	$(x-7)^2+(y-1)^2=26$	

•1 ic: state equation of a vertical line	•1 $x=7$
•2 pd: process coord. of a midpoint	•2 midpoint $=(5,4)$
•3 ss: find gradient of AC	•3 $m_{AC}=\frac{2}{3}$
•4 ic: state gradient of perpendicular	•4 $m_{\perp}=-\frac{3}{2}$
•5 ic: state equation of straight line	•5 $y-4=-\frac{3}{2}(x-5)$
•6 pd: find pt of intersection	•6 $x=7, y=1$
•7 ss: use standard form of circle equ.	•7 $(x-7)^2+(y-1)^2$
•8 ic: find radius and complete	•8 $(x-7)^2+(y-1)^2=26$
	or
	•7 $x^2+y^2-14x-2y+c=0$
	•8 $c=24$

[END OF QUESTIONS]

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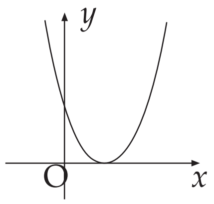
GCC Quadratics and Polynomials

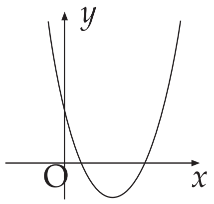
Paper 1 Section A

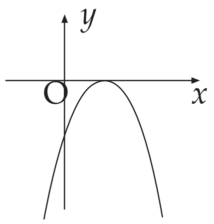
Each correct answer in this section is worth two marks.

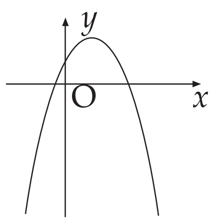
1. Which of the following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where

- $a > 0$
- $b^2 - 4ac > 0$?

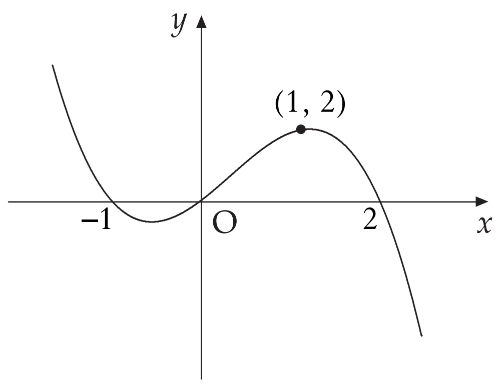
A.


B.


C.


D.


2. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A. $y = -x(x+1)(x-2)$

B. $y = -x(x-1)(x+2)$

C. $y = x(x+1)(x-2)$

D. $y = x(x-1)(x+2)$

3. If $f(x) = (x-3)(x+5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A. $-5 < x < 3$

B. $-3 < x < 5$

C. $x < -5,\ x > 3$

D. $x < -3,\ x > 5$

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4. What is the solution of $x^2 + 4x > 0$, where x is a real number?

   A. $-4 < x < 0$

   B. $x < -4,\ x > 0$

   C. $0 < x < 4$

   D. $x < 0,\ x > 4$

5. Solve $6 - x - x^2 < 0$.

   A. $-3 < x < 2$

   B. $x < -3,\ x > 2$

   C. $-2 < x < 3$

   D. $x < -2,\ x > 3$

6. The discriminant of a quadratic equation is 23.

   Here are two statements about this quadratic equation:

   I. the roots are real;

   II. the roots are rational.

   Which of the following is true?

   A. neither statement is correct

   B. only statement I is correct

   C. only statement II is correct

   D. both statements are correct

7. A function f is given by $f(x) = 2x^2 - x - 9$.

   Which of the following describes the nature of the roots of $f(x) = 0$?

   A. No real roots

   B. Equal roots

   C. Real distinct roots

   D. Rational distinct roots

8. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

   What is the value of k?

   A. $-\frac{9}{8}$

   B. $-\frac{8}{9}$

   C. $\frac{8}{9}$

   D. $\frac{9}{8}$

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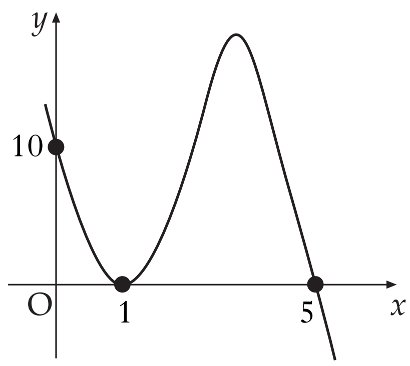
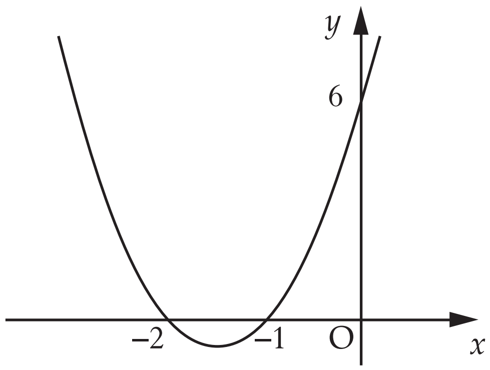
9. The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.



What are the values of k and t?

	k	t
A.	-2	-5
B.	-2	5
C.	2	-5
D.	2	5

10. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A. $y = 6(x-1)(x-2)$

B. $y = 6(x+1)(x+2)$

C. $y = 3(x-1)(x-2)$

D. $y = 3(x+1)(x+2)$

11. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x-1)$?

A. 0

B. 2

C. 3

D. 4

Questions marked '[SQA]' © SQA
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[END OF PAPER 1 SECTION A]

Paper 1 Section B

[SQA] 12. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

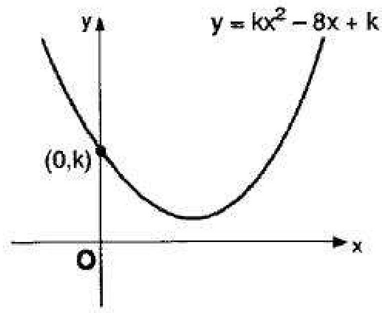
(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

[SQA] 13. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

[SQA] 14. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers. **5**

[SQA] 15. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

[SQA] 16. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis. **4**



[SQA] 17. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. **2**

18. (*a*) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 19. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

[SQA] 20. (*a*) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$. **1**
(*b*) Hence find the other roots. **3**

[SQA] 21. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.
Find the value of p and the other roots. **4**

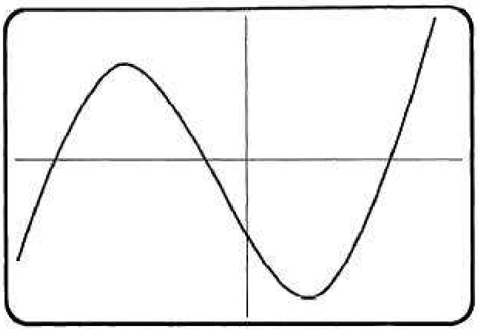
[SQA] 22. (*a*) Show that $(x - 3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**
(*b*) Hence express $f(x)$ in its fully factorised form. **2**

[SQA] 23. Express $x^4 - x$ in its fully factorised form. **4**

[SQA] 24. (*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **2**
(*b*) Show algebraically that there are no other real roots. **3**

[SQA] 25. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

[SQA] 26. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**
(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**
(*c*) Find the turning points and justify their nature. **(6)**

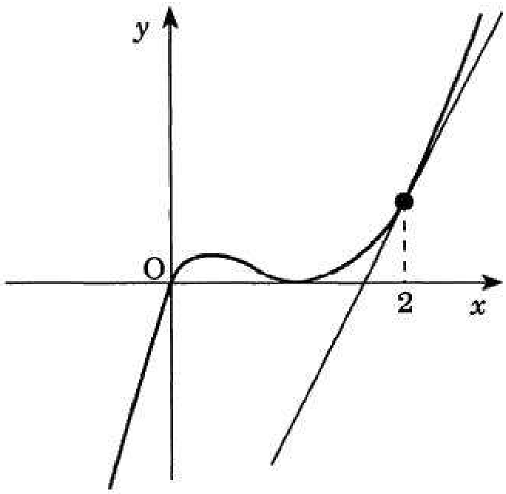
Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 27. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

[SQA] 28. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.

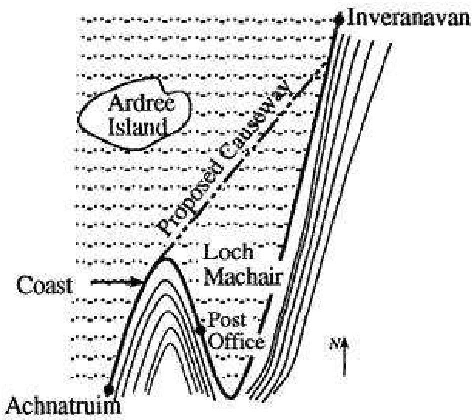


(*a*) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

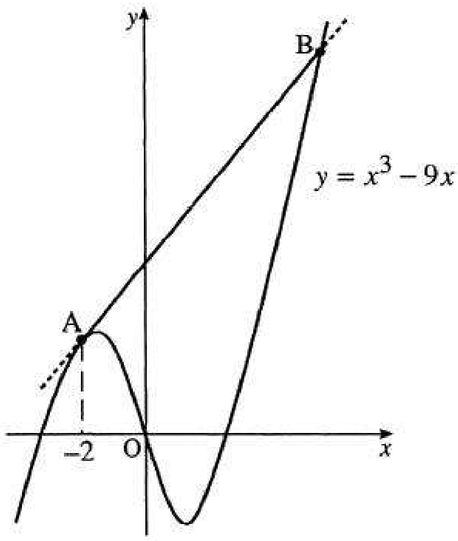
(*b*) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 29. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.

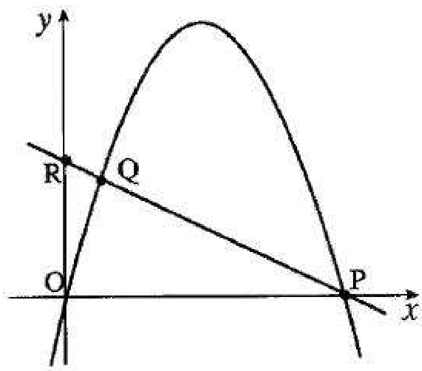


(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

[SQA] 30. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.



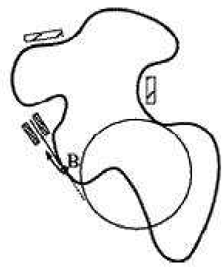
(*a*) Find the coordinates of the point P. **2**

(*b*) R is the point (0, 2). Find the equation of PR. **2**

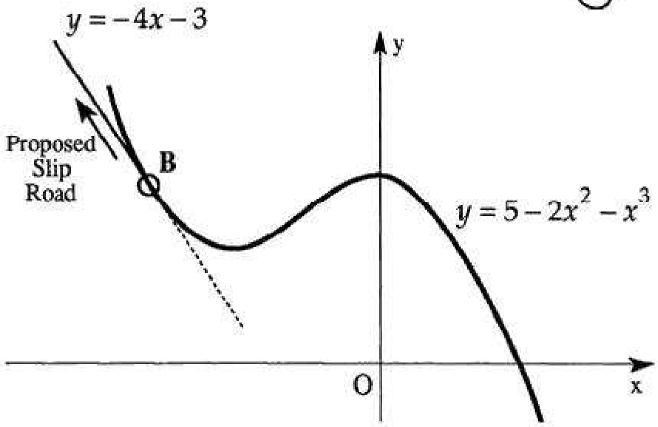
(c) The line and the parabola also intersect at Q. Find the coordinates of Q. **4**

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 31. The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

[SQA] 32. (*a*) (i) Make a sketch of the graph of $y = x^3$, where $-3 \le x \le 3$, $x \in \mathbf{R}$.

(ii) On the same diagram, draw the graph of $y = 6x + 1$. **(3)**

(*b*) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \le x \le 3$. **(1)**

(*c*) Calculate the value of the positive root, correct to 3 significant figures. **(4)**

[END OF PAPER 1 SECTION B]

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Paper 2

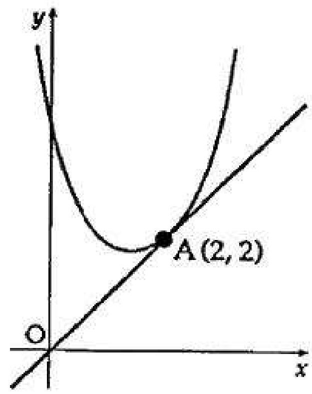
[SQA] 1.

(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots. **1**

(ii) Hence or otherwise show that the equation $x(x + 1) = 3x - 2$ has no real roots. **2**

[SQA] 2. Show that the roots of the equation $(k - 2)x^2 - (3k - 2)x + 2k = 0$ are real. **4**

[SQA] 3. **(a)** The point A(2, 2) lies on the parabola $y = x^2 + px + q$. Find a relationship between p and q. **(1)**



(b) The tangent to the parabola at A is the line $y = x$. Find the value of p. Hence find the equation of the parabola. **(6)**

(c) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value? **(2)**

[SQA] 4. The roots of the equation $(x - 1)(x + k) = -4$ are equal.

Find the values of k. **5**

Questions marked '[SQA]' © SQA
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[SQA] 5. An array of numbers such as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called a matrix. The eigenvalues of the matrix

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are defined to be the roots of the equation $(a-x)(d-x)-bc=0$.

Example

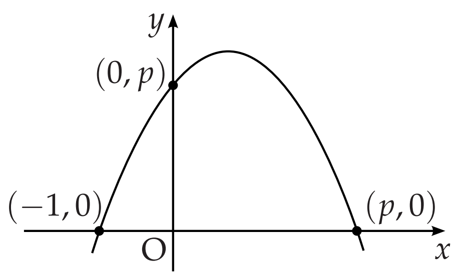
In order to find the eigenvalues of the matrix $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$
solve $(1-x)(2-x)-4\times 3=0$
solution: $2-3x+x^2-12=0$ $x^2-3x-10=0$ $(x+2)(x-5)=0$ $x=-2$ or $x=5$
so the eigenvalues of **B** are −2 and 5

(*a*) Find the eigenvalues of $\mathbf{C} = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$. **(3)**

(*b*) Find the value of t for which the eigenvalues of the matrix $\mathbf{D} = \begin{pmatrix} 3 & -1 \\ t & 1 \end{pmatrix}$ are equal. **(5)**

[SQA] 6. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

[SQA] 7. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(*a*) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. **3**

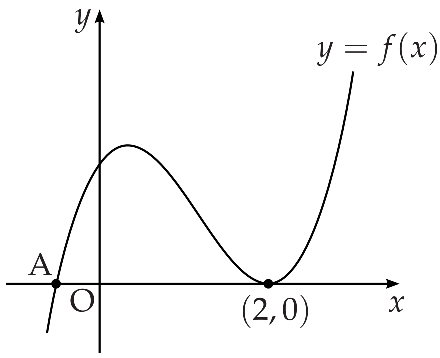
(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

[SQA] 8. (*a*) Given that $x+2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 9. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

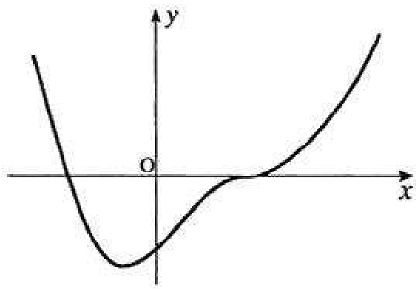
[SQA] 10. Find p if $(x + 3)$ is a factor of $x^3 - x^2 + px + 15$. **3**

[SQA] 11. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x - 2)$, the remainder is 114.

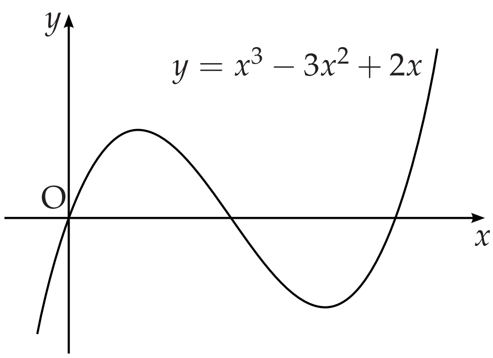
One factor of $f(x)$ is $(x + 1)$.

Find the values of p and q. **5**

[SQA] 12. The function f, whose incomplete graph is shown in the diagram, is defined by $f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature. **(8)**



[SQA] 13. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

[END OF PAPER 2]

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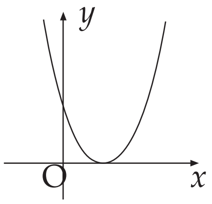
GCC Quadratics and Polynomials

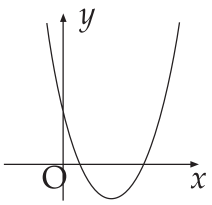
Paper 1 Section A

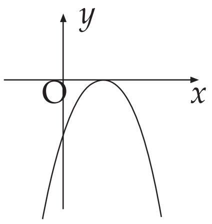
Each correct answer in this section is worth two marks.

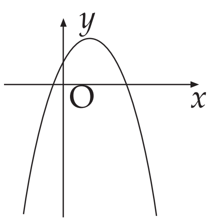
1. Which of the following diagrams shows a parabola with equation $y = ax^2 + bx + c$, where

   - $a > 0$
   - $b^2 - 4ac > 0$?

A. 

B. 

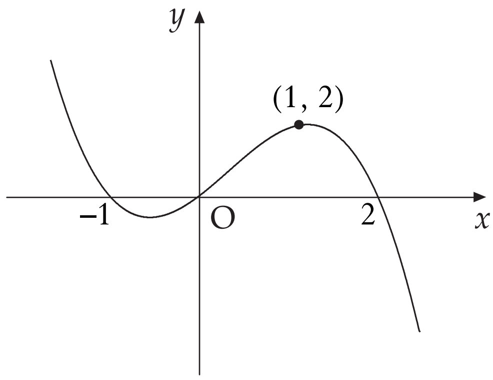
C. 

D. 

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	C	0	0	CN	A7, A15, A17	2010 P1 Q13

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

2. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A. $y = -x(x+1)(x-2)$

B. $y = -x(x-1)(x+2)$

C. $y = x(x+1)(x-2)$

D. $y = x(x-1)(x+2)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.1	C	0	0	CN	A7, A19	2011 P1 Q17

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

3. If $f(x) = (x-3)(x+5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

   A. $-5 < x < 3$

   B. $-3 < x < 5$

   C. $x < -5,\ x > 3$

   D. $x < -3,\ x > 5$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	A/B	0	0	CN	A16	2011 P1 Q18

4. What is the solution of $x^2 + 4x > 0$, where x is a real number?

   A. $-4 < x < 0$

   B. $x < -4,\ x > 0$

   C. $0 < x < 4$

   D. $x < 0,\ x > 4$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	C	0	0	CN	A16	2010 P1 Q18

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

5. Solve $6 - x - x^2 < 0$.

A. $-3 < x < 2$

B. $x < -3,\ x > 2$

C. $-2 < x < 3$

D. $x < -2,\ x > 3$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	C	0	0	NC	A16	2012 P1 Q19

6. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

I. the roots are real;

II. the roots are rational.

Which of the following is true?

