X CBSE MATHS
Free Notes

CBSE Class X

CBSE - Mathematics

Chapter as per NCERT	Topic
Real Numbers	Euclid's Division Lemma
	Fundamental Theorem of Arithmetic
	Revisiting Rational and Irrational Numbers
Polynomials	Geometrical Meaning of Zero of Polynomials
	Relationship between zeros & Co-Efficient of a Polynomial
	Division algorithm for Polynomial
Pair of Linear Equation in Two Variables	Graphical Method of Solution
	Algebraic Methods of Solving a Pair of Linear Equations
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Arithmetic Progression	General Terms
	Sum of n Terms
Triangles	Similarity of Triangles
	Criteria for Similarity of Triangles
	Areas of Similar Triangles
Co-ordinate Geometry	Distance Formula
	Section Formula
	Area of Triangle
Trigonometry	Trigonometric Ratios and Angles
	Trigonometric Ratios of Special Angles

	Trigonometric Identities
Application of Trigonometry	Heights and Distance
Circles	Tangents to a Circle
Constructions	Constructions
Areas related to Circle	Areas of Sector and Segment of a Circle
	Areas of Combinations of Plain Figures
Surface Areas and Volumes	Surface Area and Volume of Combination of Solids
	Conversion of Solid from One Shape to Another
	Frustum of a Cone
Statistics	Mean of Grouped Data
	Mode of Grouped Data
	Median of Grouped Data
	Graphical Representation of Cumulative Frequency Distribution
Probability	Probability - Theoretical Approach

Euclid's Division Lemma

Euclid's division lemma, states that for any two positive integers 'a' and 'b' we can find two whole numbers 'q' and 'r' such that

$$a = b \times q + r$$

Euclid's division lemma can be used to:

Find the **highest common factor** of any two positive integers and to show the common properties of numbers.

Finding **H.C.F** using **Euclid's division lemma:**

Suppose, we have two positive integers 'a' and 'b' such that 'a' is greater than 'b'.

Apply **Euclid's division lemma** to the given integers 'a' and 'b' to find two whole numbers 'q' and 'r' such that, 'a' is equal to 'b' multiplied by 'q' plus 'r'.

Check the value of 'r'. If 'r' is equal to zero then 'b' is the **HCF** of the given numbers. If 'r' is not equal to zero, apply **Euclid's division lemma** to the new divisor 'b' and remainder 'r'. Continue this process till the remainder 'r' becomes zero. The value of the divisor 'b' in that case is the **HCF** of the two given numbers.

Euclid's division algorithm can also be used to find some common properties of numbers

Fundamental Theorem of Arithmetic

Fundamental Theorem of Arithmetic states that:
Every **composite number** can be expressed or factorised as a **product of prime numbers** and this **factorisation** is unique except in the order of the **prime factors**.

We can write the prime **factorisation** of a number in the form of powers of its **prime factors**.

By expressing any two numbers as their **prime factors**, their **highest common factor** (**HCF**) and **lowest common multiple** (**LCM**) can be easily calculated.

The **HCF** of two numbers is equal to the product of the terms containing the least powers of common **prime factors** of the two numbers.
The **LCM** of two numbers is equal to the product of the terms containing the greatest powers of all **prime factors** of the two numbers.
Note that the product of the given numbers is equal to the product of their **HCF** and **LCM**. This result is true for all positive integers and is often used to find the **HCF** of two given numbers if their **LCM** is given and vice versa

Revisiting Rational and Irrational Numbers

A number is called a **rational number** if it can be written in the form ***a*/*b*** where *a* and b are integers and $b \neq 0$.

A number is called an **irrational number** if it cannot be written in the form *a*/*b*, where *a* and *b* are integers and $b \neq 0$.

The sum, difference, product or quotient of a **rational** and an **irrational number** is also an **irrational number**.

Rational numbers are of two types depending on whether their decimal form is terminating or recurring.

Theorem: If *p*/*q* is a **rational number**, such that the **prime factorisation** of *q* is of the form $2^a 5^b$, where *a* and *b* are positive integers, then the decimal expansion of the **rational number** *p*/*q* terminates.

Theorem: If a **rational number** is a terminating decimal, it can be written in the form *p*/*q,* where *p* and *q* are co prime and the **prime factorisation** of *q* is of the form $2^a 5^b$, where *a* and *b* are positive integers.

Theorem: If *p*/*q* is a **rational number** such that the **prime factorisation** of *q* is not of the form $2^a 5^b$ where *a* and *b* are positive integers, then the decimal expansion of the **rational number** *p*/*q* does not terminate and is recurring

Geometrical Meaning of Zero of Polynomials

A **polynomial** is an **algebraic expression** consisting of multiple terms. The **terms of a polynomial** can be **variables** or variables raised to a power of a whole number, a constant or the product of these two.

The real number that precedes the **variable** is called the **coefficient**.

A **polynomial** involving **one variable** is called a **polynomial in one variable**.

The highest power of the variable of a **polynomial** is called the **degree of the polynomial.**

Based on its degree, a **polynomial** can be called as **linear polynomial, quadratic polynomial, cubic polynomial** and so on.

The general form of a **cubic polynomial** is $\boldsymbol{ax^3 + bx^2 + cx + d}$, where a, b, c and d are real numbers and $a \neq 0$

The general form of a **quadratic polynomial** is $\boldsymbol{ax^2 + bx + c}$, where a, b and c are real numbers and $a \neq 0$

The general form of a **linear polynomial** is $\boldsymbol{ax + b}$, where a and b are real numbers and $a \neq 0$

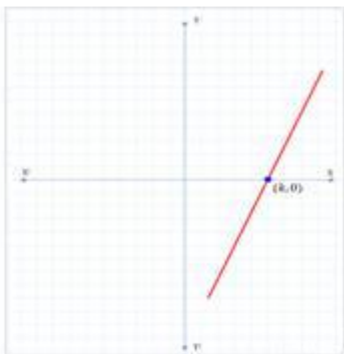
The value of a **polynomial** $p(x)$ when $x = k$ (k is a real number) is the value obtained by substituting x as k. It is denoted by $p(k)$.

The **zero of the polynomial** is defined as any real value of x, for which the value of the **polynomial** becomes zero.

A real number k is a **zero of a polynomial** $p(x)$, if $p(k) = 0$.

Geometrical Meaning of the Zeroes of a Polynomial: The **zero of the polynomial** is the x-coordinate of the point, where the graph intersects the x-axis. If a polynomial $p(x)$ intersects the x-axis at $(k, 0)$, then k is the **zero of the polynomial.**

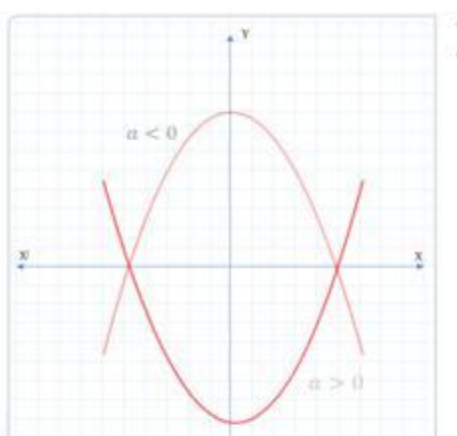
The graph of a **linear polynomial** intersects the x-axis at a maximum of one point. Therefore, a **linear polynomial** has a maximum of one zero.



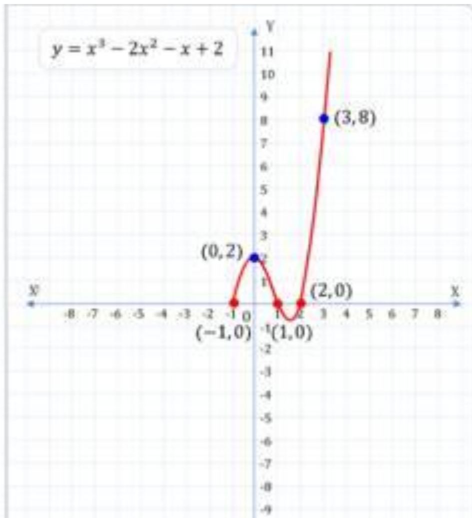
The graph of a **quadratic polynomial** intersects the x-axis at a maximum of two points. Therefore, a **quadratic polynomial** can have a maximum of two zeroes. In case of a **quadratic polynomial**, the shape of the graph is a **parabola**. The shape of the **parabola** of a **quadratic polynomial** $ax^2 + bx + c, a \neq 0$ depends on a.

If $a > 0$, then the **parabola** opens upwards.

If $a < 0$, then the **parabola** opens downwards.



The graph of a **cubic polynomial** intersects the x-axis at maximum of three points. A **cubic polynomial** has a maximum of three zeroes. In general, an **nth-degree polynomial** intersects the x-axis at a maximum of n points. Therefore, an **nth-degree polynomial** has a maximum of n zeroes.



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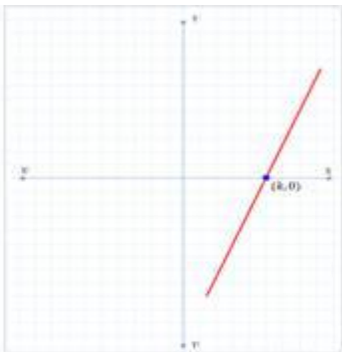

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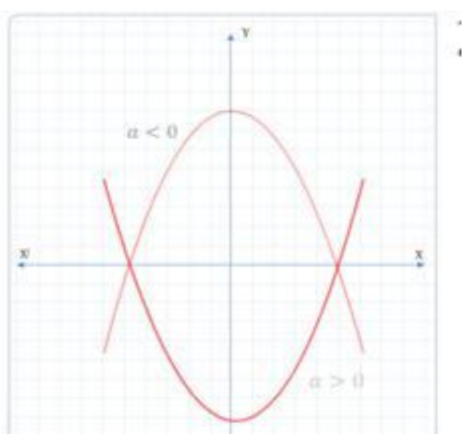

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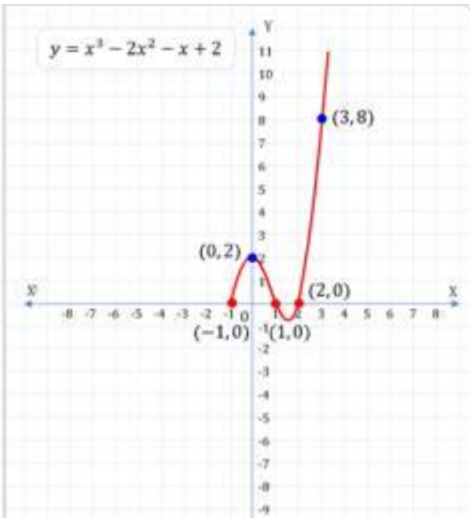


Relation between the zeroes and coefficient of a polynomial

Relation between the zeroes and coefficient of a polynomial

A **polynomial** is an algebraic expression consisting of **multiple terms**. There are various types of polynomials such as **linear**, **quadratic**, **cubic**……..