A. neither statement is correct

B. only statement I is correct

C. only statement II is correct

D. both statements are correct

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	A/B	0	0	NC	A17	2011 P1 Q9

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All others © Higher Still Notes

7. A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A. No real roots

B. Equal roots

C. Real distinct roots

D. Rational distinct roots

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	C	0	0	NC	A17	2009 P1 Q12

8. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of k?

A. $-\frac{9}{8}$

B. $-\frac{8}{9}$

C. $\frac{8}{9}$

D. $\frac{9}{8}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.1	C	0	0	CN	A18	2010 P1 Q6

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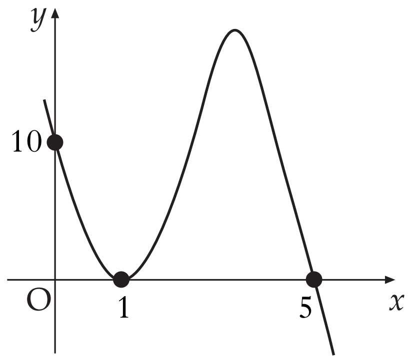
9. The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.



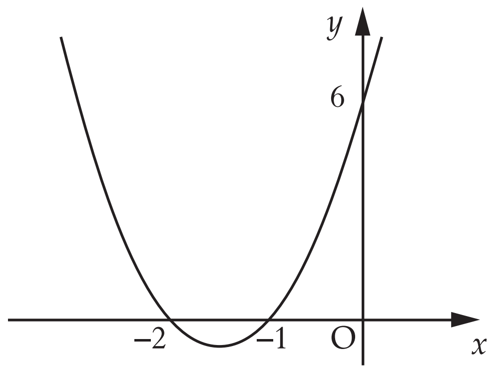
What are the values of k and t?

	k	t
A.	-2	-5
B.	-2	5
C.	2	-5
D.	2	5

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.1	C	0	0	CN	A19	2010 P1 Q16

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

10. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A. $y = 6(x-1)(x-2)$

B. $y = 6(x+1)(x+2)$

C. $y = 3(x-1)(x-2)$

D. $y = 3(x+1)(x+2)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.1	C	0	0	NC	A19	2012 P1 Q13

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11. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

    What is the remainder when $f(x)$ is divided by $(x-1)$?

    A. 0

    B. 2

    C. 3

    D. 4

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.1	C	0	0	NC	A21	2011 P1 Q7

[END OF PAPER 1 SECTION A]

Paper 1 Section B

[SQA] 12. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x-a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(*b*)	4	C	NC	A3	sketch	
(*c*)	1	C	NC	A16, A6	$-1 < x < 5$	

•1 pd: process, e.g. completing the square	•1 $a = 2$
•2 pd: process, e.g. completing the square	•2 $b = 1$
•3 ic: interpret minimum	•3 *any two from:* parabola; min. t.p. $(2,1)$; $(0,5)$
•4 ic: interpret y-intercept	•4 the remaining one from above list
•5 ss: reflect in x-axis	•5 reflecting in x-axis
•6 ss: translate parallel to y-axis	•6 translating $+10$ units, parallel to y-axis
•7 ic: interpret graph	•7 $(-1,5)$ i.e. $-1 < x < 5$

Questions marked '[SQA]' © SQA
All others © Higher Still Notes

[SQA] 13. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	2	C	NC	C7, A16		1996 P1 Q16
	2	A/B	NC	C7, A16		

- •[1] know to consider $f'(x) > 0$ stated or implied by the evidence for •[4].
- •[2] $\frac{dy}{dx} = 6x^2 - 6x - 36$
- •[3] $6(x-3)(x+2) > 0$ or by formula or completing the square
- •[4] $x < -2,\ x > 3$

[SQA] 14. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	NC	A17		1991 P1 Q18
	4	A/B	NC	A17		

- •[1] for realising "$b^2 - 4ac$" ≥ 0
- •[2] $kx^2 + 3x + (3-k) = 0$
- •[3] $\Delta = 3^2 - 4k(3-k)$
- •[4] $\Delta = (2k-3)^2$
- •[5] for stating $(2k-3)^2$ is ≥ 0 for all real k

[SQA] 15. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A18	$k = \frac{1}{4}$	2001 P1 Q2

•[1] ss: know to set disc. to zero	•[1] $b^2 - 4ac = 0$ *stated or implied by* •[2]
•[2] ic: substitute a, b and c into discriminant	•[2] $(-5)^2 - 4 \times (k+6)$
•[3] pd: process equation in k	•[3] $k = \frac{1}{4}$

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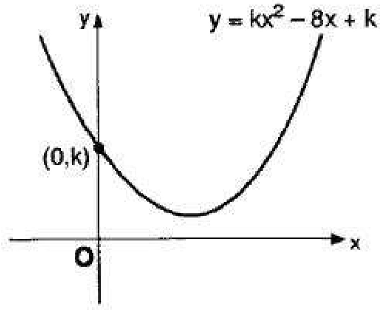
[SQA] 16. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	NC	A18		1992 P1 Q17
	3	A/B	NC	A18		

- •[1] *strat*: use discriminant
- •[2] $b^2 - 4ac < 0$
- •[3] $64 - 4k^2$
- •[4] $k = 5$

[SQA] 17. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	2	C	NC	A18		1993 P1 Q3

- •[1] discriminant $= 16 - 4 \times 2 \times k$
- •[2] $16 - 8k \geq 0$ for real roots $\Rightarrow k \leq 2$

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18. (*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.
    (ii) Hence factorise $f(x)$ fully. **5**

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. **1**

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G. **5**

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	A21	$(x-1)(x-1)(2x+5)$	2010 P1 Q22
(*b*)	1	C	CN	A22	$x = 1, -\frac{5}{2}$	
(*c*)	5	C	CN	A23	$(1, -1)$	
(*d*)	1	C	CN	A23	$(-\frac{5}{2}, -8)$	

•1 ss: know to use $x = 1$	•1 evaluating at $x = 1$...
•2 ic: complete evaluation	•2 $2 + 1 - 8 + 5 = 0$
•3 ic: state conclusion	•3 $(x-1)$ is a factor
•4 pd: find quadratic factor	•4 $(x-1)(2x^2 + 3x - 5)$
•5 pd: factorise completely	•5 $(x-1)(x-1)(2x+5)$
•6 ic: state solutions	•6 $x = 1$ and $x = -\frac{5}{2}$
•7 ss: set $y_{\text{curve}} = y_{\text{line}}$	•7 $2x^3 + x^2 - 6x + 2 = 2x - 3$
•8 ic: express in standard form	•8 $2x^3 + x^2 - 8x + 5 = 0$
•9 ss: compare with (a) or factorise	•9 $(x-1)(x-1)(2x+5) = 0$
•10 ic: identify x_G	•10 $x = 1$
•11 pd: evaluate y_G	•11 $y = -1$
•12 pd: state solution	•12 $(-\frac{5}{2}, -8)$

[SQA] 19. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1989 P1 Q2

•1 *strat*: make 2 trial divisions or 2 trial evaluations

•2 first linear factor

•3 quadratic factor

•4 other linear factors
$(x-1)(2x+1)(x+3)$

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[SQA] 20. (*a*) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$. **1**

(*b*) Hence find the other roots. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	NC	A21		1999 P1 Q1
(*b*)	3	C	NC	A21		

•[1] $f(2) = 16 + 4 - 26 + 6 = 0$
or
the appearance of a '0' at the end of the 3rd line in the table below

•[2]
$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

•[3] $2x^2 + 5x - 3$

•[4] $-3, \frac{1}{2}$

[SQA] 21. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.

Find the value of p and the other roots. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1993 P1 Q7

•[1] $f(-3) = -54 - 27 - 3p + 30$ or synth. division e.g.
$$\begin{array}{r|rrrr} -3 & 2 & -3 & p & 30 \\ & & -6 & 27 & -3p-81 \\ \hline & 2 & -9 & p+27 & -3p-51 \end{array}$$
and $-3p - 51 = 0$

•[2] $p = -17$

•[3] $2x^2 - 9x + 10$

•[4] $2, \frac{5}{2}$

[SQA] 22. (*a*) Show that $(x-3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**

(*b*) Hence express $f(x)$ in its fully factorised form. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	NC	A21		1995 P1 Q2
(*b*)	2	C	NC	A21		

•[1] $f(3) = 2 \times 3^3 + 3 \times 3^2 - 23 \times 3 - 12$ | or equivalent division

•[2] $= 0$

•[3] $2x^2 + 9x + 4$

•[4] $(x-3)(2x+1)(x+4)$

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[SQA] 23. Express $x^4 - x$ in its fully factorised form. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1996 P1 Q7

- $\bullet^1$ $x(x^3-1)$
- $\bullet^2$ synthetic division or eval. $f(k)$
- $\bullet^3$ linear factor $=(x-1)$
- $\bullet^4$ $x(x-1)(x^2+x+1)$

OR

- $\bullet^1$ synthetic division or eval. $f(k)$
- $\bullet^2$ linear factor $=(x-1)$
- $\bullet^3$ cubic factor $=(x^3+x^2+x)$
- $\bullet^4$ $x(x-1)(x^2+x+1)$

[SQA] 24. (*a*) Find a real root of the equation $2x^3-3x^2+2x-8=0$. **2**

(*b*) Show algebraically that there are no other real roots. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	NC	A21		1997 P1 Q5
(*b*)	3	C	NC	A21		

- $\bullet^1$ looking for $f(x)=\ldots\ldots=0$
- $\bullet^2$ $x=2$ **explicitly stated**
- $\bullet^3$ $2x^2+x+4$
- $\bullet^4$ $b^2-4ac=1-4\times2\times4$
- $\bullet^5$ $b^2-4ac<0$ means no real roots

[SQA] 25. Express $x^3-4x^2-7x+10$ in its fully factorised form. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	4	C	NC	A21		1998 P1 Q2

- $\bullet^1$ evaluating $f(k)$ for any integer by any method
- $\bullet^2$ find 1 value of k s.t. $f(k)=0$ *e.g.* $f(1)$ *or* $f(-2)$ *or* $f(5)$
- $\bullet^3$ quad factor *e.g.* $x^2-3x-10$
- $\bullet^4$ $(x-1)(x+2)(x-5)$

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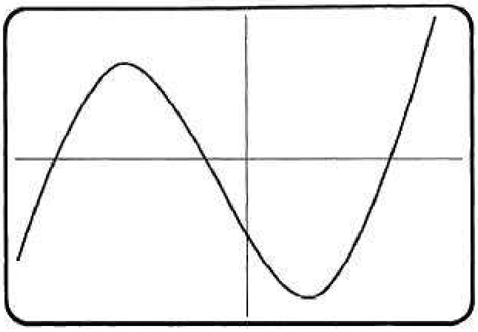
[SQA] 26. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	NC	A21		1992 P2 Q1
(*b*)	2	C	NC	A6		
(*c*)	6	C	NC	C8		

(a) •1 any linear factor
•2 corresponding quadratic factor
•3 $f(x) = (x+1)(x-4)(x+4)$

(b) •4 For all 3 points on x-axis
•5 $(0, -16)$

(c) •6 $f'(x) = 3x^2 + 2x - 16$
•7 use $f'(x) = 0$
•8 $x = 2$, and $x = -\frac{8}{3}$
•9 $y = -36$, and $y = \frac{400}{27}(14.8)$
•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	⋯	⋱	⋱	⋯	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2, -36)$

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[SQA] 27. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	NC	A21		1994 P2 Q1
(*b*)	4	C	NC	A21		

(a) •[1] strategy

eg

2	2	1	−13	a
		4	10	−6
	2	5	−3	0

or $f(2) = 0 = 16 + 4 - 26 + a$

•[2] $a = 6$

•[3] (0,6)

(b) •[4] $2x^2 + 5x - 3$

•[5] $(x+3)(2x-1)$

•[6] $x = -3,\ \frac{1}{2}$

•[7] $(-3,0),\ \left(\frac{1}{2},0\right)$

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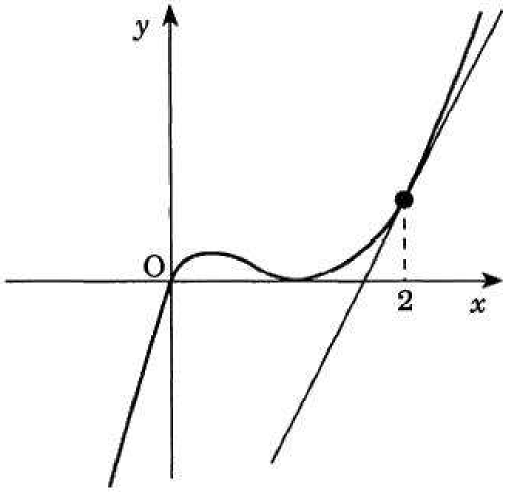
[SQA] 28. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	4	C	NC	C4, G3		1995 P2 Q2
(b)	5	C	NC	A23		

(a)
- •[1] $\frac{dy}{dx} = \ldots\ldots\ldots$
- •[2] $3x^2 - 4x + 1$
- •[3] $m_{x=2} = 5$
- •[4] $y - 2 = 5(x - 2)$

(b)
- •[5] equate 'y's
- •[6] $x^3 - 2x^2 - 4x + 8 = 0$
- •[7] e.g. synthetic division
- •[8] the appearance of:
  $x^2 - 4$
  or $x^2 - 4x + 4$
  or ± 2
  or $-2, 2, 2$
- •[9] $x = -2,\ y = -18$

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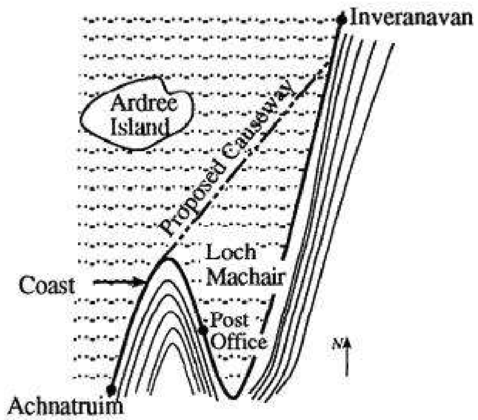
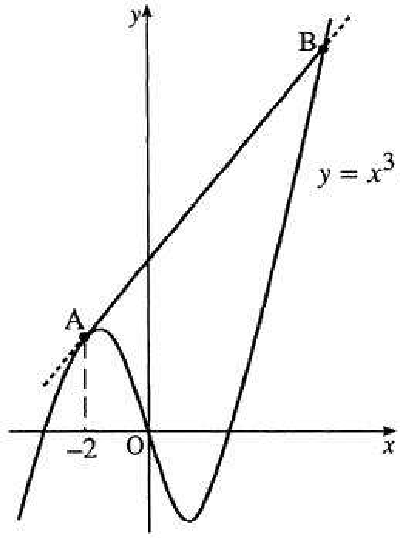
[SQA] 29. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*ai*)	1	C	NC	A6		1998 P2 Q5
(*aii*)	4	C	NC	C4, G3		
(*b*)	2	C	NC	A23		
(*b*)	5	A/B	NC	A23		

(*a*)

•1 $y_{x=-2} = 10$

•2 $\frac{dy}{dx} = \ldots\ldots$

•3 $3x^2 - 9$

•4 $m_{x=-2} = 3$

•5 $y - 10 - 3(x+2)$

(*b*)

•6 $y = 3x + 16$

•7 $3x + 16 = x^3 - 9x$

•8 $x^3 - 12x - 16 = 0$

•9 *e.g.*

−2	1	0	−12	−16
		−2	4	16
	1	−2	−8	0

•10 *e.g.* $x^2 - 2x - 8$

•11 *e.g.* $(x+2)(x-4)$

•12 B is $(4, 28)$

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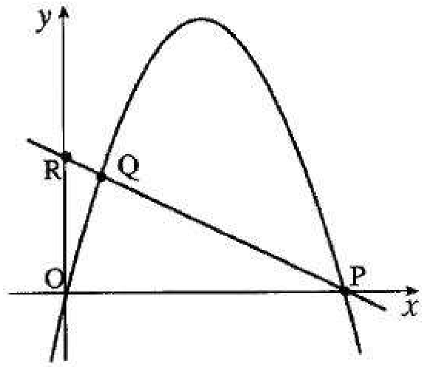
[SQA] 30. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.



(*a*) Find the coordinates of the point P. **2**

(*b*) R is the point (0, 2). Find the equation of PR. **2**

(*c*) The line and the parabola also intersect at Q. Find the coordinates of Q. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	NC	A6		1999 P2 Q4
(*b*)	2	C	NC	G3		
(*c*)	4	C	NC	A23		

(a)
- •1 $4x - x^2 = 0$ *stated or implied by* •2
- •2 $(4, 0)$

(b)
- •3 $m = -\frac{1}{2}$
- •4 $y = -\frac{1}{2}x + 2$
  - *or* $y - 2 = -\frac{1}{2}(x - 0)$
  - *or* $y - 0 = -\frac{1}{2}(x - 4)$

(c)
- •5 $4x - x^2 = 2 - \frac{1}{2}x$
- •6 *e.g.* $2x^2 - 9x + 4 = 0$
- •7 $x = \frac{1}{2}, x = 4$
- •8 Q is $\left(\frac{1}{2}, \frac{7}{4}\right)$

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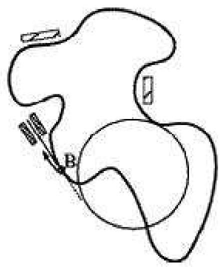
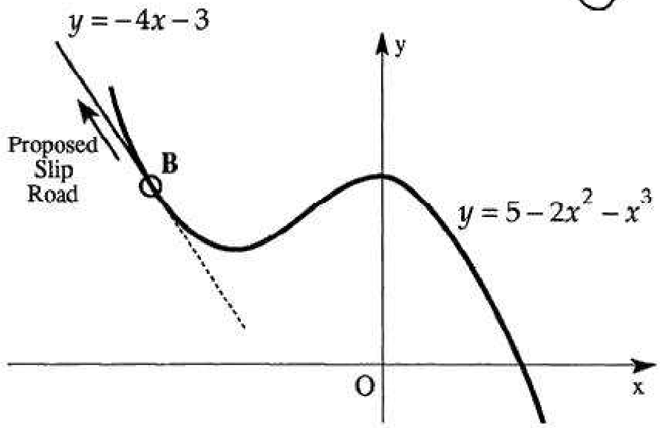
[SQA] 31. The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	7	C	NC	A23, A21		1993 P2 Q7
(*b*)	1	A/B	NC	A24		

(a) •[1] equating expressions for y

•[2] re-arranging cubic...... "..."= 0

•[3] strategy for solving cubic

•[4] first linear factor

•[5] quadratic factor

•[6] $x = -2,\ 2$

•[7] intersection at $(-2, 5)$

(b) •[8] double root $\Rightarrow$ tangency or $y'(-2) = -4 =$ gradient of line

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[SQA] 32. (*a*) (i) Make a sketch of the graph of $y = x^3$, where $-3 \leq x \leq 3$, $x \in \mathbf{R}$.

(ii) On the same diagram, draw the graph of $y = 6x + 1$. **(3)**

(*b*) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \leq x \leq 3$. **(1)**

(*c*) Calculate the value of the positive root, correct to 3 significant figures. **(4)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	NC	CGD		1989 P2 Q3
(*b*)	1	C	NC	CGD		
(*c*)	1	C	NC	A26		
(*c*)	3	A/B	NC	A26		

(*a*)
•[1] suitable choice of scales
•[2] sketch of $y = x^3$ from $x = -3$ to $x = 3$
•[3] sketch of $y = 6x + 1$ from $x = -3$ to $x = 3$

(*b*)
•[4] 3 roots

(*c*)
•[5] 1st estimate: between 2 and 3
•[6] 2nd estimate: between 2.5 and 2.6
•[7] 3rd estimate: between 2.53 and 2.534
•[8] 2.53

[END OF PAPER 1 SECTION B]

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Paper 2

[SQA] 1.

(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots. **1**

(ii) Hence or otherwise show that the equation $x(x+1) = 3x - 2$ has no real roots. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A17		1999 P1 Q8

•1 $b^2 - 4ac = 0$

•2 $x^2 + 6x + 9 = 0$

•3 $b^2 - 4ac = 36 - 36 = 0$ *OR* •3 $(x+3)(x+3) = 0$ so roots are $-3, -3$

[SQA] 2. Show that the roots of the equation $(k-2)x^2 - (3k-2)x + 2k = 0$ are real. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	CN	A17		1990 P1 Q18
	3	A/B	CN	A17		

•1 *use* discriminant Δ

•2 $\Delta = (3k-2)^2 - 8k(k-2)$

•3 $\Delta = k^2 + 4k + 4$

•4 $(k+2)^2 \geq 0$ so roots real

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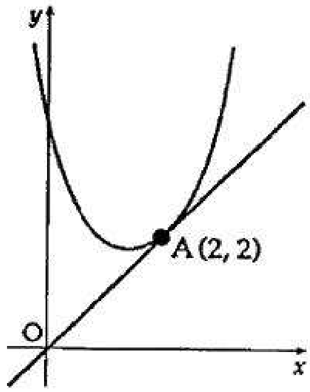
[SQA] 3. (*a*) The point A(2, 2) lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q. **(1)**



(*b*) The tangent to the parabola at A is the line $y = x$. Find the value of p. Hence find the equation of the parabola. **(6)**

(*c*) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ? **(2)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	CN	A6		1994 P2 Q9
(*b*)	2	C	CN	C4, CGD		
(*b*)	4	A/B	CN	C4, CGD		
(*c*)	2	A/B	CN	A17		

(*a*) •1 $2p + q = -2$

(*b*) •2 strategy

•3 $2x + p$

•4 gradient = 1, or equivalent

•5 $4 + p$

•6 $p = -3$

•7 $q = 4$

(*c*) •8 $\Delta = -7$

•9 $\sqrt{-7}$ means no roots

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[SQA] 4. The roots of the equation $(x-1)(x+k) = -4$ are equal.

Find the values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	CN	A18		1995 P1 Q20
	4	A/B	CN	A18	$k = -5, 3$	

•[1] $x^2 + kx - x + 4 - k = 0$

•[2] $b^2 - 4ac = 0$

•[3] $(k-1)^2 - 4(4-k)$

•[4] $k^2 + 2k - 15 = 0$

•[5] $k = -5,\ k = 3$

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[SQA] 5. An array of numbers such as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called a matrix. The eigenvalues of the matrix

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are defined to be the roots of the equation $(a-x)(d-x)-bc=0$.

Example

In order to find the eigenvalues of the matrix $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$

solve $(1-x)(2-x)-4\times3=0$

solution:

$$2-3x+x^2-12=0$$
$$x^2-3x-10=0$$
$$(x+2)(x-5)=0$$
$$x=-2 \text{ or } x=5$$

so the eigenvalues of **B** are –2 and 5

(*a*) Find the eigenvalues of $\mathbf{C} = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$. **(3)**

(*b*) Find the value of t for which the eigenvalues of the matrix $\mathbf{D} = \begin{pmatrix} 3 & -1 \\ t & 1 \end{pmatrix}$ are equal. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	CN	CGD		1993 P2 Q4
(*b*)	5	C	CN	A18, CGD		

(a)
- •[1] $(3-x)(5-x)-2\times4=0$
- •[2] $x^2-8x+7=0$
- •[3] eigenvalues are 1, 7

(b)
- •[4] $(3-x)(1-x)+t=0$
- •[5] $x^2-4x+(3+t)=0$
- •[6] $\Delta=0$ for equal roots or equiv.
- •[7] $\Delta=16-4\times1\times(3+t)$ or equiv.
- •[8] $t=1$

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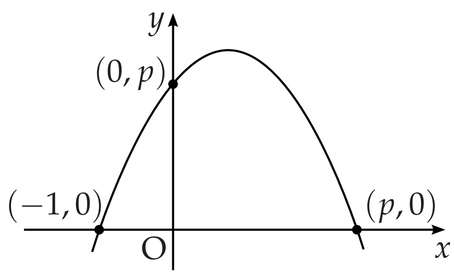
[SQA] 6. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

•1 ss: know to use discriminant •2 ic: pick out discriminant •3 pd: simplify to quadratic •4 ss: choose to draw table or graph •5 pd: complete proof using disc. ≥ 0	•1 discriminant = ... •2 disc $= (-5k)^2 - 4(1-2k)(-2k)$ •3 $9k^2 + 8k$ •4 e.g. draw a table, graph, complete the square •5 complete proof and conclusion relating to disc. ≥ 0

[SQA] 7. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(a) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. **3**

(b) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	3	A/B	CN	A19	proof	2001 P2 Q11
(b)	3	A/B	CN	A24	2	

•1 ss: use a standard form of parabola •2 ss: use 3rd point to determine k •3 pd: complete proof •4 ss: equate and simplify to zero •5 ss: use discriminant for tangency •6 pd: process	•1 $y = k(x+1)(x-p)$ •2 $k = -1$ with justification (i.e. substitute $(0, p)$) •3 $y = -1(x+1)(x-p)$ **and complete** •4 $x^2 + 2x - px = 0$ •5 $b^2 - 4ac = (2-p)^2 = 0$ **or** $(2-p)^2 - 4 \times 0 = 0$ •6 $p = 2$

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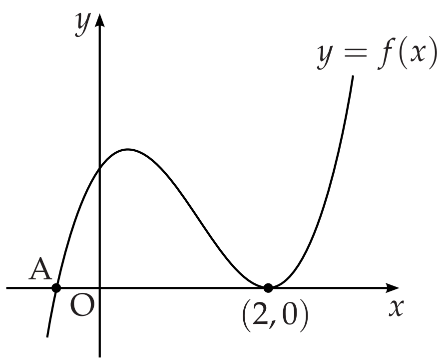
[SQA] 8. (*a*) Given that $x+2$ is a factor of $2x^3+x^2+kx+2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3+x^2+kx+2=0$ when k takes this value. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	CN	A21	$k=-5$	2001 P2 Q1
(*b*)	2	C	CN	A22	$x=-2,\frac{1}{2},1$	

•1 ss: use synth division or f(evaluation)	•1 $f(-2)=2(-2)^3+\cdots$
•2 pd: process	•2 $2(-2)^3+(-2)^2-2k+2$
•3 pd: process	•3 $k=-5$
•4 ss: find a quadratic factor	•4 $2x^2-3x+1$ **or** $2x^2+3x-2$ **or** x^2+x-2
•5 pd: process	•5 $(2x-1)(x-1)$ **or** $(2x-1)(x+2)$ **or** $(x+2)(x-1)$ **and** $x=-2,\frac{1}{2},1$

[SQA] 9. The diagram shows part of the graph of the curve with equation $y=2x^3-7x^2+4x+4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3-7x^2+4x+4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3-7x^2+4x+4<0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x=\frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2},0)$, $x<-\frac{1}{2}$	

•1 ss: know to differentiate	•1 $f'(x)=\ldots$
•2 pd: differentiate	•2 $6x^2-14x+4$
•3 ss: know to set derivative to zero	•3 $6x^2-14x+4=0$
•4 pd: start solving process of equation	•4 $(3x-1)(x-2)$
•5 pd: complete solving process	•5 $x=\frac{1}{3}$
•6 ss: strategy for cubic, e.g. synth. division	•6 $\begin{array}{c\|cccc} \cdots & 2 & -7 & 4 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \boxed{0} \end{array}$
•7 ic: extract quadratic factor	•7 $2x^2-3x-2$
•8 pd: complete the cubic factorisation	•8 $(x-2)(2x+1)(x-2)$
•9 ic: interpret the factors	•9 $A(-\frac{1}{2},0)$
•10 ic: interpret the diagram	•10 $x<-\frac{1}{2}$

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[SQA] 10. Find p if $(x+3)$ is a factor of $x^3 - x^2 + px + 15$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A21		1990 P1 Q1

- •[1] *strat*: e.g. find $f(-3)$
- •[2] $f(-3) = 0$
- •[3] $p = -7$

[SQA] 11. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x-2)$, the remainder is 114.

One factor of $f(x)$ is $(x+1)$.

Find the values of p and q. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	C	CN	A21		1991 P1 Q6

- •[1] $f(2) = 114$
- •[2] $f(-1) = 0$
- •[3] $4p + 2q = 78$
- •[4] $p - q = -15$
- •[5] $p = 8, q = 23$

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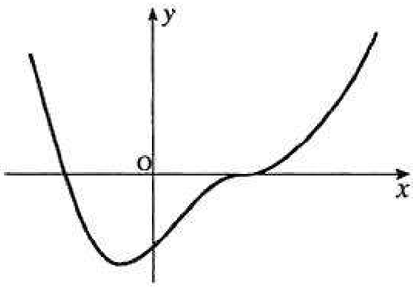
[SQA] 12. The function f, whose incomplete graph is shown in the diagram, is defined by
$f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature.



(8)

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	8	C	CN	A21, C8		1993 P2 Q1

•1 for knowing to differentiate

•2 $f'(x) = 4x^3 - 6x^2 + 2$

•3 for putting $f'(x) = 0$

•4 for factorising or checking zeros

•5 $x = -\frac{1}{2}$, $x = 1$

•6 $y = -\frac{27}{16}$, $y = 0$

•7 completed nature table

x	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	< 1	1	> 1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	╲	—	╱	╱	—	╱

•8 $(1,0)$ is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

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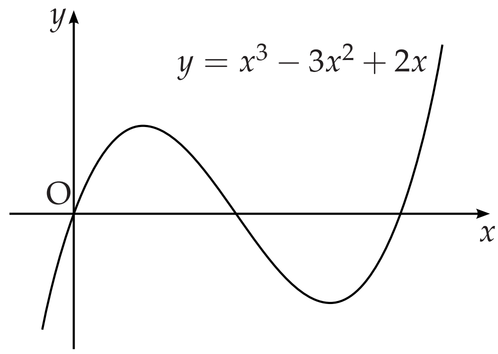
[SQA] 13. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

•1 ss: know to differentiate	•1 $y' = \dots$
•2 pd: differentiate correctly	•2 $3x^2 - 6x + 2$
•3 ss: know that gradient $= f'(1)$	•3 $y'(1) = -1$
•4 ss: know that y-coord $= f(1)$	•4 $y(1) = 0$
•5 ic: state equ. of line	•5 $y - 0 = -1(x - 1)$
•6 ss: equate equations	•6 $2x - 4 = x^3 - 3x^2 + 2x$
•7 pd: arrange in standard form	•7 $x^3 - 3x^2 + 4 = 0$
•8 ss: know how to solve cubic	•8 $\dots$ | 1 −3 0 4 / $\dots$ $\dots$ $\dots$ / $\dots$ $\dots$ $\dots$ | $\dots$
•9 pd: process	•9 identify $x = -1$ from working
•10 ic: interpret	•10 $(-1, -6)$

[END OF PAPER 2]

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GCC Vectors

[SQA] 1. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

(*a*) Find the coordinates of M, the midpoint of AB. **1**

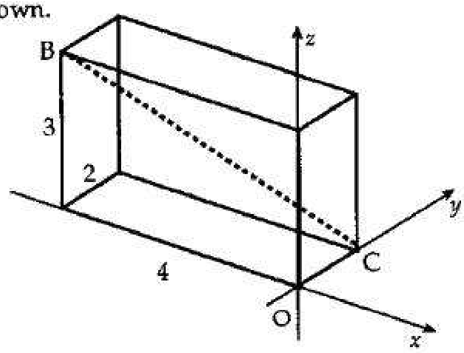
(*b*) Find the coordinates of the point T, which divides CM in the ratio 2 : 1. **3**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

[SQA] 2. A cuboid crystal is placed relative to the coordinate axes as shown.

(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $|\overrightarrow{BC}|$. 2

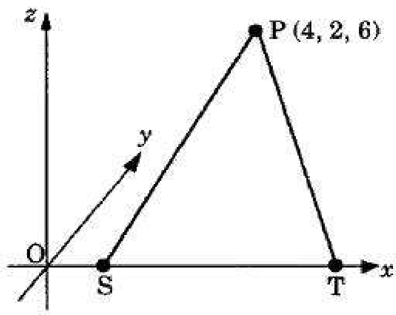


[SQA] 3. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find

(*a*) the components of vector $\overrightarrow{AB}$; 1

(*b*) the length of AB. 2

[SQA] 4. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 3



[SQA] 5. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by

$$\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}, \quad \boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}.$$

(*a*) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. 2

(*b*) Calculate $\boldsymbol{p}.\boldsymbol{r}$ 1

(*c*) Find $|\boldsymbol{r}|$. 1

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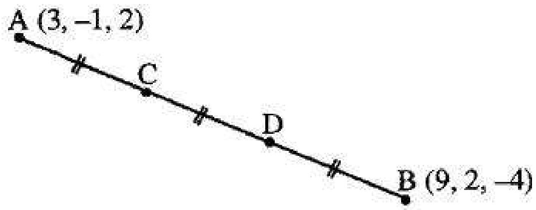
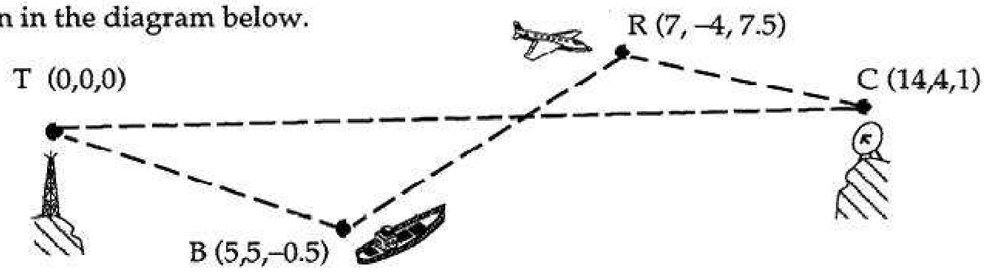
[SQA] 6. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).



(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(*b*) Find the coordinates of C and D. 2

[SQA] 7. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

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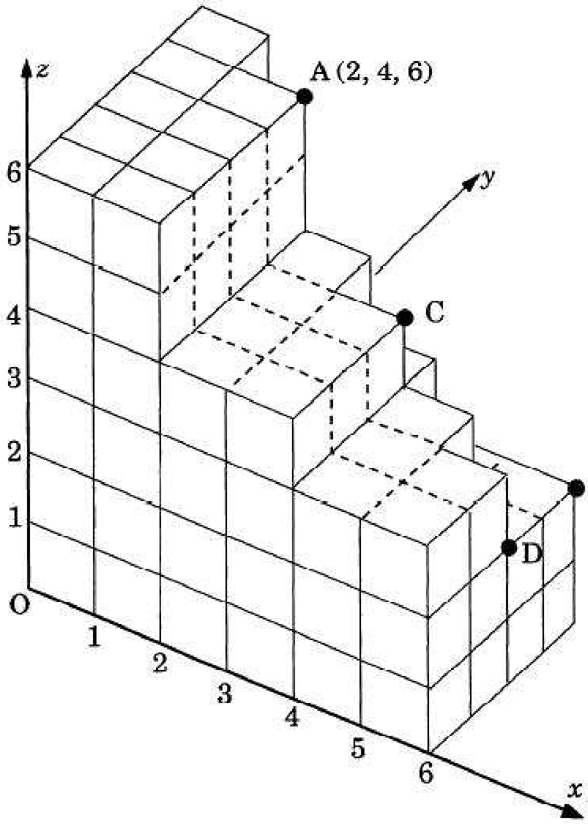
[SQA] 8. With coordinate axes as shown, the point A is (2,4,6).



(a) Write down the coordinates of B,C and D. **(3)**

(b) Show that C is the midpoint of AD. **(1)**

(c) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(d) Hence calculate the size of angle OAB. **(2)**

[SQA] 9. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

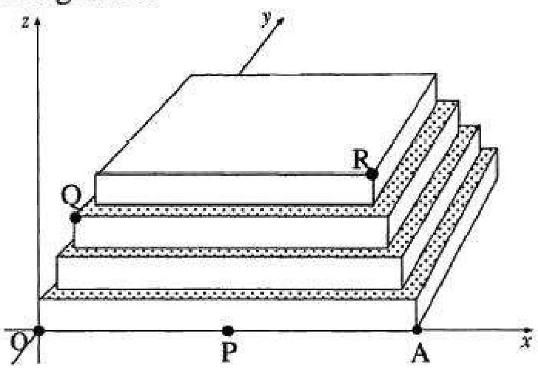


Diagram 1

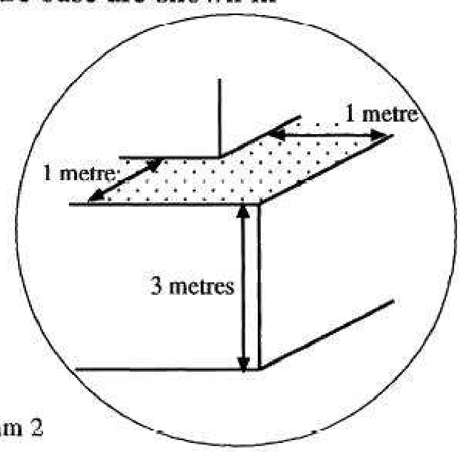


Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(a) Find the coordinates of Q and R. **(2)**
(b) Find the size of angle QPR. **(7)**

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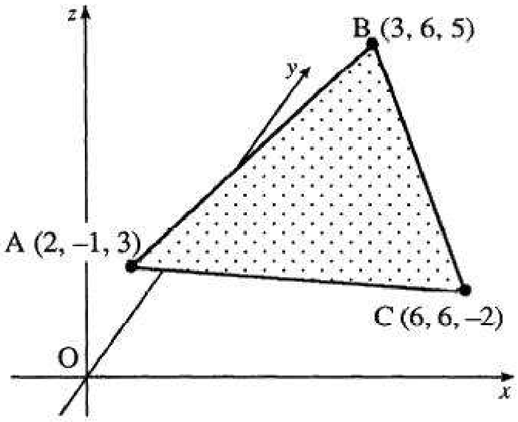
[SQA] 10. A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).