A real number k is a **zero of a polynomial** of p(x) if **p(k)= 0.**

The **general form of linear polynomial** is **p(x)=ax+b**, its zero is **–b/a** or **minus of constant term divided by coefficient of x.**

$$\alpha = \frac{-b}{a}$$

$$\alpha = -\frac{\text{Constant term}}{\text{Coefficient of } x}$$

General form of quadratic polynomial is **$ax^2 + bx + c$**. There are **two zeroes** of **quadratic polynomial**.

Factor Theorem: If a is zero of a polynomial p(x) then **(x – a) is a factor of p(x)**.

Sum of zeroes = $-\frac{coeficient\ of\ x}{coefficient\ of\ x^2}$

Product of zeroes = $-\frac{constant\ term}{coefficient\ of\ x^2}$

$$\text{The sum of zeroes} = \alpha + \beta = \frac{-b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{The product of zeroes } = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

General form of cubic polynomial of **$ax^3 + bx^2 + cx + d$** where $a \neq 0$. The **sum of zeroes of the cubic polynomial** =

$$-\frac{coeficient\ of\ x^2}{coefficient\ of\ x^3}$$

Sum of the product of zeroes taken two at a time =

$$-\frac{coeficient\ of\ x}{coefficient\ of\ x^3}$$

Product of zeroes = $-\frac{constant\ term}{coefficient\ of\ x^2}$

$$\text{The sum of zeroes } = \alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\text{The sum of the product of zeroes taken two at a time} = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\text{The product of zeroes } = \alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

Division algorithm for Polynomial

Let us consider two numbers a and b such that **a is divisible by b** then a is called is **dividend**, b is called the **divisor** and the resultant that we get on dividing a with b is called the **quotient** and here the **remainder** is zero, since a is divisible by b. Hence by **division rule** we can write,

Dividend = divisor x quotient + remainder.

This holds good even for polynomials too. Let **f(x), g(x), q(x) and r(x**) are **polynomials** then the **division algorithm for polynomials** states that "**If f(x) and g(x) are two polynomials such that degree of f(x) is greater that degree of g(x) where g(x) ≠ 0, then there exists unique polynomials q(x) and r(x) such that f(x) = g(x).q(x) + r(x) where r(x) = 0 or degree of r(x) less than degree of g(x)".**

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Division Algorithm for Polynomials: If $f(x)$ and $g(x)$ are the polynomials such that degree of $f(x) \geq$ degree of $g(x)$ where $g(x) \neq 0$, then there exists unique polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x)q(x) + r(x)$, where $r(x) = 0$ or Degree of $r(x) <$ Degree of $g(x)$.

Graphical Method of Solution

In everyday life, you will find many things that share a **one-to-one relationship** with each other, for example the quantity and cost of things, the age and the height, the altitude and the temperature. Such relationships are linear in nature and can be expressed mathematically as a **pair of linear equations in two variables.**

The general form of a **pair of linear equations in two variables** x and y as

$$\boldsymbol{a_1x + b_1y + c_1 = 0}$$

$$\boldsymbol{a_2x + b_2y + c_2 = 0}$$, Where $a_1, a_2, b_1, b_2, c_1, c_2$ are all real numbers , $a_1^{\ 2} + b_1^{\ 2} \neq 0 \ and \ a_2^{\ 2} + b_2^{\ 2} \neq 0$

We know that a **linear equation in two variables** when plotted on a graph defines a line. So, this means when a pair of **linear equations** is plotted, two lines are defined. Now, we know that two lines in a plane can intersect each other, be parallel to each other, or coincide with each other. The points where the two lines intersect are called the **solutions of the pair of linear equations.**

Condition 1: **Intersecting Lines**

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the **pair of linear equations** $\mathbf{a_1x + b_1y + c_1 = 0}$ $\mathbf{a_2x + b_2y + c_2 = 0}$ has a unique solution.

Condition 2: **Coincident Lines**

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the **pair of linear equations** $\mathbf{a_1x + b_1y + c_1 = 0}$ $\mathbf{a_2x + b_2y + c_2 = 0}$ has infinite solutions.

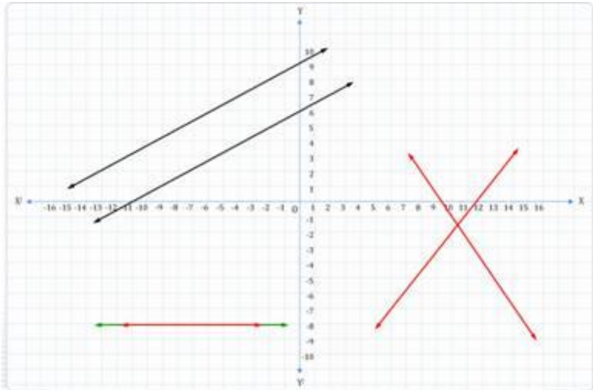
A **pair of linear equations**, which has a unique or infinite solutions are said to be a **consistent pair of linear equations.**

Condition 3: **Parallel Lines**

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then a **pair of linear equations** $\mathbf{a_1x + b_1y + c_1 = 0}$ $\mathbf{a_2x + b_2y + c_2 = 0}$ has no solution.

A pair of linear equations which has no solution is said to be an **inconsistent pair of linear equations.**

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$



Algebraic Methods of Solving a Pair of Linear Equations

To find the **solution to pair of linear equations**, graphical method may not always give the most accurate solutions. Especially, when the point representing the solution has non-integral coordinates like ($\sqrt{2}, \sqrt{3}$) or $(\frac{1}{3}, \frac{-5}{4})$.

There are three **algebraic methods** that can be used to solve a pair of linear equations namely (1) **Substitution method** (2) **Elimination method** (3) **Cross - multiplication method.**

Substitution method:

The first step to solve a **pair of linear equations** by the **substitution method** is to solve one equation for either of the variables. The choice of equation or variable in a given pair does not affect the solution for the pair of equations.

In the next step, we'll substitute the resultant value of one variable obtained in the other equation and solve for the other variable.

In the last step, we can substitute the value obtained of the variable in any one equation to find the value of the second variable.

Elimination method:

Step 1: Multiply the equations with suitable non-zero constants, so that the coefficients of one variable in both equations become equal.

Step 2: Subtract one equation from another, to eliminate the variable with equal coefficients.

Step 3: Solve for the remaining variable.

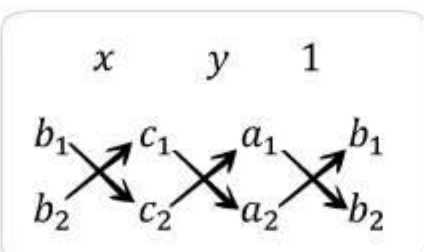
Step 4: Substitute the obtained value of the variable in one of the equations and solve for the second variable.

Cross - multiplication method

Let's consider the general form of a pair of linear equations.

$a_1x + b_1y + c_1 = 0$, and $a_2x + b_2y + c_2 = 0$. Recall that when a_1 divided by a_2 is not equal to b_1 divided by b_2, the pair of linear equations will have a unique solution.

To solve this pair of equations for x and y using cross-multiplication, we'll arrange the variables x and y and their coefficients a_1, a_2, b_1 and b_2, and the constants c_1 and c_2 as shown below



$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

Quadratic Equation

An **equation** of the form **ax^2 +bx +c =0** is called a $\boldsymbol{quadratic\ equation}$ in one variable, where a, b, c are real numbers and a ≠0. It's a $\boldsymbol{second\ degree\ polynomial}$. There are various $\boldsymbol{methods\ to\ solve\ the\ quadratic\ equation}$, they are (i) $\boldsymbol{Factorization}$ (ii) $\boldsymbol{Completing\ the\ squares}$ (iii) $\boldsymbol{Quadratic\ Formula}$ (iv) $\boldsymbol{Graphical\ Representation}$.

In factorization method the quadratic equation is solved by splitting them into factors. In $\boldsymbol{completing\ squares}$ the quadratic equation is converted into either $\boldsymbol{(a+b)2}$ or

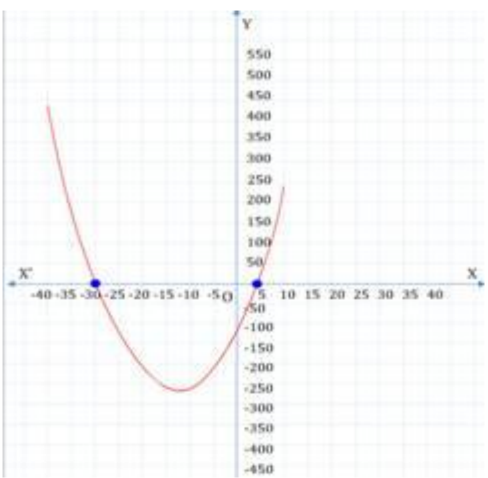
$(a - b)2$

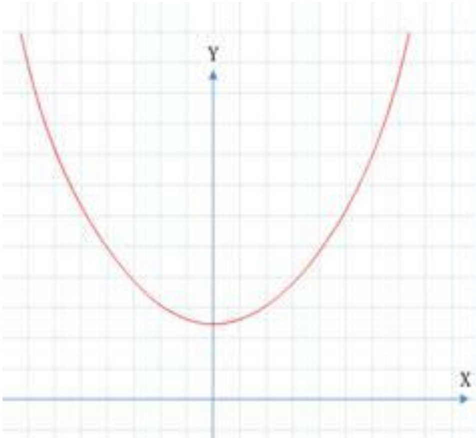
The quadratic formula to find the $roots\ of\ quadratic\ equation$ are $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $\sqrt{b^2 - 4ac}$ is called the **discriminant** of the quadratic equation denoted by **D or D.** The **sum of the roots** of the quadratic equation is $\frac{-b}{a}$ or $\frac{-coefficient\ of\ x}{coefficient\ of\ a^2}$ and the **product of the roots** of the quadratic equation is $\frac{c}{a}$ or $\frac{constant\ term}{coefficient\ of\ a^2}$**.**

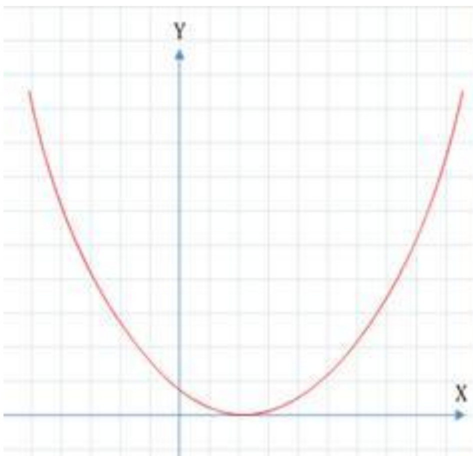
If **D=0 roots are real and equal , D>0 roots are real and unequal, D<0 roots are imaginary**.