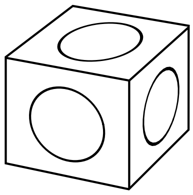


(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

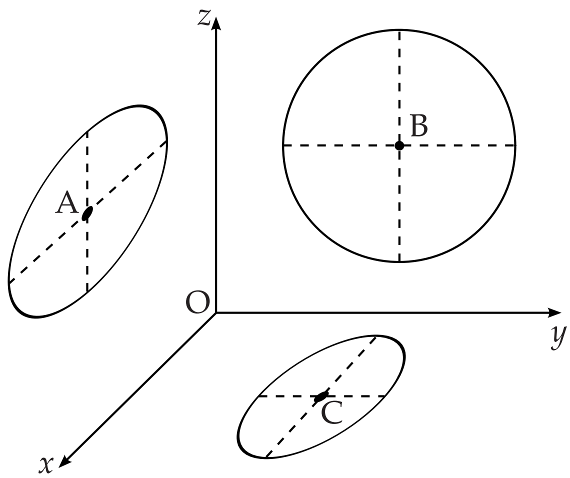
(*c*) Hence find the area of the triangle. **(2)**

[SQA] 11. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $\text{A}(6,0,7)$, $\text{B}(0,5,6)$ and $\text{C}(4,5,0)$.

Find the size of angle ABC. **7**



[SQA] 12. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows:

$$\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},\ \boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},\ \boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}.$$

(*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. **1**

(*b*) Find the value of $|2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}|$. **2**

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[SQA] 13. VABCD is a pyramid with rectangular base ABCD.

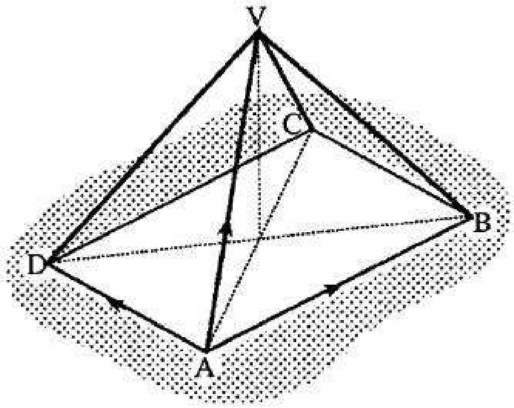
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8i + 2j + 2k$

$\overrightarrow{AD} = -2i + 10j - 2k$ and

$\overrightarrow{AV} = i + 7j + 7k$.

Express $\overrightarrow{CV}$ in component form.



3

[SQA] 14. The vector $ai + bj + k$ is perpendicular to both the vectors $i - j + k$ and $-2i + j + k$.

Find the values of a and b. **3**

[SQA] 15. Calculate the length of the vector $2i - 3j + \sqrt{3}k$. **2**

[SQA] 16. Show that the vectors $a = 2i + 3j - k$ and $b = 3i - j + 3k$ are perpendicular. **3**

[SQA] 17. The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. 2

(*b*) Find the length of PQ. 1

[SQA] 18. The vectors a, b and c are defined as follows:

$$a = 2i - k, \quad b = i + 2j + k, \quad c = -j + k.$$

(*a*) Evaluate $a.b + a.c$. 3

(*b*) From your answer to part (*a*), make a deduction about the vector $b + c$. 2

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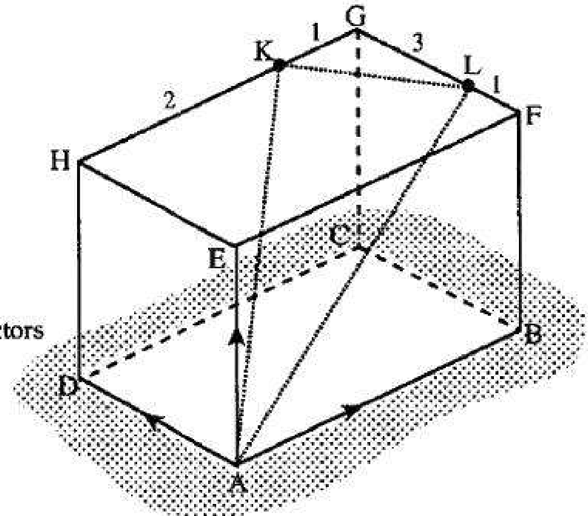
[SQA] 19. ABCDEFGH is a cuboid.



K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\vec{AB}$, $\vec{AD}$ and $\vec{AE}$ can be represented by the vectors

$\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ respectively.

(*a*) Calculate the components of $\vec{AK}$. **2**

(*b*) Calculate the components of $\vec{AL}$. **2**

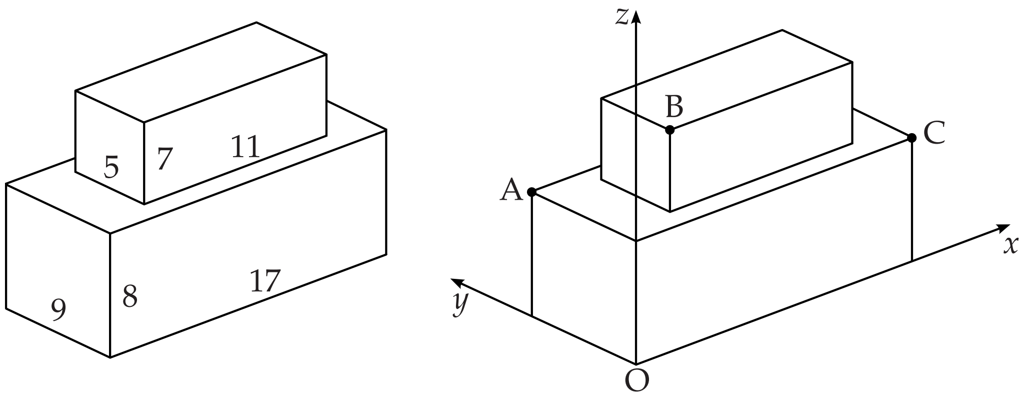
(c) Calculate the size of angle KAL. **5**

[SQA] 20. If $\boldsymbol{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, write down the components of $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$.

Hence show that $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular. **3**

[SQA] 21. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.



(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

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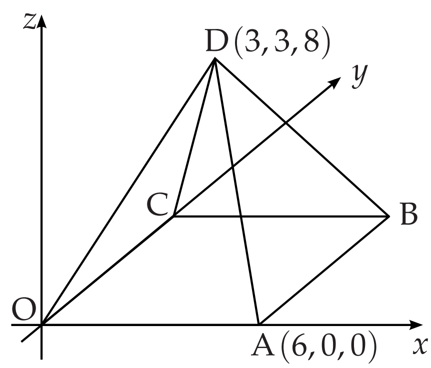
[SQA] 22. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are $(6, 0, 0)$ and $(3, 3, 8)$.

C lies on the y-axis.

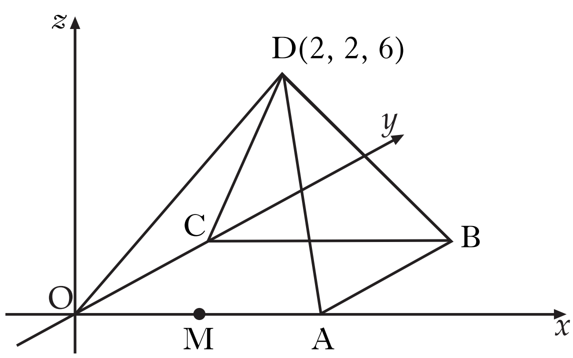


(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

23. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point $(2, 2, 6)$ and $\text{OA} = 4$ units.

M is the mid-point of OA.

(*a*) State the coordinates of B. **1**

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(*c*) Find the size of angle BDM. **5**

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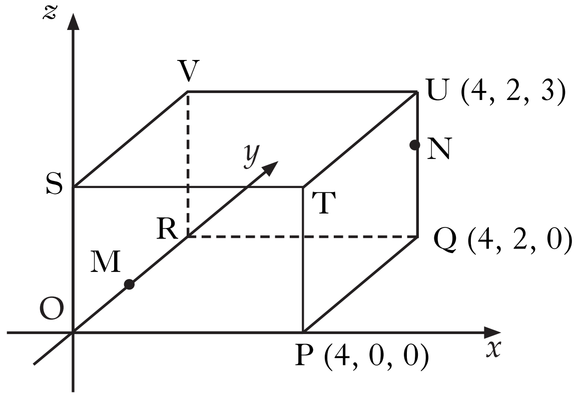
24. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$, Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



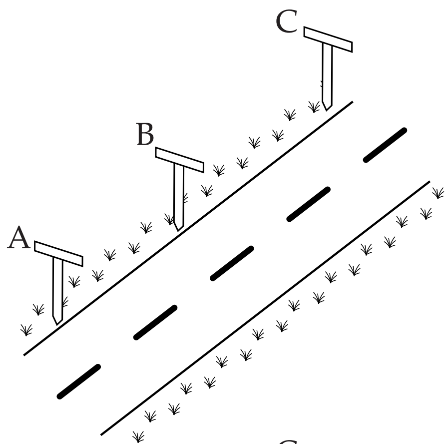
(*a*) State the coordinates of M and N. **2**

(*b*) Express the vectors $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

(*c*) Calculate the size of angle MVN. **5**

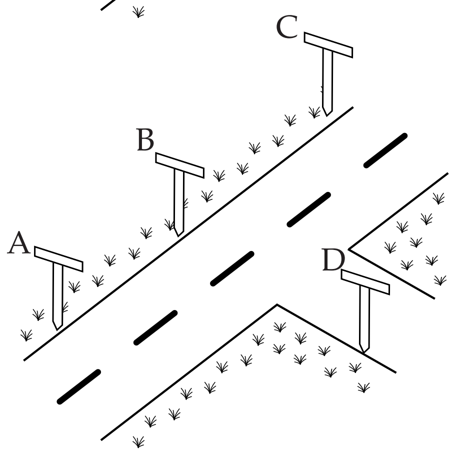
[SQA] 25. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8, -10, -2)$, $B(-2, -1, 1)$ and $C(6, 11, 5)$.

Determine whether or not the section of road ABC has been built in a straight line. **3**



(*b*) A further T-rod is placed such that D has coordinates $(1, -4, 4)$.

Show that DB is perpendicular to AB. **3**



[SQA] 26. **(a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear.** **4**

(b) Find the ration in which M divides LN. **1**

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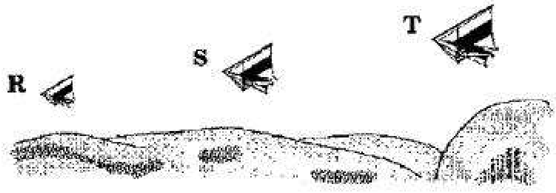
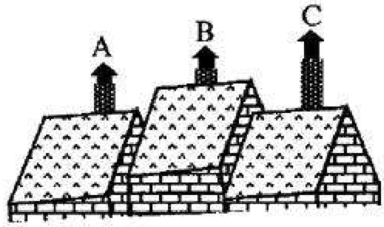
[SQA] 27. Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6).
Prove that R, S and T are collinear. **3**



[SQA] 28. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1, 4) and C(4, –9, 8).
Show that A, B and C are collinear. **3**



[SQA] 29. Show that $P(2,2,3)$, $Q(4,4,1)$ and $R(5,5,0)$ are collinear and find the ratio in which Q divides PR. **4**

[SQA] 30. A is the point $(2,-5,6)$, B is $(6,-3,4)$ and C is $(12,0,1)$. Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

[SQA] 31. D, E and F have coordinates $(10,-8,-15)$, $(1,-2,-3)$ and $(-2,0,1)$ respectively.

(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

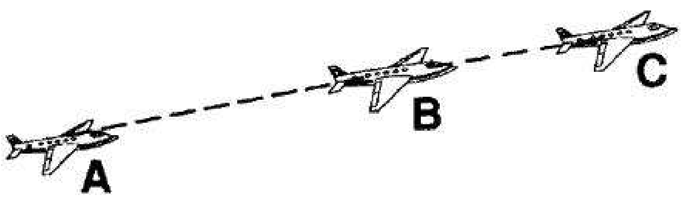
(*b*) G has coordinates $(k,1,0)$.

Given that DE is perpendicular to GE, find the value of k. **4**

[SQA] 32. The point Q divides the line joining $P(-1,-1,0)$ to $R(5,2,-3)$ in the ratio $2:1$.

Find the coordinates of Q. **3**

[SQA] 33. An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point (–1, 3, 4) and B is the point (3, 1, –2). Find the co-ordinates of the point C. **3**



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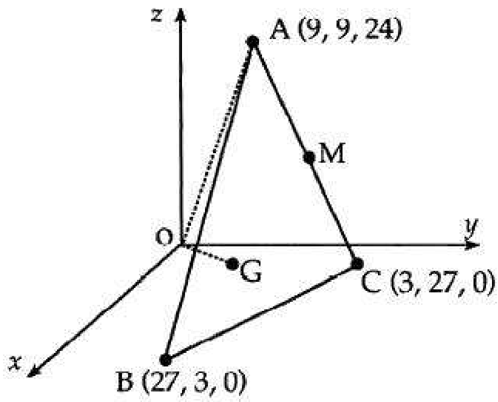
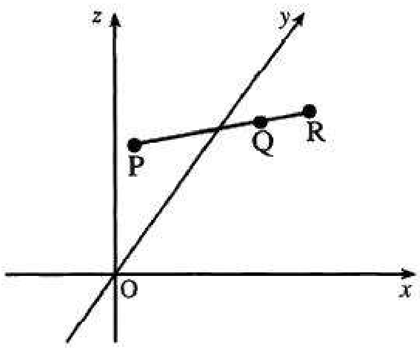
[SQA] 34. (*a*) Relative to mutually perpendicular axes Ox, Oy and Oz, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.
Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**

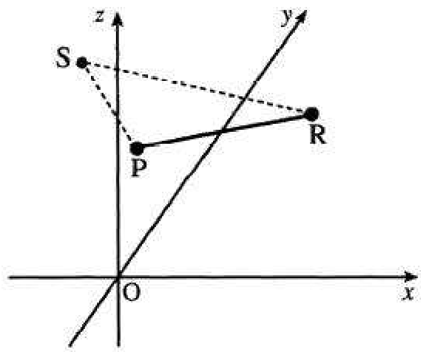


[SQA] 35. Relative to the axes shown and with an appropriate scale, P(−1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**

(*b*) Roads from P and R are built to meet at the point S (−2, 2, 5). Calculate the size of angle PSR. **(7)**



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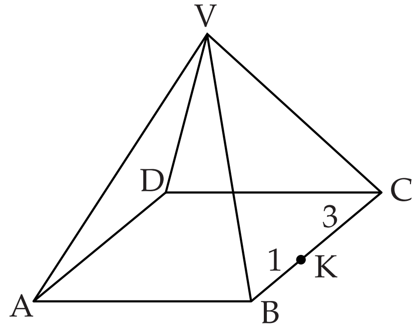
[SQA] 36. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

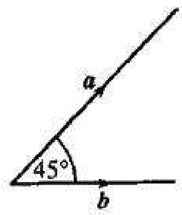
$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.



K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form. **3**

[SQA] 37. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$. These vectors are inclined at an angle of 45° to each other.



(*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$

(ii) $\boldsymbol{b}.\boldsymbol{b}$

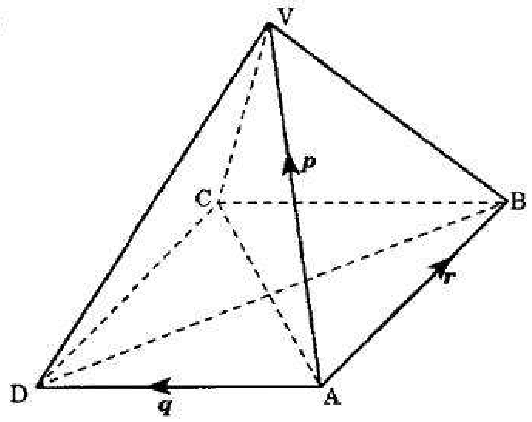
(iii) $\boldsymbol{a}.\boldsymbol{b}$ **2**

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.
Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $|\boldsymbol{p}|$. **4**

[SQA] 38. In the square-based pyramid, all the eight edges are of length 3 units.

$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$.

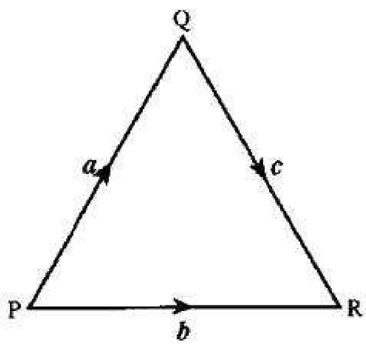
Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **4**



[SQA] 39. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$ and hence identify two vectors which are perpendicular. **4**



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[SQA] 40. For what value of t are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

[SQA] 41. A$(4,4,10)$, B$(-2,-4,12)$ and C$(-8,0,10)$ are the vertices of a right-angled triangle.

Determine which angle of the triangle is the right angle. **3**

[SQA] 42. Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$ are perpendicular. **3**

[SQA] 43. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

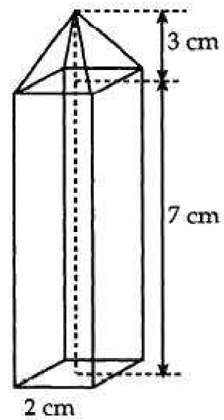


Diagram 1

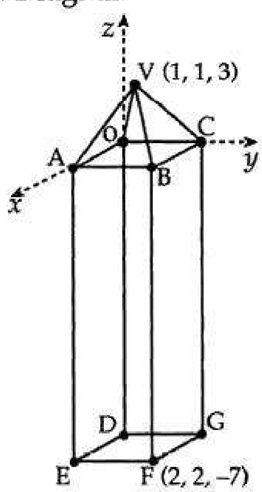


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;

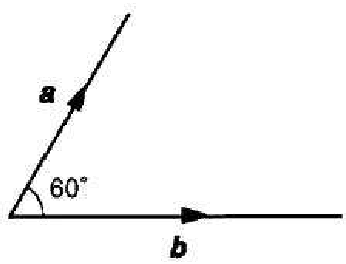
(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

[SQA] 44. The diagram shows representatives of two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, inclined at an angle of 60°.

If $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = 3$, evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b})$ **3**



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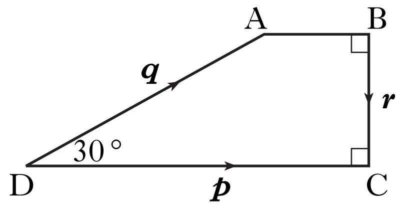
[SQA] 45. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC $= 30°$.

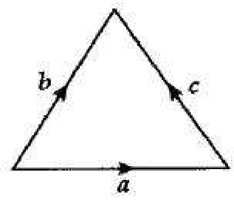
It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})$. **6**

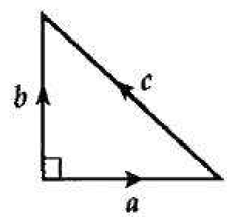
(*b*) Find $|\boldsymbol{q}+\boldsymbol{r}|$ and $|\boldsymbol{p}-\boldsymbol{q}|$. **4**



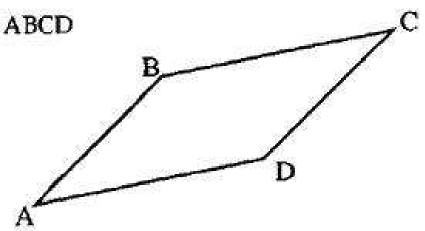
[SQA] 46. The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown.
Evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$. 5



[SQA] 47. The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.
The two equal sides have length 2 units.
Find the value of $\boldsymbol{b}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$ 5



[SQA] 48. A is the point $(2, -1, 4)$, B is $(7, 1, 3)$ and C is $(-6, 4, 2)$. If ABCD is a parallelogram, find the coordinates of D. 3



[SQA] 49. PQRS is a parallelogram with vertices $P(1,3,3)$, $Q(4,-2,-2)$ and $R(3,1,1)$.
Find the coordinates of S. **3**

[END OF QUESTIONS]

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GCC Vectors

[SQA] 1. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

(*a*) Find the coordinates of M, the midpoint of AB. **1**

(*b*) Find the coordinates of the point T, which divides CM in the ratio $2:1$. **3**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G6, G25		1989 P2 Q2
(*b*)	3	C	CN	G25		
(*c*)	4	C	CN	G23, G25		

(*a*) •1 $(6,1,1)$

(*b*) •2 e.g. $\overrightarrow{CM}=\begin{pmatrix}6\\-3\\-3\end{pmatrix}$

•3 $\overrightarrow{CT}=\begin{pmatrix}4\\-2\\-2\end{pmatrix}$

•4 $T=(4,2,2)$

(*c*) •5 e.g. $\overrightarrow{BT}=\begin{pmatrix}-4\\-1\\3\end{pmatrix}$

•6 $\overrightarrow{TD}=\begin{pmatrix}-8\\-2\\6\end{pmatrix}=2\times\overrightarrow{BT}$

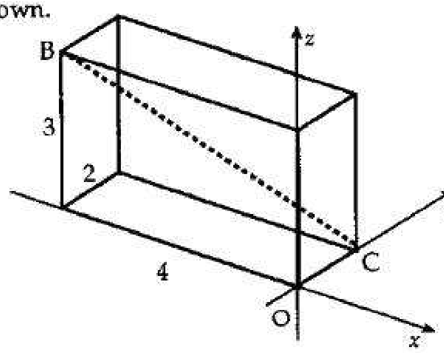
•7 TD is parallel to BT, T is common point so B, T, D collinear

•8 BT : TD = 1 : 2

[SQA] 2. A cuboid crystal is placed relative to the coordinate axes as shown.

(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $\left|\overrightarrow{BC}\right|$. **2**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G16		1990 P1 Q5
(*b*)	1	C	CN	G16		

•1 $\overrightarrow{BC}=\begin{pmatrix}4\\2\\-3\end{pmatrix}$

•2 $\sqrt{29}$

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[SQA] 3. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find

(a) the components of vector $\overrightarrow{AB}$; 1

(b) the length of AB. 2

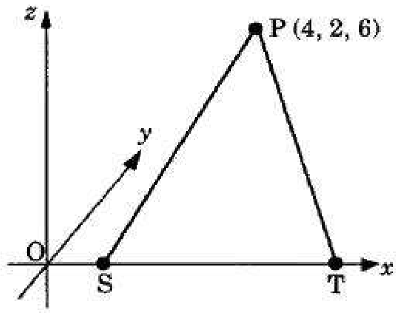
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	C	CN	G16		1993 P1 Q1
(b)	2	C	CN	G16		

•[1] $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

•[2] $\sqrt{(-3+1)^2 + (2-3)^2 + (4-2)^2}$

•[3] 3

[SQA] 4. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the x-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	A/B	CN	G16		1994 P1 Q18

•[1] $(x,0,0)$ or equiv.	*OR*	•[1] $PQ = \sqrt{40}$	*OR*	•[1] $d^2 = 7^2 - 6^2 - 2^2$	
•[2] $(x-4)^2 + 4 + 36 = 49$ or equiv.		•[2] $d = 3$		•[2] $d = 3$	
•[3] $x = 1,\ 7$		•[3] $(1,0,0),\ (7,0,0)$		•[3] $(1,0,0),\ (7,0,0)$	

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[SQA] 5. Vectors p, q and r are defined by

$$p = i + j - k, \quad q = i + 4k \quad \text{and} \quad r = 4i - 3j.$$

(*a*) Express $p - q + 2r$ in component form. 2

(*b*) Calculate $p.r$ 1

(*c*) Find $|r|$. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G16		1998 P1 Q3
(*b*)	1	C	CN	G26		
(*c*)	1	C	CN	G16		

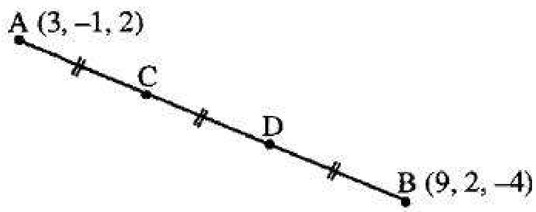
•[1] $p = \begin{pmatrix}1\\1\\-1\end{pmatrix}, q = \begin{pmatrix}1\\0\\4\end{pmatrix}, r = \begin{pmatrix}4\\-3\\0\end{pmatrix}$ s/i by •[2]

•[2] $\begin{pmatrix}8\\-5\\-5\end{pmatrix}$

•[3] 1

•[4] 5

[SQA] 6. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).



(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(*b*) Find the coordinates of C and D. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G16		1998 P1 Q5
(*b*)	2	C	CN	G16		

•[1] $\overrightarrow{AB} = \begin{pmatrix}6\\3\\-6\end{pmatrix}$

•[2] $\overrightarrow{AC} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$

•[3] $C = (5,0,0)$

•[4] $D = (7,1,-2)$

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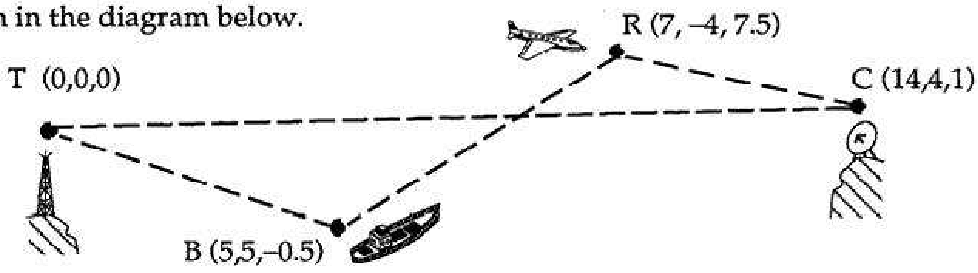
[SQA] 7. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G16		1992 P2 Q2
(*b*)	2	C	CR	G16		
(*c*)	3	C	CR	G27		
(*d*)	5	C	CR	G28		

(a) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(b) •4 $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(c) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(d) •9 Strategy: know to use $\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|\overrightarrow{TC}||\overrightarrow{RC}|}$ or equiv.

•10 $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•13 36.7°

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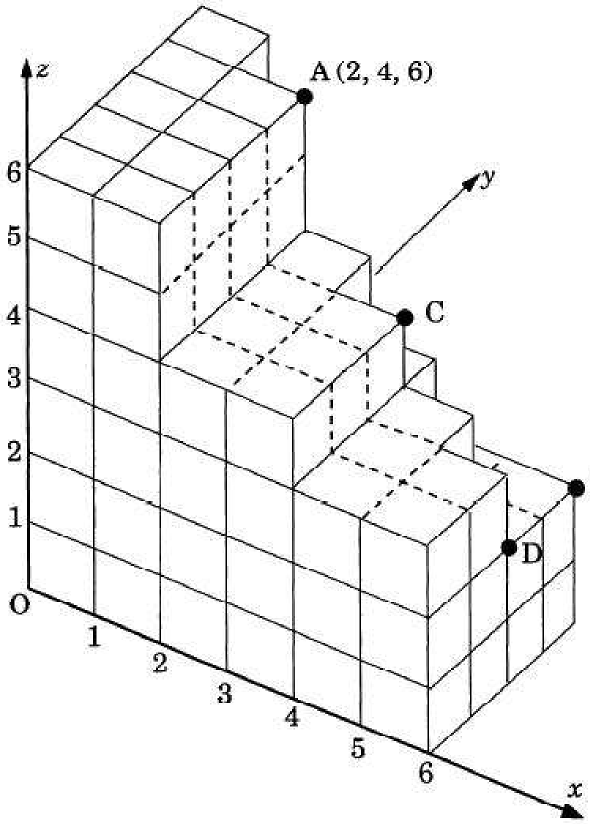
[SQA] 8. With coordinate axes as shown, the point A is (2,4,6).



(*a*) Write down the coordinates of B,C and D. **(3)**

(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(*d*) Hence calculate the size of angle OAB. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G16		1994 P2 Q3
(*b*)	1	C	CR	G25		
(*c*)	4	C	CR	G28		
(*d*)	2	C	CR	CGD		

(*a*) •1 One of B,C or D

•2 Remaining two of B, C and D

•3 B(6,4,2), C(4,3,4), D(6,2,2)

(*b*) •4 $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(*c*) •5 $\cos A\hat{O}B = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$ *or* $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•6 $\overrightarrow{OA}.\overrightarrow{OB} = 40$ *or* $AB^2 = 32$

•7 $OA = \sqrt{56} = OB$

•8 44°

(*d*) •9 strategy: e.g. use isosceles Δ

•10 68°

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[SQA] 9. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

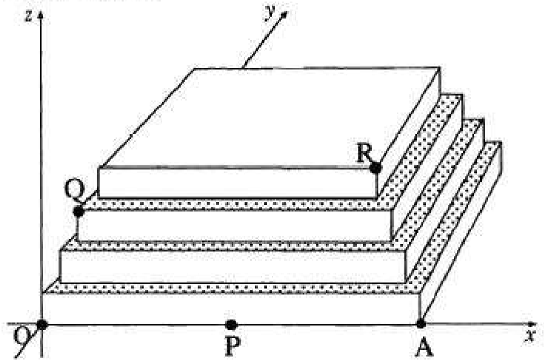

Diagram 1

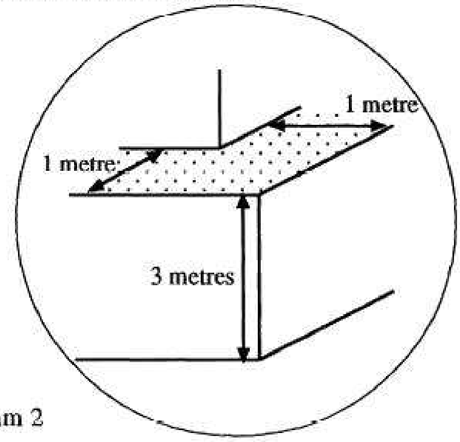

Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**
(*b*) Find the size of angle QPR. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1996 P2 Q3
(*b*)	7	C	CR	G28		

(a) •[1] $Q = (2,2,9)$

•[2] $R = (21,3,12)$

(b) •[3] $\cos\theta = \frac{a.b}{|a||b|}$ *with some subsequent use*

eg $\cos Q\hat{P}R = \frac{\vec{PQ}.\vec{PR}}{|\vec{PQ}||\vec{PR}|}$

•[4] $\vec{PQ} = \begin{pmatrix} -10 \\ 2 \\ 9 \end{pmatrix}$ •[5] $\vec{PR} = \begin{pmatrix} 9 \\ 3 \\ 12 \end{pmatrix}$

•[6] $|\vec{PQ}| = \sqrt{185}$

•[7] $|\vec{PR}| = \sqrt{234}$

•[8] $\vec{PQ}.\vec{PR} = 24$

•[9] $Q\hat{P}R = 83 \cdot 4^\circ$

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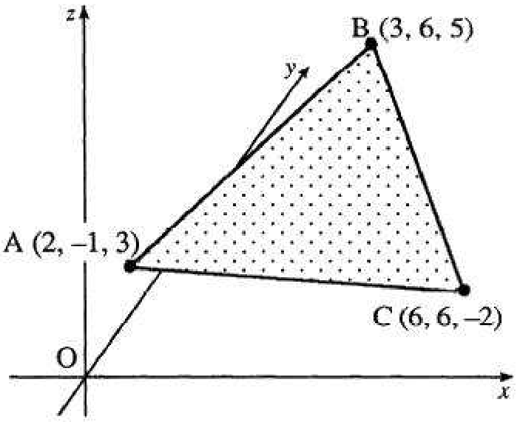
[SQA] 10. A triangle ABC has vertices
A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1998 P2 Q1
(*b*)	5	C	CR	G28		
(*c*)	2	C	CR	CGD		

(a) •1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•2 $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

(b) •3 $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •4 to •7*

•4 $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•5 $\overrightarrow{AB} = \sqrt{54}$

•6 $\overrightarrow{AC} = \sqrt{90}$

•7 $B\hat{A}C = 51 \cdot 9°$

(c) •8 **identify** 2 sides and included angle *e.g.* $\sqrt{54}$, $\sqrt{90}$, $B\hat{A}C$

•9 $27 \cdot 4$

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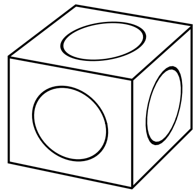
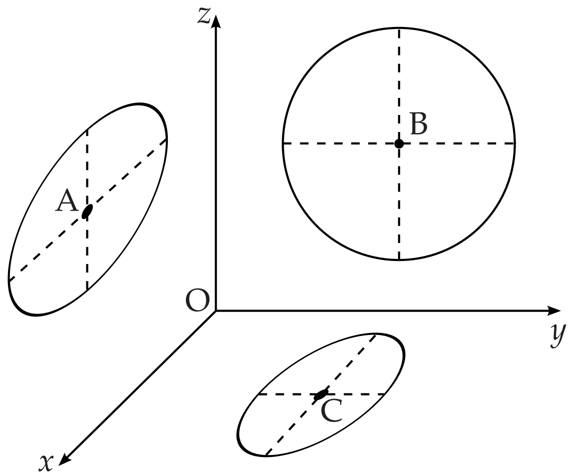
[SQA] 11. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6, 0, 7), B(0, 5, 6) and C(4, 5, 0).

Find the size of angle ABC. **7**





Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CR	G17, G16, G22		2001 P2 Q4
	2	A/B	CR	G26, G28	71·5°	

•1 ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$	•1 use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ *stated or implied by* •7
•2 ic: state vector e.g. $\overrightarrow{BA}$	•2 $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$
•3 ic: state a consistent vector e.g. $\overrightarrow{BC}$	•3 $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$
•4 pd: process $\lvert\overrightarrow{BA}\rvert$	•4 $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$
•5 pd: process $\lvert\overrightarrow{BC}\rvert$	•5 $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$
•6 pd: process scalar product	•6 $\overrightarrow{BA}.\overrightarrow{BC} = 18$
•7 pd: find angle	•7 $A\hat{B}C = 71{\cdot}5°$

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[SQA] 12. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows:

$$\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},\ \boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},\ \boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}.$$

(*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. **1**

(*b*) Find the value of $|2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}|$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G18		1989 P1 Q3
(*b*)	2	C	CN	G16		

•[1] $6i - 7j + 6k$

•[2] $\sqrt{6^2 + (-7)^2 + 6^2}$

•[3] 11

[SQA] 13. VABCD is a pyramid with rectangular base ABCD.

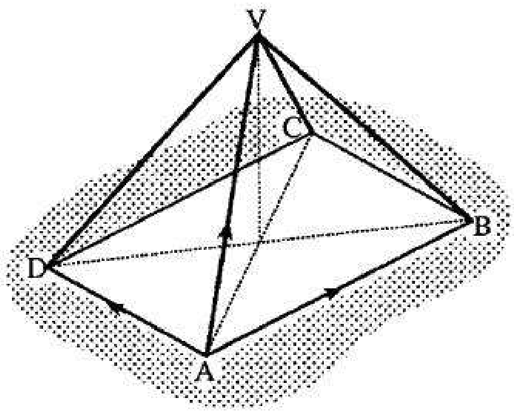
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8i + 2j + 2k$

$\overrightarrow{AD} = -2i + 10j - 2k$ and

$\overrightarrow{AV} = i + 7j + 7k.$

Express $\overrightarrow{CV}$ in component form.



3

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18		1999 P1 Q6

•[1] pathway for $\overrightarrow{CV}$: $\overrightarrow{CV} = \overrightarrow{CA} + \overrightarrow{AV}$

•[2] e.g. $\overrightarrow{CB} = 2i - 10j + 2k$

or $\overrightarrow{BA} = -8i - 2j - 2k$

or $\overrightarrow{AC} = 6i + 12j$

•[3] $\begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$

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[SQA] 14. The vector $a\boldsymbol{i} + b\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to both the vectors $\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $-2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$.