The **graph of a quadratic equation** is a **parabola**. It depends on the value of 'a', if a>0 the **parabola opens upwards**, a<0 **parabola opens downwards**.

If **D>0 parabola intersects x-axis at two distinct points**, **D<0 parabola does not intersect the parabola** and **D=0 parabola touches x-axis at only one point**.







General Terms

We come across various **patterns** in our daily life. **Arithmetic Progression** in short **AP** is a **sequence** of numbers or **terms** in which each term except the **first term** is obtained by adding a **fixed number** or constant to the preceding term. This constant or fixed number is called **common difference** denoted by **d**.

The **general term** or **nth term of AP** is given by $t_n = a + (n - 1)d$, where a is the **first term**, d is the **common difference** and n is the **number of term**.

Common difference is given by $d = t_2 - t_1 = t_3 - t_2 = \ldots\ldots$

The nth term is also denoted with **l or b**.

General term of AP is $t_n = a + (n - 1)d$

Sum of n Terms

An **AP** is a **sequence** of numbers or terms in which each term except the first one is obtained by adding a **fixed number** or **constant** to the term **preceding term**.

The **first term** is denoted with **'a'** and the **fixed number** is called the **common**

difference denoted by ‘**d**’.

The common difference is the **difference between two successive terms** that is $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$

The **sum of first n terms** of an AP is given by **Sn =** $\frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}[a + l]$

or

$\frac{n}{2}[first\ term + last\ term]$

where **l = a + (n – 1)d** is called the **last term**.

Similarity of Triangles

In general, we come across several objects which have something common between them. Observing them closely, we can see that some of them have same shape but may have different or same size.

For example, if we consider the photographs a person developed from same negative, they all look same in all respect except for their size. Such objects are called **similar objects**.

Two line segments of different sizes, two circles of different radii, two squares of different sizes, two rectangles of different dimensions – come under **similar figures**. One smaller circle can be got by shrinking a larger circle. One bigger square can be got by stretching a smaller square. Then, what about the **similarity of triangles**? Is it true to say any two given **triangles are similar**? The answer is NO. This is true only when the triangles are equilateral. For all other triangles, we have the following statement which lays down the condition for the **similarity of two triangles**.
"Two triangles are said to be **similar** if their **corresponding angles are equal** and their **sides are proportional**"

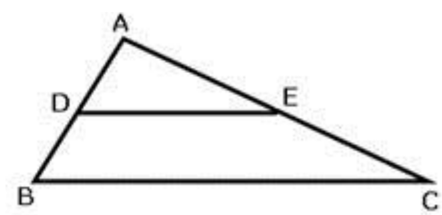
We use the symbol $\sim$ for the similarity of two triangles. We write $\Delta ABC \sim \Delta A'B'C'$ for the **similarity** of ΔABC and $\Delta A'B'C'$. Further, we follow that the vertices $A \leftrightarrow A', B \leftrightarrow B' \text{ and } C \leftrightarrow C'$, the angles $\angle A = \angle A', \angle B = \angle B' \text{ and } \angle C = \angle C'$ and
$\dfrac{AB}{A'B'} = \dfrac{BC}{B'C'} = \dfrac{AC}{A'C'}$. This ratio is called **Scale factor**.

OBJECTS THAT ARE OF THE SAME SHAPE BUT DIFFER IN SIZE ARE SAID TO BE SIMILAR.

TWO TRIANGLES ARE SIMILAR, IF THEIR CORRESPONDING:

- ANGLES ARE EQUAL
- SIDES ARE PROPORTIONAL

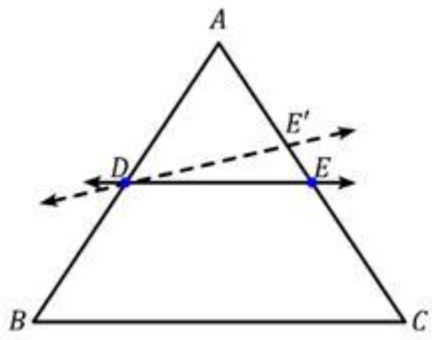
Basic proportionality theorem or Thales' theorem: If a line is drawn parallel to one side of a triangle and it intersects the other two sides in two distinct points then it divides the two sides in the same ratio.



In the ΔABC, if DE||BC, then $\frac{AD}{DB} = \frac{AE}{EC}$.

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

In the ΔABC, if D and E are two points on AB and AC respectively such that, $\frac{AD}{DB} = \frac{AE}{EC}$, then DE||BC



In ΔABC, line DE intersects the sides AB and AC in the same ratio.

$DE \parallel BC$

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Two line segments of different sizes, two circles of different radii, two squares of different sizes, two rectangles of different dimensions – come under **similar figures**. One smaller circle can be got by shrinking a larger circle. One bigger square can be got by stretching a smaller square. Then, what about the **similarity of triangles**? Is it true to say any two given **triangles are similar**? The answer is NO. This is true only when the triangles are equilateral. For all other triangles, we have the following statement which lays down the condition for the **similarity of two triangles**.
"Two triangles are said to be **similar** if their **corresponding angles are equal** and their **sides are proportional**"

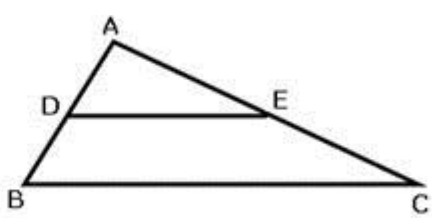
We use the symbol $\sim$ for the similarity of two triangles. We write $\Delta ABC \sim \Delta A'B'C'$ for the **similarity** of ΔABC and $\Delta A'B'C'$. Further, we follow that the vertices $A \leftrightarrow A', B \leftrightarrow B'$ and $C \leftrightarrow C'$, the angles $\angle A = \angle A', \angle B = \angle B'$ and $\angle C = \angle C'$ and
$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AC}{A'C'}$. This ratio is called **Scale factor**.

OBJECTS THAT ARE OF THE SAME SHAPE BUT DIFFER IN SIZE ARE SAID TO BE SIMILAR.

TWO TRIANGLES ARE SIMILAR, IF THEIR CORRESPONDING:

- ANGLES ARE EQUAL
- SIDES ARE PROPORTIONAL

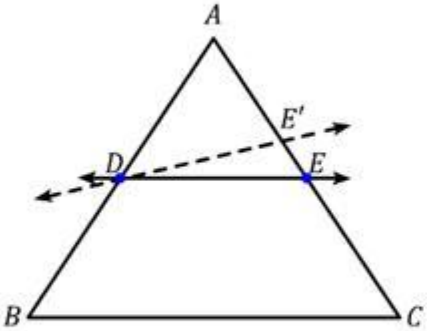
Basic proportionality theorem or Thales' theorem: If a line is drawn parallel to one side of a triangle and it intersects the other two sides in two distinct points then it divides the two sides in the same ratio.



In the ΔABC, if DE||BC, then $\frac{AD}{DB} = \frac{AE}{EC}$.

Converse of Basic Proportionality Theorem: If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

In the ΔABC, if D and E are two points on AB and AC respectively such that, $\frac{AD}{DB} = \frac{AE}{EC}$, then DE||BC



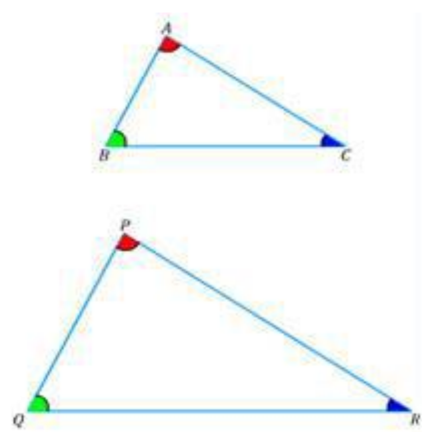
In ΔABC, line DE intersects the sides AB and AC in the same ratio.

$DE \parallel BC$

Criteria for Similarity of Triangles

In general, we come across several objects which have something common between them. Observing them closely, we can see that some of them have **same shape** but may have different or **same size** such figures are called **similar figures**.
In case of triangles "**Two triangles are said to be similar** if their **corresponding angles** are equal and corresponding **sides are proportional**". By using **AAA similarity theorem**, **SSS similarity theorem** and **SAS similarity theorem** we can prove **two triangles are similar**.