Find the values of a and b. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18	$a = 2, b = 3$	1990 P1 Q12

•[1] $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = a - b + 1$ *or* $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -2a + b + 1$

•[2] $a - b + 1 = 0$ *or* $-2a + b + 1 = 0$

•[3] $a = 2$ *and* $b = 3$

[SQA] 15. Calculate the length of the vector $2\boldsymbol{i} - 3\boldsymbol{j} + \sqrt{3}\boldsymbol{k}$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	C	CN	G18	4	1995 P1 Q1

•[1] $\sqrt{2^2 + (-3)^2 + (\sqrt{3})^2}$ stated or implied by •[2]

•[2] 4

[SQA] 16. Show that the vectors $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ are perpendicular. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18, G27	$\boldsymbol{a}.\boldsymbol{b} = \cdots = 0$	1991 P1 Q3

•[1] *strat:* $\boldsymbol{a}.\boldsymbol{b} = \ldots\ldots\ldots$

•[2] $\boldsymbol{a}.\boldsymbol{b} = 0 \Rightarrow$ perpendicularity explicitly stated

•[3] $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} . \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} = 6 - 3 - 3 = 0$

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[SQA] 17. The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. 2

(*b*) Find the length of PQ. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G18, G16		1997 P1 Q4
(*b*)	1	C	CN	G16		

•[1] $q - p = 8i - 4j + k$

or $p = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$, $q = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$

•[2] $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$

•[3] 9

[SQA] 18. The vectors *a*, *b* and *c* are defined as follows:

$$a = 2i - k, \quad b = i + 2j + k, \quad c = -j + k.$$

(*a*) Evaluate $a.b + a.c$. 3

(*b*) From your answer to part (*a*), make a deduction about the vector $b + c$. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CN	G18, G26		1993 P1 Q12
(*b*)	2	A/B	CN	G27		

•[1] $a = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, c = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

•[2] $a.b = 1$

•[3] $a.c = -1$

•[4] $a.b + a.c = a.(b + c)$

•[5] $a \perp b + c$

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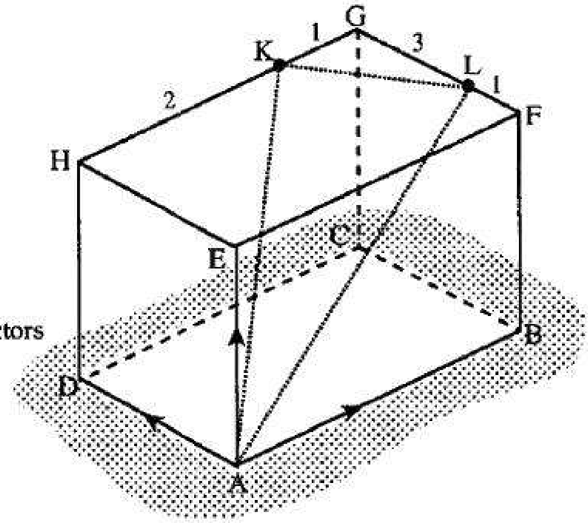
[SQA] 19. ABCDEFGH is a cuboid.

K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\vec{AB}$, $\vec{AD}$ and $\vec{AE}$ can be represented by the vectors $\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\begin{pmatrix}-8\\4\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-3\\5\end{pmatrix}$ respectively.



(*a*) Calculate the components of $\vec{AK}$. **2**

(*b*) Calculate the components of $\vec{AL}$. **2**

(c) Calculate the size of angle KAL. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G20		1999 P2 Q3
(*b*)	2	C	CN	G20		
(*c*)	5	C	CN	G28		

(*a*)
- •[1] obtaining for example $\begin{pmatrix}2\\4\\2\end{pmatrix}$
- •[2] $\vec{AK}=\begin{pmatrix}-5\\5\\11\end{pmatrix}$

(*b*)
- •[3] obtaining for example $\begin{pmatrix}-2\\1\\1\end{pmatrix}$
- •[4] $\vec{AL}=\begin{pmatrix}2\\4\\9\end{pmatrix}$

(*c*)
- •[5] *strategy e.g.* $\cos K\hat{A}L=\frac{\vec{AK}.\vec{AL}}{|AK|\times|AL|}$
- •[6] 109
- •[7] $\sqrt{171}$
- •[8] $\sqrt{101}$
- •[9] $\hat{A}=34.0$

OR

- •[5] *strategy e.g.* $\cos K\hat{A}L=\frac{AK^2+AL^2-KL^2}{2AK\times AL}$
- •[6] $\sqrt{54}$
- •[7] $\sqrt{171}$
- •[8] $\sqrt{101}$
- •[9] $\hat{A}=34.0$

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[SQA] 20. If $u = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, write down the components of $u + v$ and $u - v$.

Hence show that $u + v$ and $u - v$ are perpendicular. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G20, G27		1994 P1 Q7

•[1] $\mathbf{u}+\mathbf{v} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ and $\mathbf{u}-\mathbf{v} = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$

•[2] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v}) = 8 - 16 + 8$

•[3] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v}) = 0$ so $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are perpendicular

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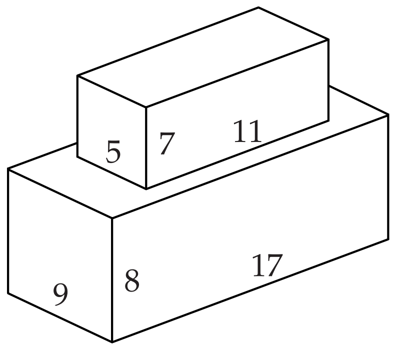
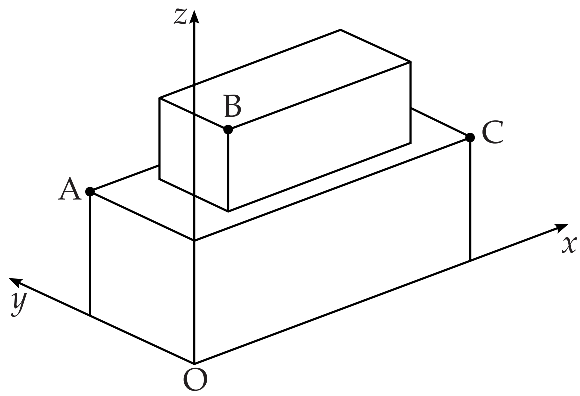
[SQA] 21. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.





(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$B(3, 2, 15)$	2000 P2 Q9
(*b*)	6	C	CR	G28	$92{\cdot}5°$	

•[1] ic: interpret 3-d representation	•[1] $B = (3, 2, 15)$ *treat* $\begin{pmatrix}3\\2\\15\end{pmatrix}$ *as bad form*
•[2] ss: know to use scalar product	•[2] $\cos A\hat{B}C = \dfrac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$
•[3] pd: process vectors	•[3] $\overrightarrow{BA} = \begin{pmatrix}-3\\7\\-7\end{pmatrix}$
•[4] pd: process vectors	•[4] $\overrightarrow{BC} = \begin{pmatrix}14\\-2\\-7\end{pmatrix}$
•[5] pd: process lengths	•[5] $\lvert\overrightarrow{BA}\rvert = \sqrt{107}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{249}$
•[6] pd: process scalar product	•[6] $\overrightarrow{BA}.\overrightarrow{BC} = -7$
•[7] pd: evaluate scalar product	•[7] $A\hat{B}C = 92{\cdot}5°$

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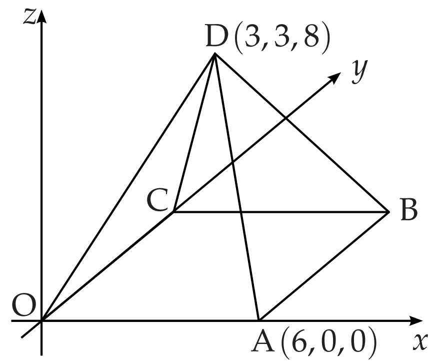
[SQA] 22. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are $(6,0,0)$ and $(3,3,8)$.

C lies on the y-axis.



(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$(6,6,0)$	2002 P2 Q2
(*b*)	2	C	CN	G17	$\overrightarrow{DA} = \begin{pmatrix}3\\-3\\-8\end{pmatrix}$, $\overrightarrow{DB} = \begin{pmatrix}3\\3\\-8\end{pmatrix}$	
(*c*)	4	C	CR	G28	$38{\cdot}7°$	

•[1] ic: interpret diagram	•[1] $B = (6,6,0)$
•[2] ic: write down components of a vector	•[2] $\overrightarrow{DA} = \begin{pmatrix}3\\-3\\-8\end{pmatrix}$
•[3] ic: write down components of a vector	•[3] $\overrightarrow{DB} = \begin{pmatrix}3\\3\\-8\end{pmatrix}$
•[4] ss: use e.g. scalar product formula	•[4] $\cos A\hat{D}B = \frac{\overrightarrow{DA}.\overrightarrow{DB}}{\lvert\overrightarrow{DA}\rvert\lvert\overrightarrow{DB}\rvert}$
•[5] pd: process lengths	•[5] $\lvert\overrightarrow{DA}\rvert = \sqrt{82}$, $\lvert\overrightarrow{DB}\rvert = \sqrt{82}$
•[6] pd: process scalar product	•[6] $\overrightarrow{DA}.\overrightarrow{DB} = 64$
•[7] pd: process angle	•[7] $A\hat{D}B = 38{\cdot}7°$

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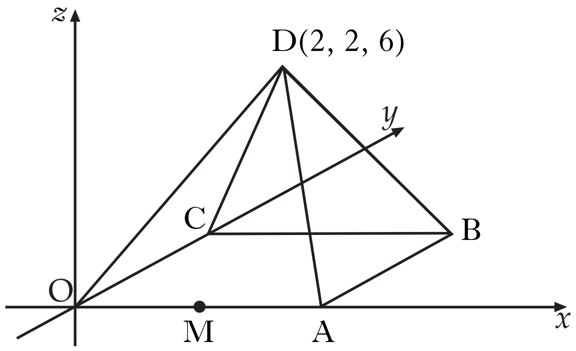
23. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point $(2, 2, 6)$ and OA $= 4$ units.

M is the mid-point of OA.

(*a*) State the coordinates of B. **1**

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(*c*) Find the size of angle BDM. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$(4, 4, 0)$	2011 P2 Q1
(*b*)	3	C	CN	G20, G22	$\overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}, \overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	
(*c*)	5	C	CN	G28	40·3°	

•[1] ic: state coordinates of B	•[1] $(4, 4, 0)$
•[2] pd: state components of $\overrightarrow{DB}$	•[2] $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$
•[3] ic: state coordinates of M	•[3] $(2, 0, 0)$
•[4] pd: state components of $\overrightarrow{DM}$	•[4] $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$
•[5] ss: know to use scalar product	•[5] $\cos BDM = \frac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert \lvert\overrightarrow{DM}\rvert}$
•[6] pd: find scalar product	•[6] $\overrightarrow{DB} \cdot \overrightarrow{DM} = 32$
•[7] pd: find magnitude of a vector	•[7] $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$
•[8] pd: find magnitude of a vector	•[8] $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$
•[9] pd: evaluate angle BDM	•[9] 40·3° or 0·703 rads

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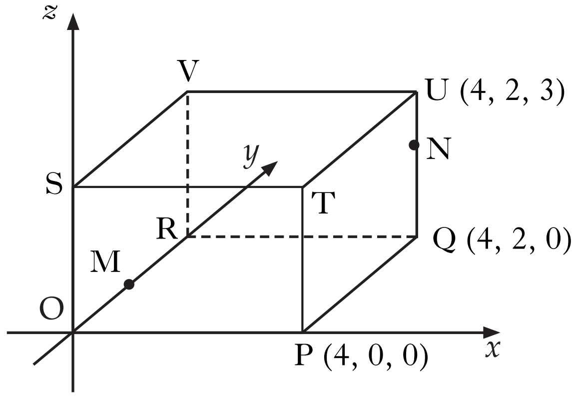
24. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4,0,0)$, Q is $(4,2,0)$ and U is $(4,2,3)$.

M is the midpoint of OR.

N is the point on UQ such that $\text{UN} = \frac{1}{3}\text{UQ}$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express the vectors $\overrightarrow{\text{VM}}$ and $\overrightarrow{\text{VN}}$ in component form. **2**

(*c*) Calculate the size of angle MVN. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G22, G25	$M(0,1,0), N(4,2,2)$	2010 P2 Q1
(*b*)	2	C	CN	G17	$\overrightarrow{\text{VM}} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \overrightarrow{\text{VN}} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$	
(*c*)	5	C	CN	G28	$76{\cdot}7°$ or $1{\cdot}339$ rad	

•[1] ic: interpret midpoint for M	•[1] $(0,1,0)$
•[2] ic: interpret ratio for N	•[2] $(4,2,2)$
•[3] ic: interpret diagram	•[3] $\overrightarrow{\text{VM}} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$
•[4] pd: process vectors	•[4] $\overrightarrow{\text{VN}} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$
•[5] ss: know to use scalar product	•[5] $\cos \text{M}\hat{\text{V}}\text{N} = \dfrac{\overrightarrow{\text{VM}}.\overrightarrow{\text{VN}}}{\lvert\overrightarrow{\text{VM}}\rvert\lvert\overrightarrow{\text{VN}}\rvert}$
•[6] pd: find scalar product	•[6] $\overrightarrow{\text{VM}}.\overrightarrow{\text{VN}} = 3$
•[7] pd: find magnitude of a vector	•[7] $\lvert\overrightarrow{\text{VM}}\rvert = \sqrt{10}$
•[8] pd: find magnitude of a vector	•[8] $\lvert\overrightarrow{\text{VN}}\rvert = \sqrt{17}$
•[9] pd: evaluate angle	•[9] $76{\cdot}7°$ or $1{\cdot}339$ rads or $85{\cdot}2$ grads

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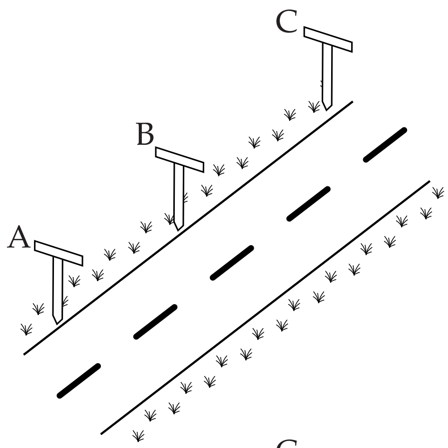
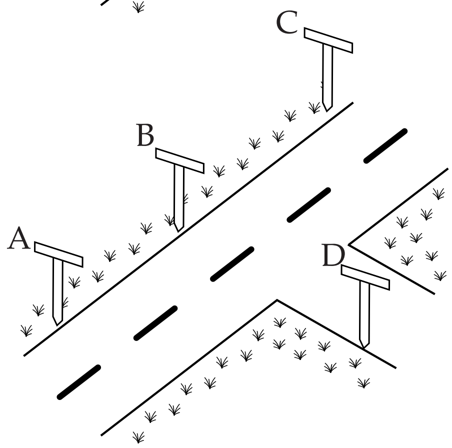
[SQA] 25. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8,-10,-2)$, $B(-2,-1,1)$ and $C(6,11,5)$.

Determine whether or not the section of road ABC has been built in a straight line. **3**



(*b*) A further T-rod is placed such that D has coordinates $(1,-4,4)$.

Show that DB is perpendicular to AB. **3**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CN	G23	the road ABC is straight	2001 P1 Q3
(*b*)	3	C	CN	G27, G17	proof	

•[1] ic: interpret vector (e.g. $\overrightarrow{AB}$) •[2] ic: interpret multiple of vector •[3] ic: complete proof •[4] ic: interpret vector (i.e. $\overrightarrow{BD}$) •[5] ss: state requirement for perpend. •[6] ic: complete proof	•[1] e.g. $\overrightarrow{AB} = \begin{pmatrix}6\\9\\3\end{pmatrix}$ •[2] e.g. $\overrightarrow{BC} = \begin{pmatrix}8\\12\\4\end{pmatrix} = \frac{4}{3}\overrightarrow{AB}$ **or** $\overrightarrow{AB} = 3\begin{pmatrix}2\\3\\1\end{pmatrix}$ and $\overrightarrow{BC} = 4\begin{pmatrix}2\\3\\1\end{pmatrix}$ •[3] a common direction exists **and** a common point exists, so A, B, C collinear •[4] $\overrightarrow{BD} = \begin{pmatrix}3\\-3\\3\end{pmatrix}$ •[5] $\overrightarrow{AB}.\overrightarrow{BD} = 0$ •[6] $\overrightarrow{AB}.\overrightarrow{BD} = 18-27+9=0$ **or** •[5] $\overrightarrow{AB}.\overrightarrow{BD} = 18-27+9$ •[6] $\overrightarrow{AB}.\overrightarrow{BD} = 0$ so AB is at right angles to BD

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[SQA] 26. (a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. 4

(b) Find the ration in which M divides LN. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G23		1991 P1 Q7
(*b*)	1	C	CN	G25		

•[1] $\overrightarrow{LM} = \begin{pmatrix} 12 \\ -8 \\ 4 \end{pmatrix}$ or equivalent combinations for (a)

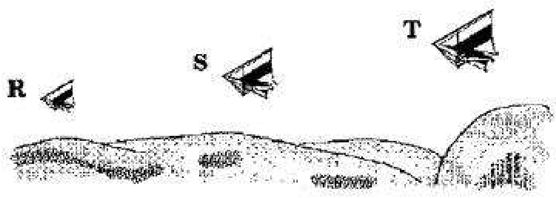
•[2] $\overrightarrow{MN} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

•[3] $\overrightarrow{LM} = 4\overrightarrow{MN}$

•[4] vectors are parallel and have common point so L, M, N are collinear

•[5] 4:1

[SQA] 27. Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6). Prove that R, S and T are collinear. 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G23		1994 P1 Q4

•[1] $\overrightarrow{ST} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ or equivalent **and** $\overrightarrow{RS} = \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$ or equivalent

•[2] $\overrightarrow{RS} = 3\overrightarrow{ST}$ or equiv.

•[3] RS // ST **and** S is common.

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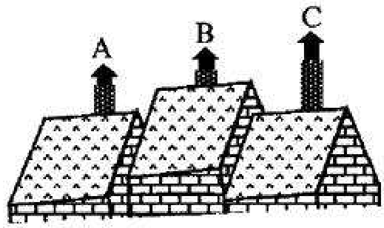
[SQA] 28. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, −1, 4) and C(4, −9, 8).
Show that A, B and C are collinear. **3**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G23		1997 P1 Q2

•[1] $\overrightarrow{AB}=\begin{pmatrix}1\\-4\\2\end{pmatrix}$

•[2] $\overrightarrow{BC}=\begin{pmatrix}2\\-8\\4\end{pmatrix}$ AND $\overrightarrow{BC}=2\times\overrightarrow{AB}$

•[3] $\overrightarrow{AB}\,||\,\overrightarrow{BC}$ & B is common hence A, B, C collinear

[SQA] 29. Show that $P(2,2,3)$, $Q(4,4,1)$ and $R(5,5,0)$ are collinear and find the ratio in which Q divides PR. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	C	CN	G23, G25	$\overrightarrow{QR}=\frac{1}{2}\overrightarrow{PQ}$, $PQ:QR=2:1$	1990 P1 Q4

•[1] $\overrightarrow{PQ}=\begin{pmatrix}2\\2\\-2\end{pmatrix}$

•[2] $\overrightarrow{QR}=\begin{pmatrix}1\\1\\-1\end{pmatrix}=\frac{1}{2}\overrightarrow{PQ}$ or equivalent

•[3] vectors parallel and have pt in common so pts collinear

•[4] $PQ:QR=2:1$

[SQA] 30. A is the point $(2,-5,6)$, B is $(6,-3,4)$ and C is $(12,0,1)$. Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	4	C	CN	G23, G25	2 : 3 or equivalent	1996 P1 Q6

•[1] $\overrightarrow{AB}=\begin{pmatrix}4\\2\\-2\end{pmatrix}$ *or* $\overrightarrow{AC}=\begin{pmatrix}10\\5\\-5\end{pmatrix}$ *or* $\overrightarrow{BC}=\begin{pmatrix}6\\3\\-3\end{pmatrix}$

•[2] $\overrightarrow{AB}=2\begin{pmatrix}2\\1\\-1\end{pmatrix}$ *and* $\overrightarrow{BC}=3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or equivalent

•[3] $AB\,||\,BC$ and B is point in common

•[4] 2:3 (*or* equivalent e.g. $1:1\frac{1}{2}$)

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[SQA] 31. D, E and F have coordinates $(10, -8, -15)$, $(1, -2, -3)$ and $(-2, 0, 1)$ respectively.

(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

(*b*) G has coordinates $(k, 1, 0)$.

Given that DE is perpendicular to GE, find the value of k. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G23, G24	3 : 1	2009 P1 Q22
(*b*)	4	C	CN	G27	$k = 7$	

•[1] ss: use vector approach •[2] ic: compare two vectors •[3] ic: complete proof •[4] ic: state ratio •[5] ss: use vector approach •[6] ss: know scalar product = 0 for ⊥ vectors •[7] pd: start to solve •[8] pd: complete	•[1] $\overrightarrow{DE} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$ or $\overrightarrow{EF} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$ •[2] 2nd column vector and $(\overrightarrow{DE}) = 3\overrightarrow{EF}$ •[3] $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have common point and common direction; hence D, E and F are collinear •[4] 3 : 1 •[5] $\overrightarrow{GE} = \begin{pmatrix} 1-k \\ -3 \\ -3 \end{pmatrix}$ •[6] $\overrightarrow{DE}.\overrightarrow{GE} = 0$ •[7] $-9(1-k) + 6 \times (-3) + 12 \times (-3)$ •[8] $k = 7$

[SQA] 32. The point Q divides the line joining $P(-1, -1, 0)$ to $R(5, 2, -3)$ in the ratio $2 : 1$.

Find the coordinates of Q. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	NC	G25	$(3, 1, -2)$	2002 P1 Q2

•[1] pd: find vector components •[2] ss: use parallel vectors •[3] pd: process vectors	•[1] $\overrightarrow{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ •[2] $\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{PR}$ •[3] $Q = (3, 1, -2)$

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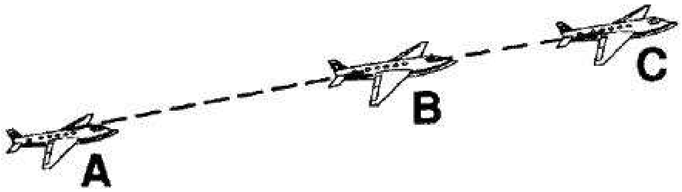
[SQA] 33. An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point (–1, 3, 4) and B is the point (3, 1, –2). Find the co-ordinates of the point C. 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G25		1992 P1 Q15

•[1] $\vec{AB} = \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$

•[2] $\vec{BC} = \vec{AB}$

•[3] $(5, 0, -5)$

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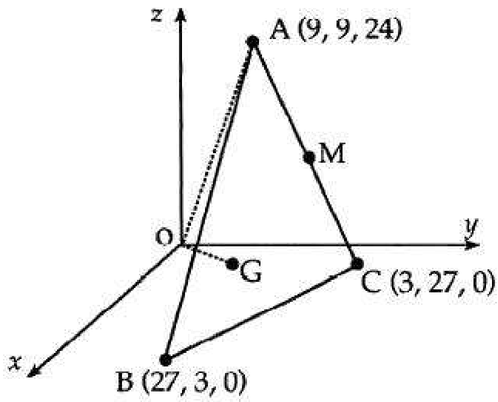
All others © Higher Still Notes

[SQA] 34. (*a*) Relative to mutually perpendicular axes Ox, Oy and Oz, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.

Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G25		1990 P2 Q4
(*b*)	5	C	CR	G28		

(*a*) •[1] M = (6, 18, 12)

•[2] e.g. $\overrightarrow{BG} = \frac{2}{3}\begin{pmatrix}-21\\15\\12\end{pmatrix}$

•[3] G = (13, 13, 8)

(*b*) •[4] $\cos A\hat{O}G = \dfrac{\overrightarrow{OA}.\overrightarrow{OG}}{|\overrightarrow{OA}|\,|\overrightarrow{OG}|}$

•[5] $\overrightarrow{OA} = \begin{pmatrix}9\\9\\24\end{pmatrix}$ and $\overrightarrow{OG} = \begin{pmatrix}13\\13\\8\end{pmatrix}$

•[6] $\overrightarrow{OA}.\overrightarrow{OG} = 426$

•[7] $|\overrightarrow{OA}| = \sqrt{738}$ and $|\overrightarrow{OG}| = \sqrt{402}$

•[8] 38.5°

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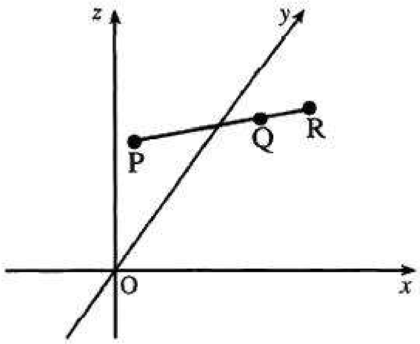
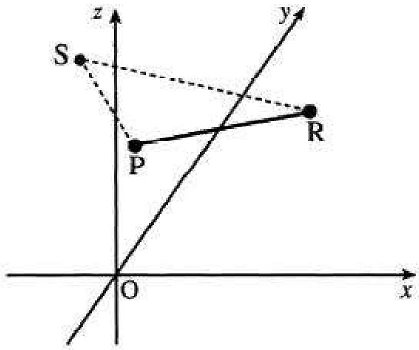
[SQA] 35. Relative to the axes shown and with an appropriate scale, P(−1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**

(*b*) Roads from P and R are built to meet at the point S (−2, 2, 5). Calculate the size of angle PSR. **(7)**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G25		1997 P2 Q2
(*b*)	7	C	CR	G28		

(a)

•1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•3 $R = (7, -1, 6)$

(b)

•4 $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

•5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•7 $|SP| = \sqrt{11}$ •8 $|SR| = \sqrt{91}$

•9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•10 $P\hat{S}R = 84 \cdot 6°$

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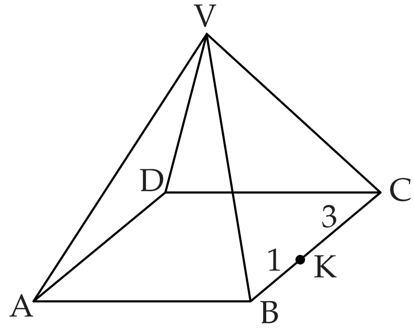
[SQA] 36. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.



K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G25, G21, G20	$\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$	2000 P1 Q7

•[1] ss: recognise crucial aspect •[2] ic: interpret ratio •[3] pd: process components	•[1] $\overrightarrow{VK} = \overrightarrow{VA} + \overrightarrow{AB} + \overrightarrow{BK}$ **or** $\overrightarrow{VK} = \overrightarrow{VB} + \overrightarrow{BK}$ •[2] $\overrightarrow{BK} = \frac{1}{4}\overrightarrow{BC}$ **or** $\frac{1}{4}\overrightarrow{AD}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ **or** $\begin{pmatrix} -1 \\ -7 \\ -17 \end{pmatrix}$ •[3] $\overrightarrow{VK} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$

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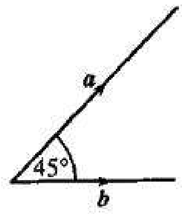
[SQA] 37. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$. These vectors are inclined at an angle of 45° to each other.



(*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$ (ii) $\boldsymbol{b}.\boldsymbol{b}$ (iii) $\boldsymbol{a}.\boldsymbol{b}$ **2**

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$. Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $|\boldsymbol{p}|$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G26		1999 P1 Q17
(*b*)	4	A/B	CN	G29, G30		

•1 $a.a = 9$ and $b.b = 8$

•2 $a.b = 6$

•3 $(2a + 3b).(2a + 3b)$

•4 $4a.a + 9b.b + 12a.b$

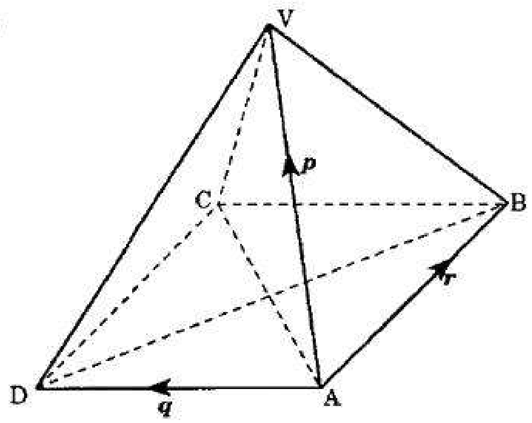
•5 180

•6 $\sqrt{180}$

[SQA] 38. In the square-based pyramid, all the eight edges are of length 3 units.

$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$.

Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$. **4**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	CN	G26		1995 P1 Q16
	3	A/B	CN	G29, G26		

•1 $\boldsymbol{p}.\boldsymbol{q} + \boldsymbol{p}.\boldsymbol{r}$

•2 $V\hat{A}D = 60^\circ$ or equiv.

•3 $|\boldsymbol{p}||\boldsymbol{q}|\cos V\hat{A}D + |\boldsymbol{p}||\boldsymbol{r}|\cos V\hat{A}B$

•4 9

•1 $r = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, q = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$

•2 $p = \begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ \frac{3}{\sqrt{2}} \end{pmatrix}$

•3 $\left(-\frac{3}{2}\right) \times (-3) + \left(\frac{3}{2}\right) \times 3 + \frac{3}{\sqrt{2}} \times 0$

•4 9

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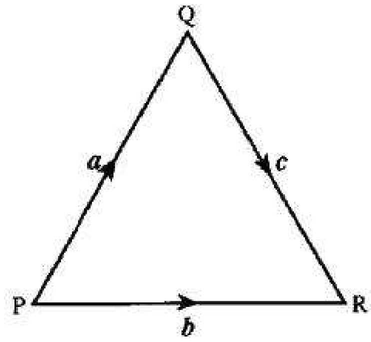
[SQA] 39. PQR is an equilateral triangle of side 2 units.

$\vec{PQ} = \boldsymbol{a}$, $\vec{PR} = \boldsymbol{b}$ and $\vec{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. **4**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	CN	G26		1997 P1 Q13
	3	A/B	CN	G29, G27		

- •1 $\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$
- •2 $\boldsymbol{a}.\boldsymbol{b} = 2\times2\times\frac{1}{2}$
- •3 $\boldsymbol{a}.\boldsymbol{c} = 2\times2\times-\frac{1}{2}$
- •4 0 and $\boldsymbol{a}$ is perpendicular to $(\boldsymbol{b}+\boldsymbol{c})$

[SQA] 40. For what value of t are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	C	CN	G27	$t = 4$	2000 P2 Q7

•1 ss: know to use scalar product •2 ic: interpret scalar product	•1 $\boldsymbol{u}.\boldsymbol{v} = 2t - 20 + 3t$ •2 $\boldsymbol{u}.\boldsymbol{v} = 0 \Rightarrow t = 4$

[SQA] 41. A$(4,4,10)$, B$(-2,-4,12)$ and C$(-8,0,10)$ are the vertices of a right-angled triangle.

Determine which angle of the triangle is the right angle. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G27		1989 P1 Q6

- •1 $\vec{AB} = \begin{pmatrix} -6 \\ -8 \\ 2 \end{pmatrix}, \vec{BC} = \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}, \vec{AC} = \begin{pmatrix} -12 \\ -4 \\ 0 \end{pmatrix}$
- •2 $\left|\vec{AC}\right|$ is longest so $\vec{AB}.\vec{CB} = -36+32+4=0$
- •3 $A\hat{B}C = 90°$

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[SQA] 42. Find the value of k for which the vectors $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix}$ are perpendicular. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G27	$k = 3$	1995 P1 Q4

•1 $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} . \begin{pmatrix} -4 \\ 3 \\ k-1 \end{pmatrix} = 0$

•2 $1 \times -4 + 2 \times 3 + -1(k-1)$

•3 3

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[SQA] 43. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

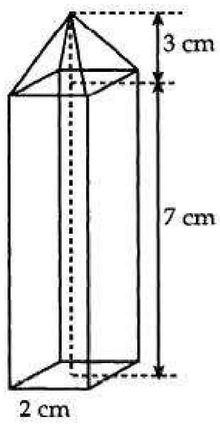

Diagram 1

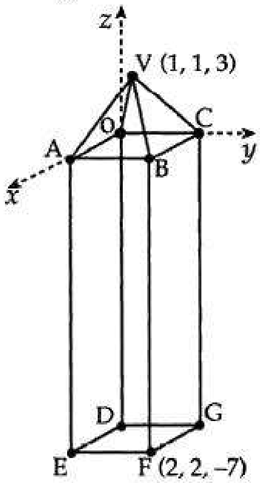

Diagram 2

(*a*) Find
  (i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;
  (ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.
Calculate the area of the glass triangle VEF. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	7	C	CR	G28, G16		1991 P2 Q5
(*b*)	3	C	CR	CGD		

(*a*)
•1 $\overrightarrow{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•2 $E = (2, 0, -7)$

•3 $\overrightarrow{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•4 $\cos E\hat{V}F = \frac{\overrightarrow{VE}.\overrightarrow{VF}}{|\overrightarrow{VE}||\overrightarrow{VF}|}$ This may appear as $\frac{100}{102}$ after the completion of •5 and •6.

•5 $\overrightarrow{VE}.\overrightarrow{VF} = 100$

•6 $|\overrightarrow{VE}||\overrightarrow{VF}| = 102$

•7 $11.4°$

(*b*)
•8 $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•9 $\frac{1}{2} \times 102 \times \sin 11.4°$

•10 10.02

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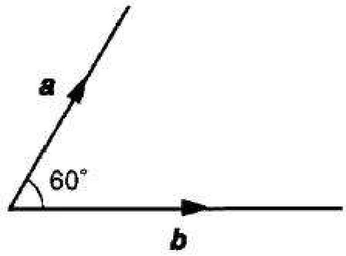
[SQA] 44. The diagram shows representatives of two vectors, a and b, inclined at an angle of 60°.

If $|a| = 2$ and $|b| = 3$, evaluate $a.(a+b)$ 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G29, G26		1992 P1 Q18

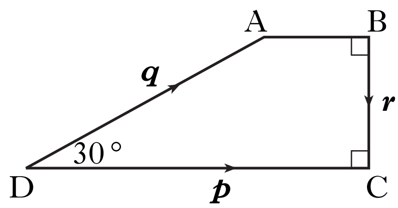
•[1] $a.a + a.b$
•[2] $2 \times 3 \times \cos 60°$
•[3] 4

[SQA] 45. Vectors p, q and r are represented on the diagram shown where angle ADC = 30°.

It is also given that $|p| = 4$ and $|q| = 3$.

(*a*) Evaluate $p.(q+r)$ and $r.(p-q)$. 6

(*b*) Find $|q+r|$ and $|p-q|$. 4



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	6	B	CN	G29, G26	$6\sqrt{3}, \frac{9}{4}$	2009 P2 Q7
(*b*)	2	A	CR	G21, G30	$\lvert q+r\rvert = \frac{3\sqrt{3}}{2}$	
(*b*)	2	B	CR	G21, G30	$\lvert p-q\rvert = \sqrt{(4-\frac{3\sqrt{3}}{2})^2 + (\frac{3}{2})^2}$	

•[1] ss: use distributive law	•[1] $p.q + p.r$
•[2] ic: interpret scalar product	•[2] $4 \times 3 \cos 30°$
•[3] pd: processing scalar product	•[3] $6\sqrt{3} (\approx 10{\cdot}4)$
•[4] ic: interpret perpendicularity	•[4] $p.r = 0$
•[5] ic: interpret scalar product	•[5] $-\lvert r\rvert \times 3 \cos 120°$
•[6] pd: complete processing	•[6] $r = \frac{3}{2}$ and $\frac{9}{4}$
•[7] ic: interpret vectors on a 2-D diagram	•[7] $q + r \equiv$ from D to the proj. of A onto DC
•[8] pd: evaluate magnitude of vector sum	•[8] $\lvert q+r\rvert = \frac{3\sqrt{3}}{2}$
•[9] ic: interpret vectors on a 2-D diagram	•[9] $p - q = \overrightarrow{AC}$
•[10] pd: evaluate magnitude of vector difference	•[10] $\lvert p-q\rvert = \sqrt{(4-\frac{3\sqrt{3}}{2})^2 + (\frac{3}{2})^2} (\approx 2{\cdot}05)$

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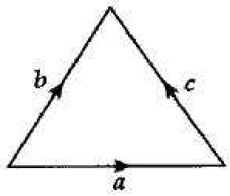
[SQA] 46. The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown. Evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$. 5



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	NC	A6		1989 P1 Q9
	4	A/B	NC	G29, G26		

•[1] $\boldsymbol{a}.\boldsymbol{a}+\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$

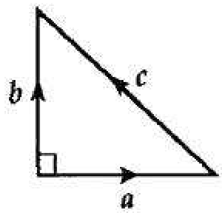
•[2] $\boldsymbol{a}.\boldsymbol{a}=|\boldsymbol{a}||\boldsymbol{a}|\cos 0$

•[3] $\boldsymbol{a}.\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos 60$

•[4] $\boldsymbol{a}.\boldsymbol{c}=|\boldsymbol{a}||\boldsymbol{c}|\cos 120$

•[5] 4

[SQA] 47. The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. The two equal sides have length 2 units. Find the value of $\boldsymbol{b}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$ 5



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CN	G29, G27		1991 P1 Q17

•[1] $\boldsymbol{b}.\boldsymbol{a}+\boldsymbol{b}.\boldsymbol{b}+\boldsymbol{b}.\boldsymbol{c}$