AAA similarity theorem or criterion: If the **corresponding angles of two triangles are equal**, then their **corresponding sides are proportional** and the **triangles are similar.**

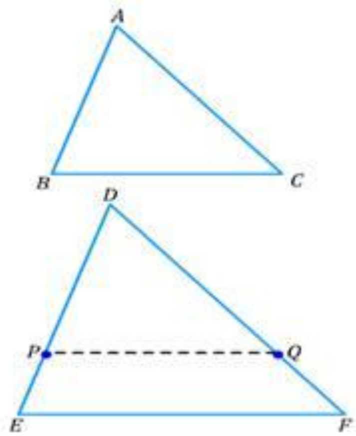


In DABC and DPQR, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ then $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ and DABC ~ DPQR

SSS similarity theorem or criteria: **If sides of one triangle are proportional to the sides of the other triangles, then their corresponding angles are equal and the triangles are similar.**

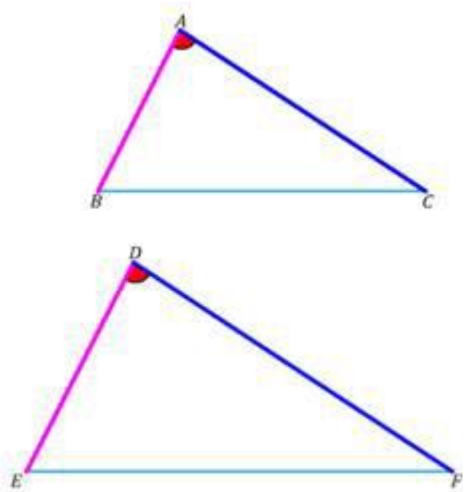
In DABC and DDEF, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ then $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and DABC ~ DDEF

SSS similarity theorem or criteria



SAS similarity theorem or criteria: **If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the triangles are similar.**

In DABC and DDEF, $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$ then DABC ~ DDEF

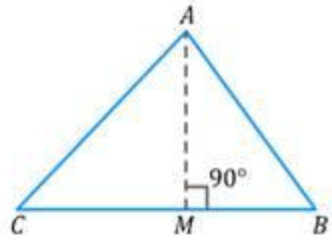


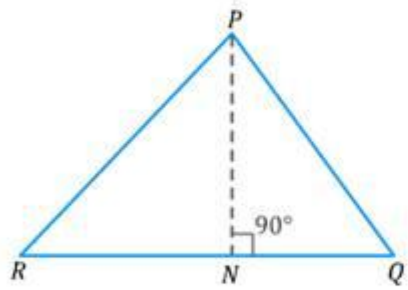
Areas of Similar Triangles

We know that **two triangles** are **similar** if their **corresponding angles** are equal and **corresponding sides** are **proportional**. If two triangles are similarly then the **ratio of the areas** of **two similar triangles** is equal to the **ratio of squares of the corresponding sides**.

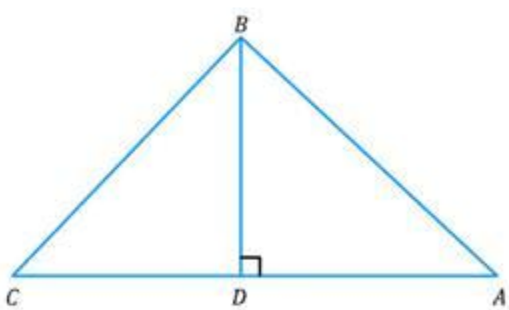
If **one of the angle is 90 degrees** then it is a **right angled triangle**. If a **perpendicular drawn form vertex** of the right angle of a **right triangle** to the **hypotenuse**, then triangles on both sides of the **perpendicular** are similar to the whole triangle and to each other.

In a right angle triangle, **the square of the hypotenuse** is equal to **sum of the squares of other two sides**, this is known as **Pythagoras theorem**. The **converse of Pythagoras** theorem states that in a triangle if the **square of one side** is equal to **sum of the squares of other two sides** then the angle opposite to the first side is a right angle.





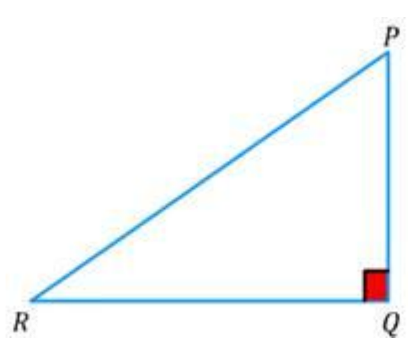
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$



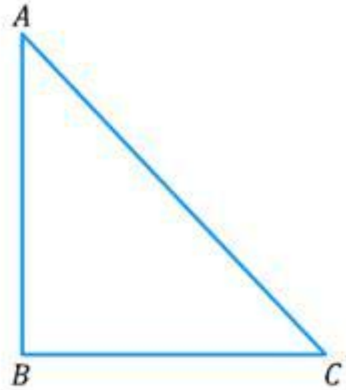
In $\Delta ABC,\ \angle B = 90^\circ\ \ BD \perp AC$

then

$\Delta ABC \sim \Delta ADB \sim \Delta BDC$



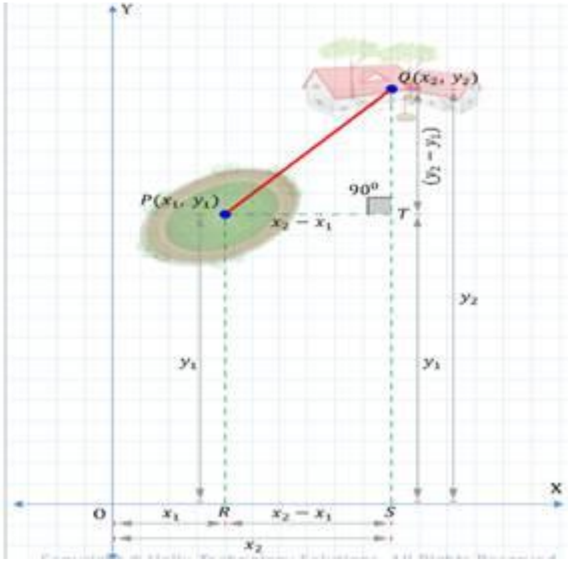
In $\Delta PQR, \angle PQR = 90^\circ$ then
$PR^2 = PQ^2 + QR^2$

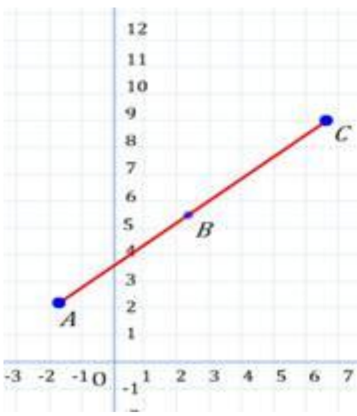


In $\Delta ABC,\ AC^2 = AB^2 + BC^2$ then $\angle B = 90^\circ$

Distance Formula

The **distance** between **two points (x1, y1) and (x2, y2)** is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ whereas **distance** between **any point (x, y) and origin** is $\sqrt{x^2 + y^2}$ on the **graph**. If the A, B and C are **three collinear points** then **AB + BC = AC** or **AB + AC =BC** or **AC + BC = AB.**





If three points on a graph are given and the sum of the distances of two points from a third point is equal to the distance between the first two points, the three points on the graph are collinear.

If $AB + BC = AC$ then A, B and C are collinear.

Section Formula

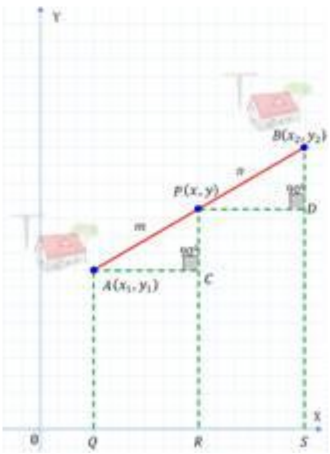
Given **two end points** of **line segment A(x_1, y_1) and B (x_2, y_2)** you can determine the coordinates of the **point P(x, y)** that divides the given **line segment** in the **ratio m:n** using **Section Formula** given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

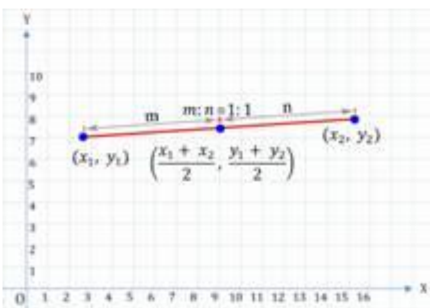
The **midpoint** of a line segment divides it into **two equal parts** or in the **ratio 1:1**. The midpoint of **line segment joining the points (x_1, y_1) and (x_2, y_2)** is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

The line joining the **vertex** to the **midpoint of opposite side of a triangle** is called

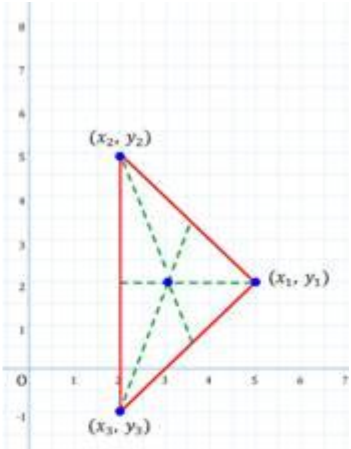
Median. **Three medians** can be drawn to a triangle and the **point of concurrency of medians** of a triangle is called **Centroid** denoted with **G.**

If **A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3)** are vertices of a triangle then its **centroid G** is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$**. The centroid of a triangle divides the median in the ratio 2:1**.





The midpoint of line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.



Centroid of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Centroid of a triangle divides the median in the ratio 2:1.

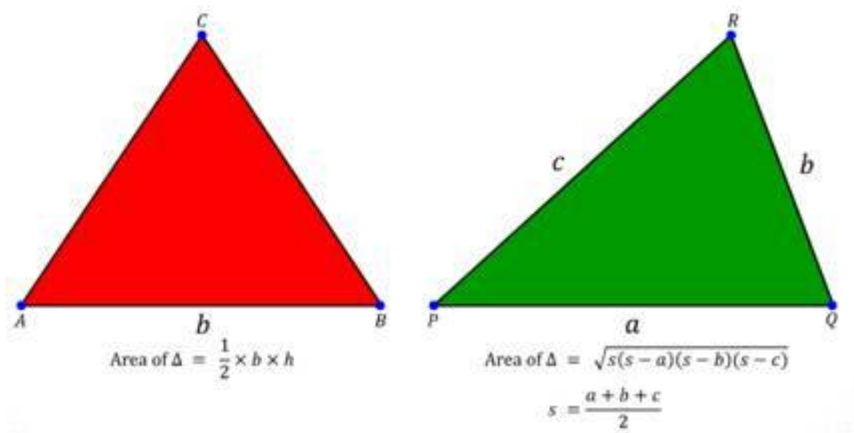
Area of Triangle

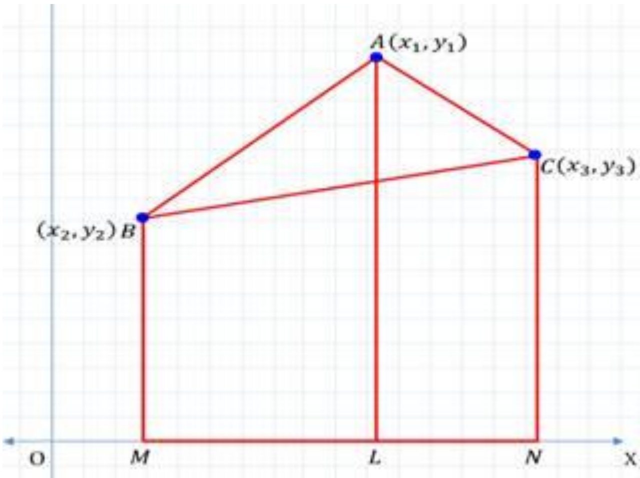
Area of triangle is half the product of its base and corresponding altitude if the base and altitude are given.

If **a, b and c are sides of a triangle** then its **area** is given by **herons formula** $\sqrt{s(s-a)(s-b)(s-c)}$.

we can calculate the **area of a triangle even if its three vertices are given**. If **(x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle**

then its area is given by $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$ square units.
If the area of triangle is zero then the points are called collinear points or vice-versa.





Area of triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ square units.

If area of triangle is zero then the points are collinear.

Trigonometric Ratios and Angles

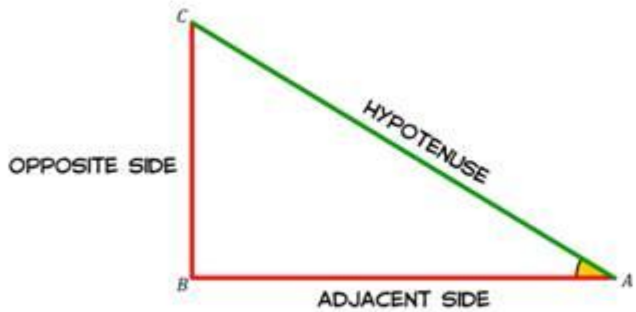
Trigonometry is the branch of mathematics that deals with the **study of relationships** between **sides and angles** of a triangle. It is derived from Greek words, **Tri meaning Three , Gon means Angle and Metron means Measure.**

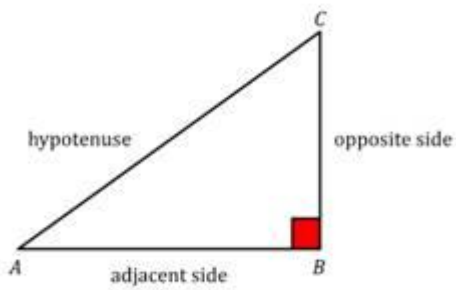
The **ratio of lengths of two sides** of a right angled triangle is called a **Trigonometric Ratio**. There are six trigonometric ratios

1. **Sine of angle** θ or in short denoted as **sinθ** is the ratio of **opposite to θ to the hypotenuse**.
2. **Cosine** of angle θ or in short **cosθ** is the ratio of **adjacent to θ to the hypotenuse**.
3. **Tangent** of angle θ or **tanθ** is the ratio of **opposite side to θ to the adjacent side to θ**.
4. **Cotangent** of angle θ or **cotθ** is the ratio of **adjacent side to θ to the opposite side to θ.**
5. **Secant** of angle θ or **secθ** is the ratio of **hypotenuse to the adjacent side to θ.**
6. **Cosecant** of angle θ or **cosecθ** is the ratio of **hypotenuse to the opposite side to θ.**

The relationships between the trigonometric ratios are:

1. $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\sin\theta \times \operatorname{cosec}\theta = 1$.
2. $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cos\theta \times \sec\theta = 1$.
3. $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$, $\tan\theta \times \cot\theta = 1$.

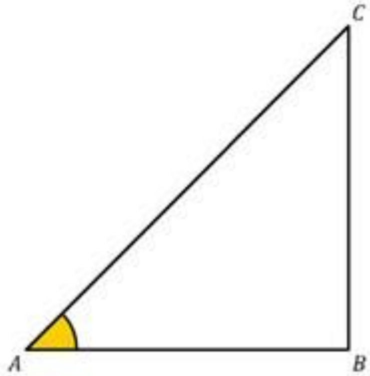




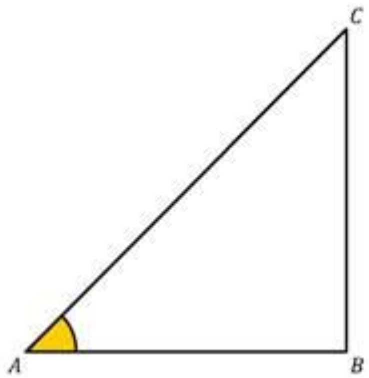
Trigonometric Ratios					
sin	cos	tan	cosec	sec	cot
$\frac{\text{opposite side}}{\text{hypotenuse}}$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	$\frac{\text{opposite side}}{\text{adjacent side}}$	$\frac{\text{hypotenuse}}{\text{opposite side}}$	$\frac{\text{hypotenuse}}{\text{adjacent side}}$	$\frac{\text{adjacent side}}{\text{opposite side}}$

Trigonometric Ratios of Special Angles

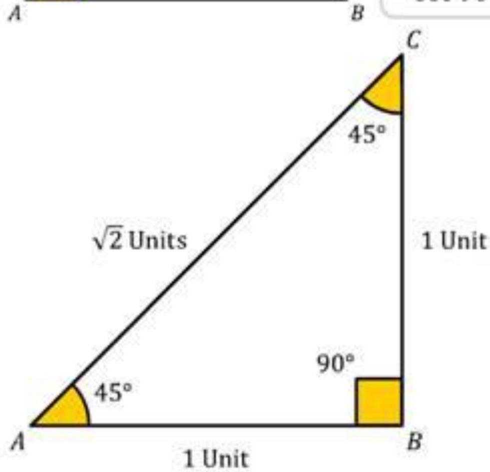
We have already learnt that trigonometry is the study of relationships between the sides and angles of a triangle. The ratios of the sides in a right triangle with respect to its acute angles are called trigonometric ratios of the angle. The angles $0°, 30°, 45°, 60°$ and $90°$ are useful angles in trigonometry, and their numerical values are easy to remember. When two angles add up to 900, then any one angle is the complement of the other. Trigonometric ratios of complementary angles help in simplifying problems.



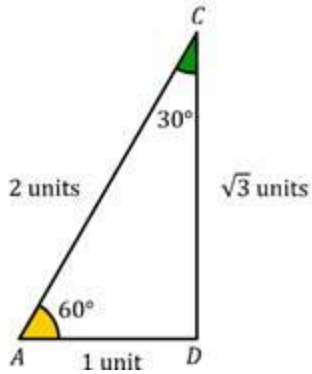
sin 0°	0
cosec 0°	Undefined
tan 0°	0
cot 0°	Undefined
cos 0°	1
sec 0°	1



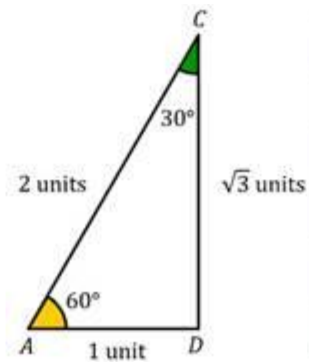
sin 90°	1
cosec 90°	1
tan 90°	Undefined
cot 90°	0
cos 90°	0
sec 90°	Undefined



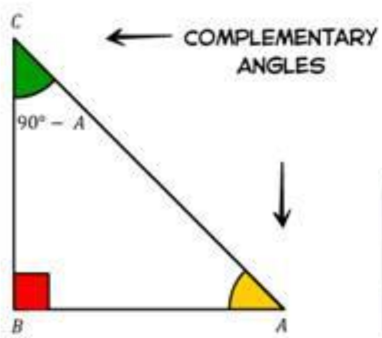
sin 45°	$\frac{1}{\sqrt{2}}$
cosec 45°	$\sqrt{2}$
tan 45°	1
cot 45°	1
cos 45°	$\frac{1}{\sqrt{2}}$
sec 45°	$\sqrt{2}$



sin 60°	$\frac{\sqrt{3}}{2}$
cosec 60°	$\frac{2}{\sqrt{3}}$
tan 60°	$\sqrt{3}$
cot 60°	$\frac{1}{\sqrt{3}}$
cos 60°	$\frac{1}{2}$
sec 60°	2



sin 30°	$\frac{1}{2}$
cosec 30°	2
tan 30°	$\frac{1}{\sqrt{3}}$
cot 30°	$\sqrt{3}$
cos 30°	$\frac{\sqrt{3}}{2}$
sec 30°	$\frac{2}{\sqrt{3}}$



$\sin(90° - \theta) = \cos\theta$	$\cos(90° - \theta) = \sin\theta$
$\text{cosec}(90° - \theta) = \sec\theta$	$\sec(90° - \theta) = \text{cosec}\,\theta$
$\tan(90° - \theta) = \cot\theta$	$\cot(90° - \theta) = \tan\theta$

Trigonometric Identities

A **trigonometric identity** is an equation involving **trigonometric ratios** of an angle, where the equation holds true for a defined range of values of the angle.
For the right triangle ABC, let 00≤ A ≤900