•[2] $\boldsymbol{b}.\boldsymbol{a}=0$

•[3] $\boldsymbol{b}.\boldsymbol{b}=4$

•[4] $|\boldsymbol{c}|=2\sqrt{2}$

•[5] $\boldsymbol{b}.\boldsymbol{c}=4$

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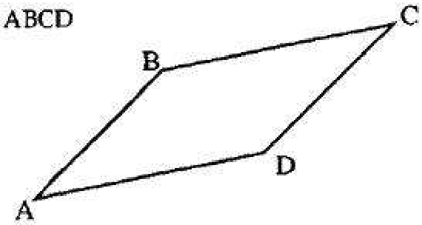
[SQA] 48. A is the point $(2,-1,4)$, B is $(7,1,3)$ and C is $(-6,4,2)$. If ABCD is a parallelogram, find the coordinates of D. **3**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G30		1994 P1 Q3

•1 $\vec{OD} = \vec{OA} + \vec{AD}$ or equivalent, stated or implied by •3

•2 $\vec{BC} = \begin{pmatrix} -13 \\ 3 \\ -1 \end{pmatrix}$ *or* $\vec{CB}$ or $\vec{AB}$ or $\vec{BA}$

•3 D= (−11, 2, 3)

OR

•1 $\vec{OD} = \vec{OM} + \vec{MD}$, M is midpoint of AC

•2 $\vec{BM} = \begin{pmatrix} -9 \\ \frac{1}{2} \\ 0 \end{pmatrix}$

•3 D = (−11, 2, 3)

[SQA] 49. PQRS is a parallelogram with vertices $P(1,3,3)$, $Q(4,-2,-2)$ and $R(3,1,1)$.
Find the coordinates of S. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G30		1989 P1 Q4

•1 $\vec{QP} = \begin{pmatrix} -3 \\ 5 \\ 5 \end{pmatrix}$

•2 $R = (3,1,1)$ and $\vec{RS} = \begin{pmatrix} -3 \\ 5 \\ 5 \end{pmatrix}$ stated or implied by •3

•3 $S = (0,6,6)$

[END OF QUESTIONS]

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GCC Basic Differentiation

[SQA] 1. If $y = x^2 - x$, show that $\frac{dy}{dx} = 1 + \frac{2y}{x}$. **3**

[SQA] 2. Given $f(x) = 3x^2(2x-1)$, find $f'(-1)$. **3**

[SQA] 3. Find $\frac{dy}{dx}$ where $y = \frac{4}{x^2} + x\sqrt{x}$. **4**

[SQA] 4. Find $f'(4)$ where $f(x) = \frac{x-1}{\sqrt{x}}$. **5**

[SQA] 5. Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$. **3**

[SQA] 6. Differentiate $2\sqrt{x}(x+2)$ with respect to x. **4**

[SQA] 7. Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$. **4**

[SQA] 8. The point P$(-1, 7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

[SQA] 9. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **4**

[SQA] 10. A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

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[SQA] 11. A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$.

(*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). **3**

(*b*) Find the speed of the ball after 2 seconds.

Explain your answer in terms of the movement of the ball. **2**

[SQA] 12. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

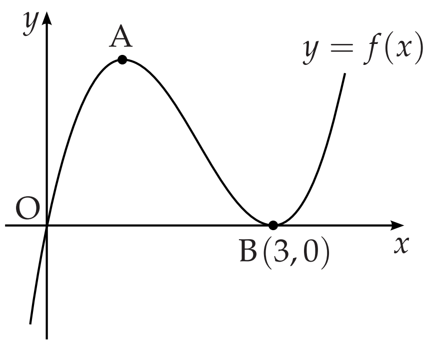
(*a*) Find the speed of the ball after 1 second. **3**

(*b*) For how many seconds is the ball travelling upwards? **2**

[SQA] 13. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing? **5**

[SQA] 14. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

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[SQA] 15. A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbb{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **3**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **7**

(*c*) Sketch the curve $y = f(x)$. **2**

[SQA] 16. A curve has equation $y = x^4 - 4x^3 + 3$.

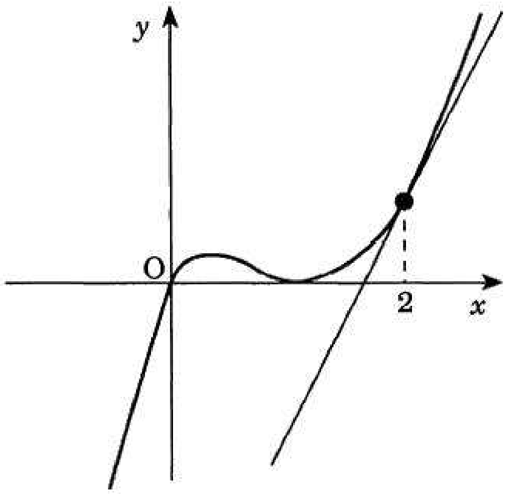
(*a*) Find algebraically the coordinates of the stationary points. **6**

(*b*) Determine the nature of the stationary points. **2**

[SQA] 17. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

[SQA] 18. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(*a*) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(*b*) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

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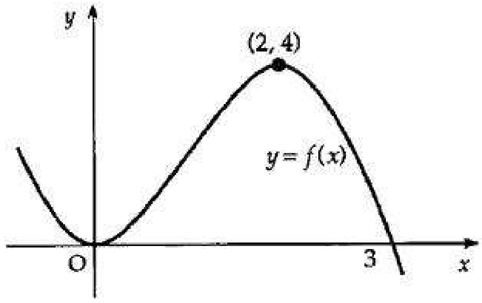
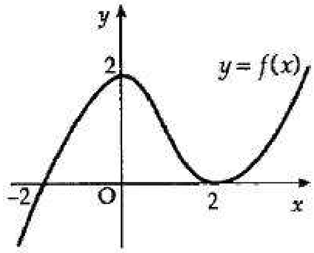
[SQA] 19. The diagram shows a sketch of a cubic function f with stationary points at $(0, 0)$ and $(2, 4)$.
Sketch the graph of the derived function f'. 3



[SQA] 20. The diagram shows the graph of $y = f(x)$, where $-2 \leq x \leq 3$.
On separate diagrams, sketch the graphs of

(a) $y = -f(x)$; 2

(b) $y = f'(x)$. 3



[END OF QUESTIONS]

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GCC Basic Differentiation

[SQA] 1. If $y = x^2 - x$, show that $\frac{dy}{dx} = 1 + \frac{2y}{x}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	1	C	NC	C1		1989 P1 Q12
	2	A/B	NC	A6, CGD		

•[1] $\frac{dy}{dx} = 2x - 1$

•[2] $RHS = 1 + \frac{2\left(x^2 - x\right)}{x}$

•[3] $1 + 2(x-1)$ and complete

[SQA] 2. Given $f(x) = 3x^2(2x-1)$, find $f'(-1)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	NC	C1		1999 P1 Q5

•[1] $6x^3 - 3x^2$

•[2] $18x^2 - 6x$

•[3] 24

[SQA] 3. Find $\frac{dy}{dx}$ where $y = \frac{4}{x^2} + x\sqrt{x}$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C1		1995 P1 Q7

•[1] $4x^{-2}$ stated or implied by •[3]

•[2] $+x^{\frac{3}{2}}$ stated or implied by •[4]

•[3] $-8x^{-3}$

•[4] $+\frac{3}{2}x^{\frac{1}{2}}$

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[SQA] 4. Find $f'(4)$ where $f(x) = \dfrac{x-1}{\sqrt{x}}$. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	C1		1996 P1 Q9

•1 $\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$ or $x \times x^{-\frac{1}{2}} - 1 \times x^{-\frac{1}{2}}$

•2 $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

•3 $\frac{1}{2}x^{-\frac{1}{2}}$

•4 $\frac{1}{2}x^{-\frac{3}{2}}$

•5 $\frac{5}{16}$

[SQA] 5. Given that $y = 2x^2 + x$, find $\dfrac{dy}{dx}$ and hence show that $x\left(1 + \dfrac{dy}{dx}\right) = 2y$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	NC	C1		1997 P1 Q8

•1 $\frac{dy}{dx} = 4x + 1$

•2 $LHS = x(1 + 4x + 1)$ or $RHS = 2\left(2x^2 + x\right)$

•3 completes proof

[SQA] 6. Differentiate $2\sqrt{x}(x+2)$ with respect to x. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C1		1998 P1 Q14

•1 know to expand

•2 $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$

•3 $3x^{\frac{1}{2}}$

•4 $2x^{-\frac{1}{2}}$

[SQA] 7. Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G2		1989 P1 Q13

•1 $\frac{dy}{dx} = 3x^2 - 4$

•2 $\frac{dy}{dx}_{x=2} = 8$

•3 $\tan\theta = 8$

•4 $83°$

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[SQA] 8. The point $P(-1,7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1999 P1 Q9

•1 $\frac{dy}{dx} = \ldots\ldots$
•2 $10x$
•3 -10
•4 $y - 7 = -10(x - (-1))$

[SQA] 9. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1992 P1 Q1

•1 $y' = 15x^2 - 12x$
•2 $y'(1) = 3$
•3 $y(1) = -1$
•4 $y - (-1) = 3(x - 1)$

[SQA] 10. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	C	CN	C4, C5	$y = 2x - 12$	2001 P2 Q2

•1 ic: find corresponding y-coord.	•1 $(4, -4)$ *stated or implied by* •6
•2 ss: express in standard form	•2 $-16x^{-\frac{1}{2}}$
•3 ss: start to differentiate	•3 $\frac{dy}{dx} = 1\ldots$
•4 pd: diff. fractional negative power	•4 $\ldots + 8x^{-\frac{3}{2}}$
•5 ss: find gradient of tangent	•5 $m_{x=4} = 2$
•6 ic: write down equ. of tangent	•6 $y - (-4) = 2(x - 4)$

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[SQA] 11. A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$.

(*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). **3**

(*b*) Find the speed of the ball after 2 seconds.

Explain your answer in terms of the movement of the ball. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	1	C	NC	C6		1995 P1 Q21
(*a*)	2	A/B	NC	C6		
(*b*)	2	A/B	NC	A6		

•[1] knows to differentiate
•[2] $20 - 10t$
•[3] 20
•[4] speed = 0
•[5] ball stationary at top of flight

[SQA] 12. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

(*a*) Find the speed of the ball after 1 second. **3**

(*b*) For how many seconds is the ball travelling upwards? **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	1	C	CN	C6, C6		1998 P1 Q17
(*a*)	2	A/B	CN	C6, C6		
(*b*)	2	A/B	CN	C6, C6		

•[1] $\frac{dh}{dt} = \ldots\ldots$
•[2] $19.6 - 9.8t$
•[3] 9.8
•[4] $\frac{dh}{dt} = 0$
•[5] $t = 2$

Alternative

•[4] $h(t)$ is a parabola which is **symmetric** about its **maximum**
•[5] (*e.g.*) $h(1) = 15{\cdot}9$, $h(2) = 20.8$, $h(3) = 15{\cdot}9$ so $t = 2$

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[SQA] 13. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing? **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C7		1990 P1 Q16
	3	A/B	NC	C7		

- •[1] $f'(x) = x^2 - 4x - 5$
- •[2] *use* $f'(x) > 0$
- •[3] zeros at $x = 5$ **and** $x = -1$
- •[4] *strat.* e.g. for $-1 < x < 5$ test $x = 0$
- •[5] $x < -1,\ x > 5$

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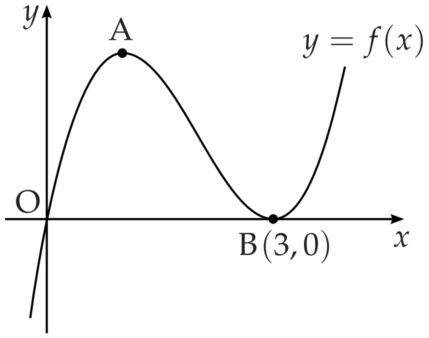
[SQA] 14. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x+2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	$A(1,4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•1 ss: know to differentiate •2 pd: differentiate correctly •3 ss: know gradient = 0 •4 pd: process •5 ic: interpret transformation •6 ic: interpret transformation •7 ic: interpret sketch	•1 $\frac{dy}{dx} = \ldots$ •2 $\frac{dy}{dx} = 3x^2 - 12x + 9$ •3 $3x^2 - 12x + 9 = 0$ •4 $A = (1, 4)$ translate $f(x)$ 4 units up, 2 units left •5 sketch with coord. of $A'(-1, 8)$ •6 sketch with coord. of $B'(1, 4)$ •7 $4 < k < 8$ (accept $4 \leq k \leq 8$)

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[SQA] 15. A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbb{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **3**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **7**

(*c*) Sketch the curve $y = f(x)$. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	C	NC	A6		1990 P2 Q1
(*b*)	7	C	NC	C8		
(*c*)	2	C	NC	C10		

(*a*) $\bullet^1$ $x = 1, -2$

$\bullet^2$ $(1,0)$ and $(-2,0)$

$\bullet^3$ $(0,2)$

(*b*) $\bullet^4$ $f(x) = x^3 - 3x + 2$

$\bullet^5$ $f'(x) = 3x^2 - 3$

$\bullet^6$ $f'(x) = 0$ **stated explicitly**

$\bullet^7$ $x = 1$ and -1

$\bullet^8$

x	-1^-	-1	-1^+	1^-	1	1^+
$f'(x)$	$+$	0	$-$	$-$	0	$+$

$\bullet^9$ max at $(-1, 4)$

$\bullet^{10}$ min at $(1,0)$

(*c*) $\bullet^{11}$ correct shape of sketch

$\bullet^{12}$ correct annotation of sketch(max, min, 2 axes intersections)

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[SQA] 16. A curve has equation $y = x^4 - 4x^3 + 3$.

(*a*) Find algebraically the coordinates of the stationary points. **6**

(*b*) Determine the nature of the stationary points. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	6	C	NC	C8		1996 P2 Q1
(*b*)	2	C	NC	C8		

(a)

- •1 $\frac{dy}{dx} =$
- •2 $4x^3 - 12x^2$
- •3 $= 0$ stated explicitly
- •4 *e.g.* $4x^2(x-3)$
- •5 $x = 0,\ 3$
- •6 $y = 3,\ -24$

(b)

•7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0		0	$+$

•8 pt of inflection at $x = 0$
minimum at $x = 3$

[SQA] 17. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C8, C7		1999 P1 Q16
	3	A/B	NC	C8, C7		

- •1 $\frac{dy}{dx} = \ldots\ldots$
- •2 $6x^2 + 6x + 4$
- •3 *e.g.* "$b^2 - 4ac$" $= \ldots\ldots$
- •4 -60 *or* -15 (from $3x^2 + 3x + 2$)
- •5 Δ negative so no st. points

OR

- •1 $\frac{dy}{dx} = \ldots\ldots$
- •2 $6x^2 + 6x + 4$
- •3 *e.g.* complete square......
- •4 $S = 6\left(x + \frac{1}{2}\right)^2 + 2\frac{1}{2}$
- •5 $S \geq 2\frac{1}{2}$ so no st. points

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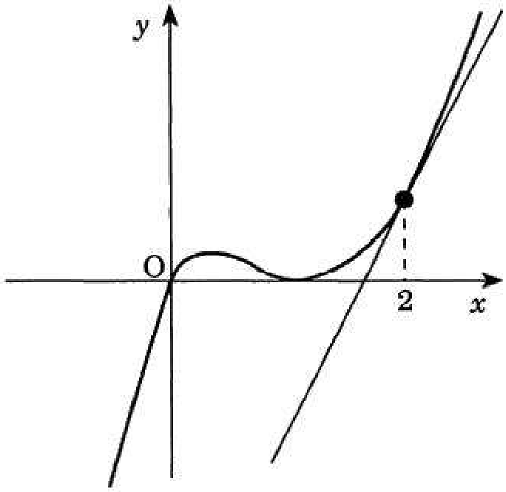
[SQA] 18. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	4	C	NC	C4, G3		1995 P2 Q2
(b)	5	C	NC	A23		

(a) •1 $\frac{dy}{dx} = \ldots\ldots\ldots$

•2 $3x^2 - 4x + 1$

•3 $m_{x=2} = 5$

•4 $y - 2 = 5(x - 2)$

(b) •5 equate 'y's

•6 $x^3 - 2x^2 - 4x + 8 = 0$

•7 e.g. synthetic division

•8 the appearance of:

$x^2 - 4$

or $x^2 - 4x + 4$

or ± 2

or $-2, 2, 2$

•9 $x = -2,\ y = -18$

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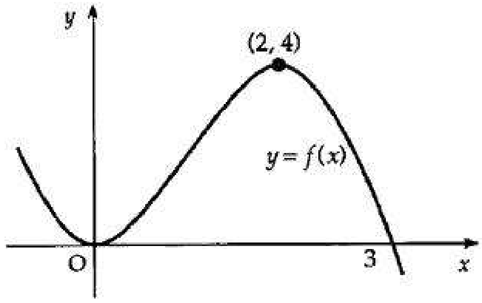
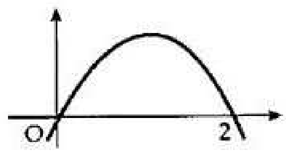
[SQA] 19. The diagram shows a sketch of a cubic function f with stationary points at $(0, 0)$ and $(2, 4)$.
Sketch the graph of the derived function f'. 3

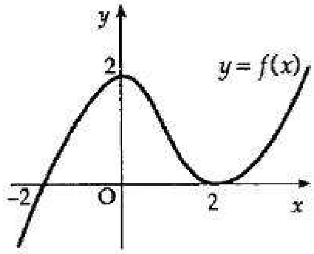


Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	2	C	NC	A3, C11		1990 P1 Q11
	1	A/B	NC	A3		

- •[1] know that there are exactly two zeros
- •[2] 0 and 2
- •[3] any parabola with max t.p.

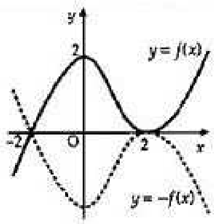


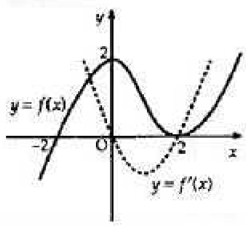
[SQA] 20. The diagram shows the graph of $y = f(x)$, where $-2 \le x \le 3$.
On separate diagrams, sketch the graphs of
(a) $y = -f(x)$; 2
(b) $y = f'(x)$. 3



Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	NC	A3		1991 P1 Q9
(b)	3	C	NC	A3, C11		

- •[1] for correct shape
- •[2] for annotation
- •[3] $f'(0) = 0$
- •[4] $f'(2) = 0$
- •[5] for correct shape





[END OF QUESTIONS]

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