$\cos^2 A + \sin^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$

$\sin^2 A = 1 - \cos^2 A$`

$1 + \tan^2 A = \sec^2 A$

$\tan^2 A = \sec^2 A – 1$

$\sec^2 A - \tan^2 A = 1$

$\cot^2 A + 1 = \text{cosec}^2 A$

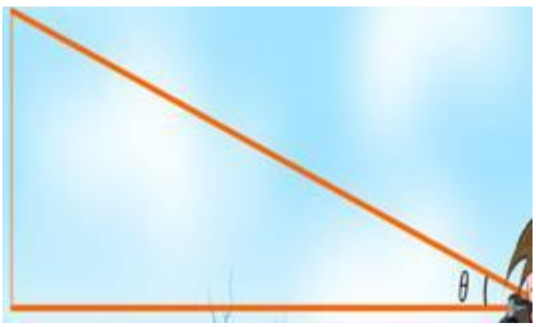
$\cot^2 A = \text{cosec}^2 A – 1$

$\text{cosec}^2 A - \cot^2 A = 1$

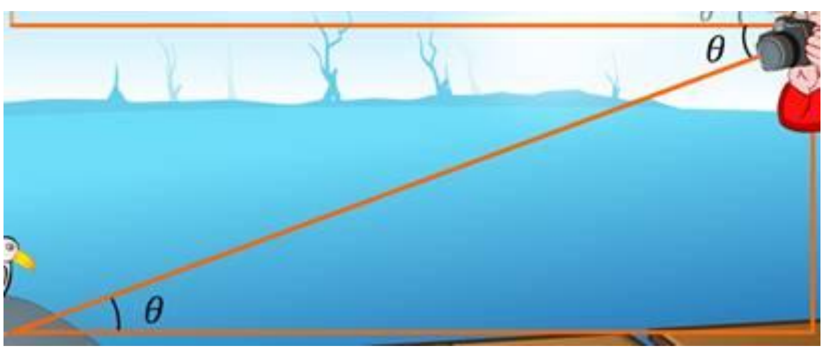
Heights and Distance

Trigonometry is the study of **relationships between the sides and angles** of a triangle. It is used in **geography** and in **navigation**. It is also used in **constructing**

maps, **determine the position of an Island** in relation to the **longitudes and latitudes**.
In this chapter **trigonometry** is used for finding the **heights and distances** of various objects without actually measuring it.
The **line joining** the **observer's eye** and the **object observed** is known as **Line of Sight**.
The **angle between the horizontal line and the line of sight** which is above the **observer's eye** is known as **Angle of Elevation**.
The **angle between the horizontal line and the line of sight** which is below the **observer's eye** is called **Angle of Depression.**



Angle of elevation: The angle between the horizontal line and the line of sight which is above the observer's eye.

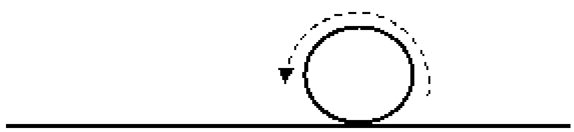


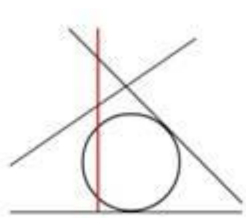
Angle of depression: The angle between the horizontal line and line of sight which is below the observer's eye.

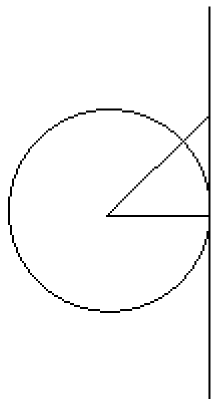
Tangents to a Circle

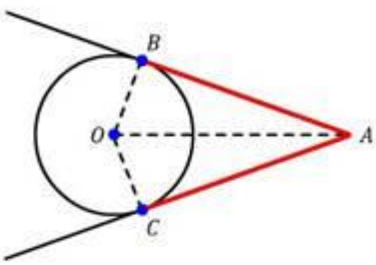
A **straight line intersects** a **circle** at one or two point. The **tangent to a circle** is a line that touches the circle at one point. **Secant** intersects the circle at two points. The point at which the straight line **touches** the circle is called the **point of contact** or **point of tangency**.

Some **properties of tangents** to a circle are **Infinite number of tangents** can be drawn to a circle but **only one tangent** can be drawn at any given point on a circle. From an **external point** we can draw **two tangents** of equal length. The **radius of the circle** is **perpendicular** to the tangent at its **point of contact** and the **tangents** drawn at the **extremities** of the **diameter of a circle are parallel.**







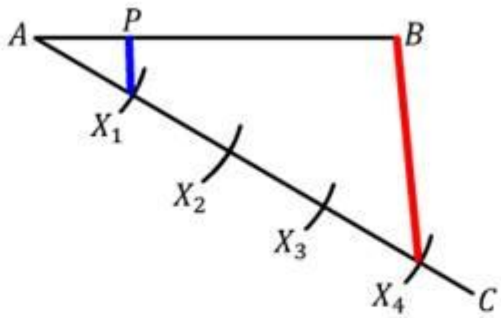

B
O
A
C

Constructions

There are two ways to **divide a given line segment** in a given ration.

First method: Let AB be the given line segment

Step 1: Draw a segment AC of any convenient length making an acute angle with the given line segment AB. Mark a few points at an equal distance from each other on AC. The number of points depends on the ratio in which you have to divide the given line segment. If the ratio is x is to y, the number of points is x plus y.
Step 2: Now, using compass to any small convenient length mark four points say X1, X2, X3 and X4 on AC, so that $AX_1 = X_1X_2 = X_2X_3 = X_3X_4$. Make sure that the distance to which the compass is open is not disturbed as you mark the points.
Step 3: Draw a line to join the points X_4 and B.
Step 4: Using a pair of set-squares draw a line parallel to X4B from the point X_1 to intersect the given line segment AB at point P.
Now the point P divides the line segment AB in the ratio x is to y.



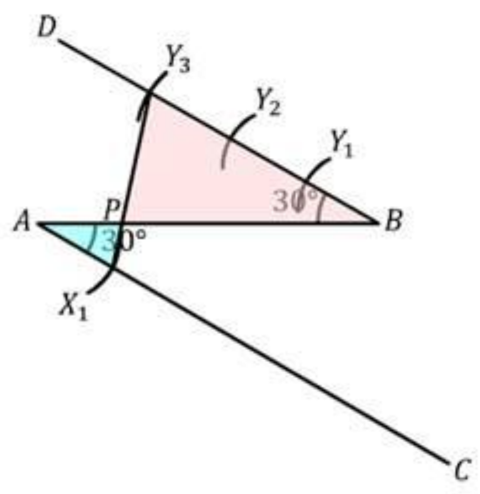
Second Method : Let AB be the given line segment

Step1:Draw a segment AC of a convenient length, making an acute angle with the given line segment AB

Step2: Draw a segment BD of any convenient length making the same angle with AB as AC on the opposite side of AC.

Step 3: Now, using compass to any small convenient length mark x number points on AC and y number points on BD or vice versa such that $AX_1 = X_1 X_2 = X_2X_3 = --- = BY_1 = Y_1Y_2 = Y_2Y_3 = ---$

Step4: Join X_x to Y_y to intersect AB at point P. Now Point P divides the line segment AB in the ratio x is to y.



You will find a **scale factor** on most maps. A **scale factor** determines how large or small an object is in comparison with another object.
Constructing a smaller (larger) triangle similar to a given triangle: Let ABC be a given triangle.

Step1: Draw a segment of a convenient length AX making an acute angle, with a side AB of the given triangle.
Step2: Now, using compass you have to mark a few points say X_1, X_2, X_3 ... at an equal distance from each other on AX. The number of points depends on the given scale factor. If the scale factor is x by y, the number of points is equal to the larger value

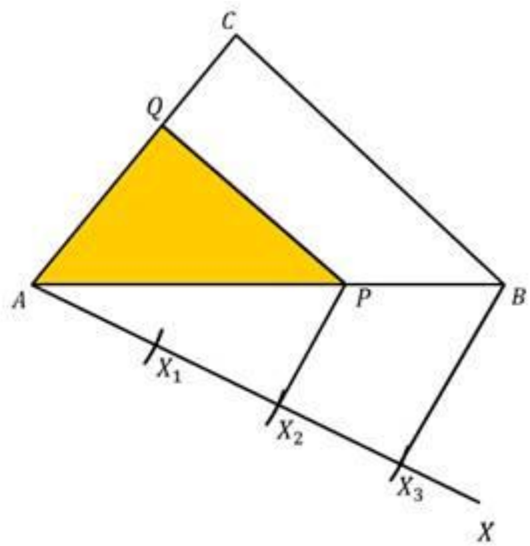
amongst x and y.

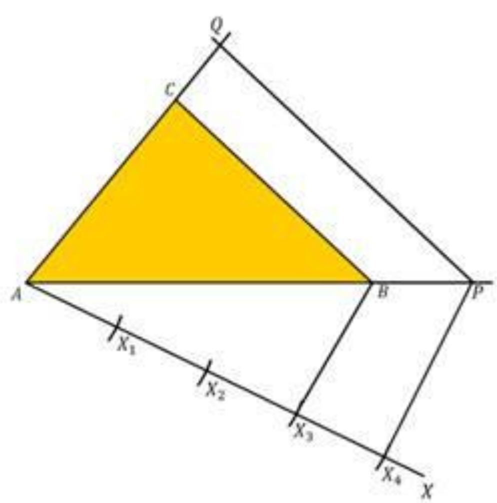
Step 3: Draw a line to join the points Xy and B.

Step 4: Draw a line parallel to XyB from Xx to intersect AB at P.

Step 5: Draw a line parallel to side BC at P to intersect side AC at Q.

Now, the triangle APQ is similar to the given triangle ABC.





Construction of Tangents from an External Point to a Circle: Let P be a point outside the circle having centre O.

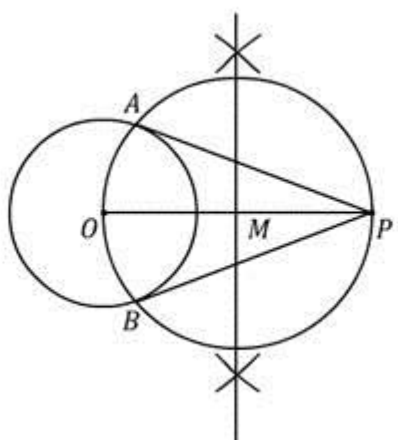
Step 1: Join OP.

Step 2: Draw perpendicular bisector of OP.

Step 3: Draw a circle with a centre M and radius MO to intersect the given circle at points A and B.

Step 4: Join AP and BP.

These are the two required tangents.



Lesson Demo

Areas of Sector and Segment of a Circle

We know that the **arc of a circle** is a part of the circumference of the circle.

The circular region enclosed by two radii and the corresponding arc of a circle is called the **sector of a circle.**

The angle subtended by the corresponding arc of the sector at the centre of the circle is called the **angle of the sector**.

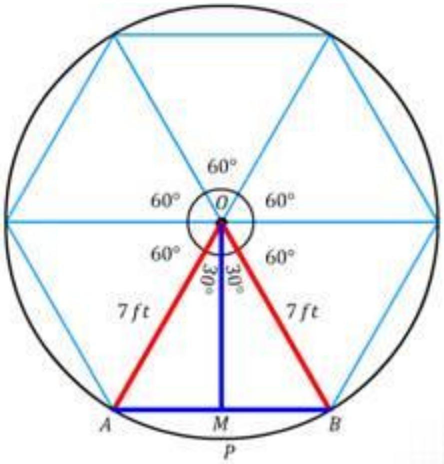
Area of a Sector of angle $\boldsymbol{\theta = \frac{\theta}{360} \times \pi r^2}$

Area of a Sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

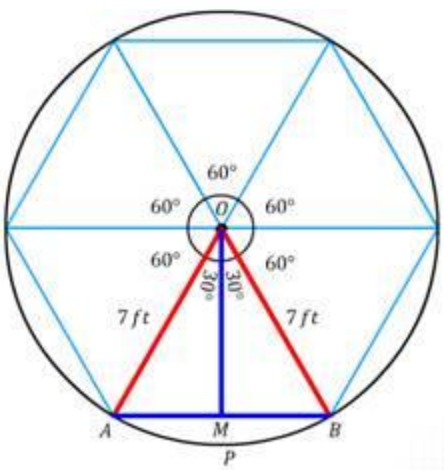
Length of an arc of angle $\boldsymbol{\theta = \frac{\theta}{360} \times 2\pi r}$

Length of an arc of angle $\theta = \frac{\theta}{360} \times 2\pi r$

The circular region enclosed between a chord and the corresponding arc is called the **segment of a circle.** The area of a segment is the area of the corresponding sector minus the area of the corresponding triangle.



Area of Segment APB = Area of Sector $OAPB$ – Area of $\Delta OAB = \frac{\theta}{360} \times \pi r^2$ – Area of ΔOAB

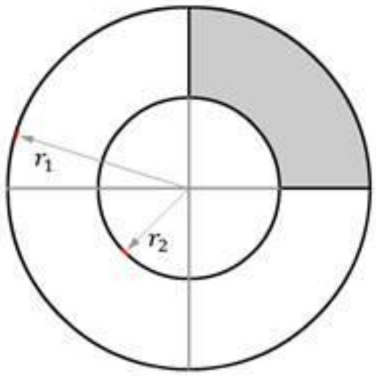


The area of a sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$.

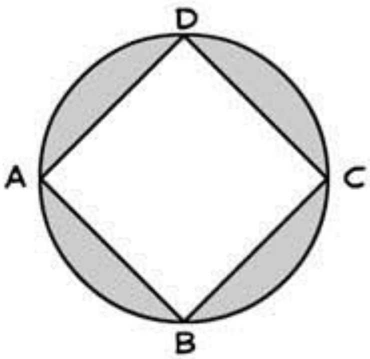
Areas of Combinations of Plain Figures

A collection of points that are on a plane is known as a plane figure.
We can calculate the area of the figure, which has a non-standard form
Involving basic geometric figures such as circles, squares, triangles, rectangles etc.

Area of the figure= Area of the rectangle + 2 × Area of semi circles



Area of the required region= $\frac{3}{4}(\pi r_1{}^2 - \pi r_1{}^2)$



Area of the required region= $\pi r^2 - a^2$

Surface Area and Volume of Combination of Solids

We know that **Curved surface area** (**CSA**) is the **area of the curved surfaces** for the solid.
The remaining surfaces of the solid are the flat or the non-curved surfaces. In a cone, the flat surface is the circular area of the base.

In a cylinder, the flat surfaces are the top and bottom circular areas. A sphere has no flat surface, whereas a cuboid and a cube have only flat surfaces.

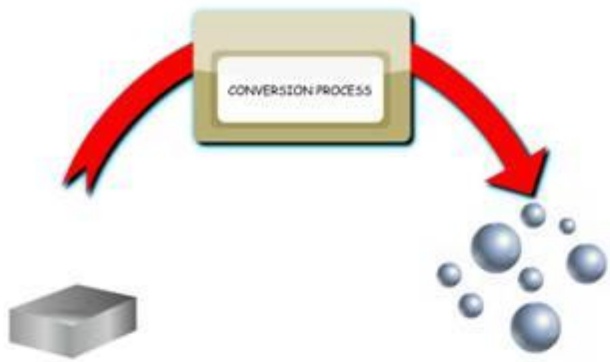
The flat surface area and the CSA of a solid together add up to the total surface area, TSA, for the solid.

Total Surface Area (TSA) = Curved Surface Area (CSA) + Areas of Flat Surfaces A simple approach used for calculating the surface area and volume for complex shapes is Split up the complex solid shapes into fundamental solid shapes. Apply the standard formulae to the fundamental shapes to calculate areas or volumes and Add the results of the calculation.

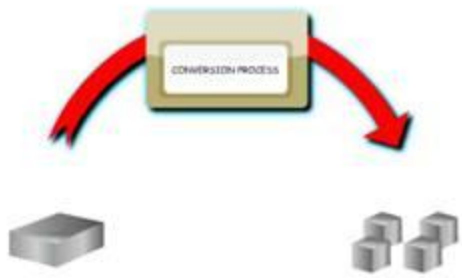
Conversion of Solid from One Shape to Another

When we cut a watermelon into slices, we are converting a solid shape into other solid shapes. Regardless of the size and shape of the slices, there is one fact that holds true of the whole process. The volume of all the slices together exactly equals the volume of the original watermelon.

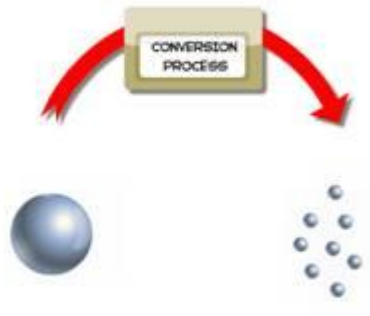
When we convert a solid of a given shape to a solid of another shape, the surface area usually changes. However, the volume is preserved.



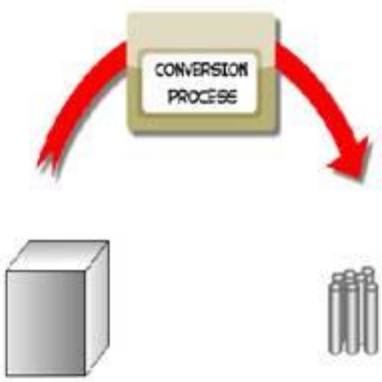
Volume of object1 = n x Volume of object2



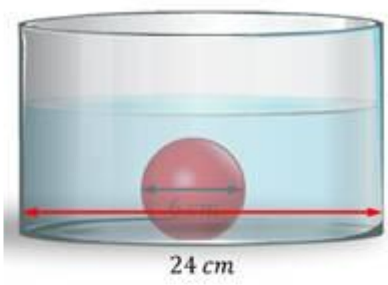
surface area of cuboid = 2 {(l x b) + (l x h) + (h x b)}



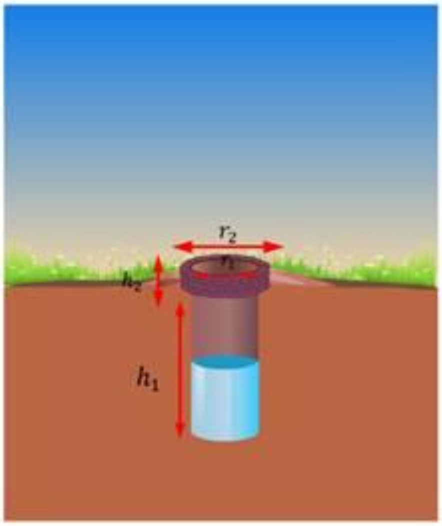
Volume of the metal sphere = n × Volume of a marble



Volume of copper cube = n × Volume of the pipe



Volume of cylindrical vessel of radius r2 and height h = Volume of the spherical ball



Volume of the embankment = $\pi(r_2^2 - r_1^2)h$

Frustum of a Cone

When we cut a right circular cone with a plane parallel to the base of the cone, then the solid shape between the plane and the base of the cone is called the **frustum of a cone**.
Flower pots have a shape as the **frustum of a cone**.

The **surface area** and the **volume of the frustum of a cone** can be calculated using standard formulae.

CSA of the frustum of a cone = $\pi l(r_1 + r_2)$, where

r1 = radius of the larger circular face of the frustum,

r2 = radius of the smaller circular face of the frustum, and

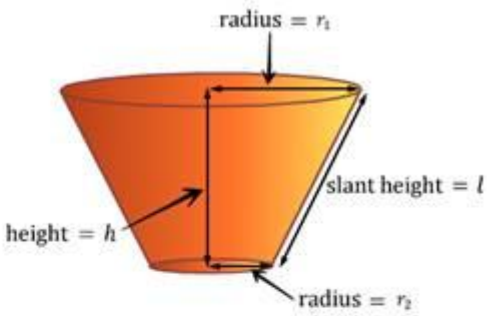
l = slant height of the frustum.

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

h is the height of the frustum.

TSA of a frustum of a cone = $\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Volume of the frustum of a cone = $(1/3)\,\pi h\,(r1^2 + r2^2 + r_1 r_2)$



FRUSTUM OF CONE

CSA of the frustum of a cone = $\pi l(r_1 + r_2)$,

Slant height of the frustum of a cone = $l = \sqrt{h^2 + (r_1 - r_2)^2}$

TSA of a frustum of a cone = $\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

Volume of the frustum of a cone = $(1/3)\ \pi h\ (r_1^2 + r_2^2 + r_1 r_2)$

Mean of Grouped Data

In **Statistics** Mean, **Median** and Mode are known as the **measures of central tendencies**. Median is the **middle most value** of the observations when the observations either arranged in **increasing or decreasing order**

Preparing **cumulative frequency distribution table** is the first step in calculating the **median** of the grouped data. The **cumulative frequency** of a class is obtained by adding the **frequencies** of all classes **preceding the given class**. To calculate median either **more than or less than cumulative frequency** is used.

If the data is converted into **frequency distribution table** it is known as **grouped data**.

The median for the grouped data is given by

$$l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

Where l is **lower class limit** of **median class,**

n is **total number of observation**s

cf is the **cumulative frequency** of class **preceding the median class**,

f is the **frequency** of median class and h is **class size.**

Number of Trees Planted (Class - Interval)	Number of Schools (Frequency) (f)	Cumulative Frequency (cf)
More than or equal to 5	12	12 + 8 + 14 + 20 + 6 = 60
More than or equal to 25	8	8 + 14 + 20 + 6 = 48
More than or equal to 45	14	14 + 20 + 6 = 40
More than or equal to 65	20	20 + 6 = 26
More than or equal to 85	6	6

THIS TABLE REPRESENTS THE CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'MORE THAN' TYPE.

Number of Trees Planted (Class - Interval)	Number of Schools (Frequency) (f)	Cumulative Frequency (cf)
5 - 25	12	12
25 - 45	8	20
45 - 65	14	34
65 - 85	20	54
85 - 105	6	60

THIS TABLE REPRESENTS THE CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'LESS THAN' TYPE.

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

Where l is **lower class limit** of **median class,**

n is **total number of observation**s

cf is the **cumulative frequency** of class **preceding the median class**,

f is the **frequency** of median class and h is **class size.**

Mode of Grouped Data

In **Statistics** Mean, Median and **Mode** are known as the **measures of central tendencies**. The mode of a given set of data is the observation with the maximum frequency.

The first step towards finding the mode of the grouped data is to locate the class interval with the maximum frequency. The class interval corresponding to the maximum frequency is called the modal class. The mode of this data is calculated using the formula

Mode = $l + h \times (f_1 - f_0) / (2f_1 - f_0 - f_2)$,

Where, l is the lower class limit of the modal class

"h" stands for the class size assuming that all class intervals have the same class size.

"f_1" stands for the frequency of the modal class.

f_0 stands for the frequency of the class preceding or just before the modal class.

f_2 stands for the frequency of the class succeeding or just after the modal class.

Number of Trees Planted (Class - Interval)	Number of Schools (Frequency: f_i)
5 - 25	12
25 - 45	8
45 - 65	14
65 - 85	20
85 - 105	6

The class interval corresponding to the maximum frequency is called the modal class.

$$\text{Mode} = l + h \times \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}$$

Where,

- $l \rightarrow$ lower class limit of the modal class.
- $h \rightarrow$ class size.
- $f_1 \rightarrow$ frequency of the modal class.
- $f_0 \rightarrow$ frequency of the class preceding or just before the modal class.
- $f_2 \rightarrow$ frequency of the class succeeding or just after the modal class.

In this case,

$l = 65$

$h = 20$

$f_1 = 20$

$f_0 = 14$

$f_2 = 6$

Median of Grouped Data

In **Statistics** Mean, **Median** and Mode are known as the **measures of central tendencies**. Median is the **middle most value** of the observations when the observations are either arranged in **increasing or decreasing order.**

Preparing a **cumulative frequency distribution table** is the first step in calculating the **median** of the grouped data. The **cumulative frequency** of a class is obtained by adding the **frequencies** of all the classes **preceding the given class**. To calculate the median either the **more than or less than cumulative frequency** is used.

If the data is converted into a **frequency distribution table** it is known as **grouped data**. The median for the grouped data is given by

$$l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

Where *l* is **lower class limit** of **median class,**

n is **total number of observations**

cf is the **cumulative frequency** of the class **preceding the median class**, and

f is the **frequency** of the median class and *h* is the **class size.**

Number of Trees Planted (Class - Interval)	Number of Schools (Frequency) (f)	Cumulative Frequency (cf)
More than or equal to 5	12	12 + 8 + 14 + 20 + 6 = 60
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More than or equal to 65	20	20 + 6 = 26
More than or equal to 85	6	6

THIS TABLE REPRESENTS THE CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'MORE THAN' TYPE.

Number of Trees Planted (Class - Interval)	Number of Schools (Frequency) (f)	Cumulative Frequency (cf)
5 - 25	12	12
25 - 45	8	20
45 - 65	14	34
65 - 85	20	54
85 - 105	6	60

THIS TABLE REPRESENTS THE CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'LESS THAN' TYPE.

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

Where *l* is the **lower class limit** of the **median class,**

n is **total number of observation**s

cf is the **cumulative frequency** of the class **preceding the median class**, and

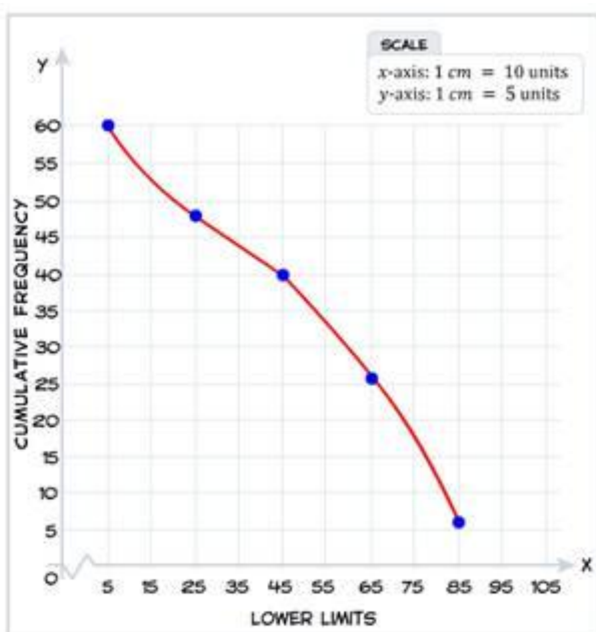
f is the **frequency** of the median class and *h* is the **class size.**

Graphical Representation of Cumulative Frequency Distribution

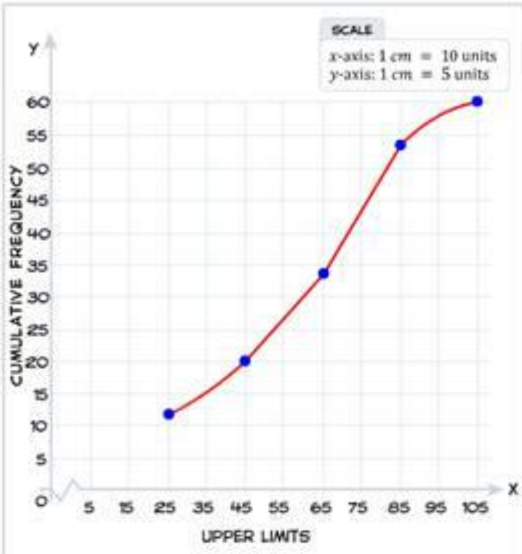
A **curve** that represents the **cumulative frequency distribution** of grouped data is called an **ogive** or **cumulative frequency curve**. The two **types of Ogives** are **more than type ogive** and **less than type ogive.**

An ogive representing a cumulative frequency distribution of '**more than' type** is called a **more than ogive**. An ogive representing a cumulative frequency distribution of **'less than' type** is called a **less than ogive**.

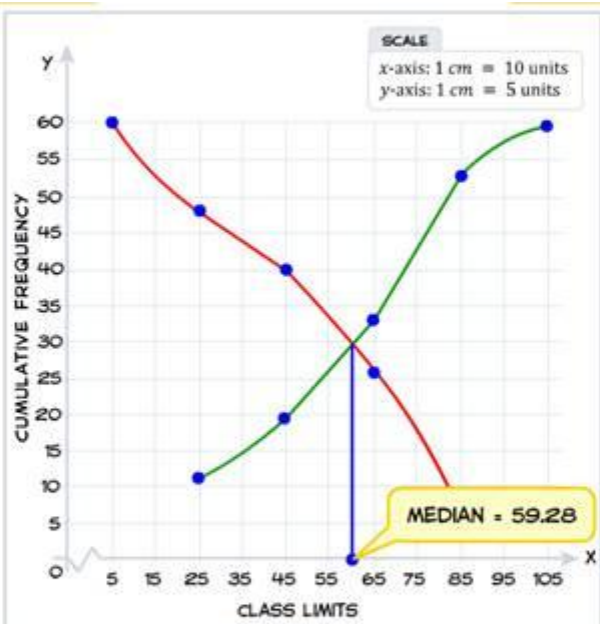
Ogives can be used to find the **median of a grouped data**. The median of grouped data can be obtained graphically by **plotting the Ogives** of the less than type and more than type and locate **the point of intersection of both the Ogives.** The x-coordinate of the point of intersection of two Ogives gives the median of the grouped data.



AN OGIVE REPRESENTING A CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'MORE THAN' TYPE IS CALLED A 'MORE THAN' OGIVE.



AN OGIVE REPRESENTING A CUMULATIVE FREQUENCY DISTRIBUTION OF THE 'LESS THAN' TYPE IS CALLED A 'LESS THAN' OGIVE.



Probability - Theoretical Approach

In our general life we come across the words likely, may be, probably, chance, hope, it may possible e.t.c. All these are synonyms to **probability**.

The **probability** that is based on the observations of an **experiment** is called an experimental or empirical probability. As the number of observations in an experiment increases, the **experimental probability** gets closer to the **theoretical probability**.

The results of an **experiment** are called **outcomes**.

The **possible outcomes** of an **experiment** are called its **elementary events**.

The **sum of the probabilities** of **elementary events** of an **experiment** is one.

These **outcomes** are said to be **equally likely** if each **outcome** has the same chance of happening.

Events that are **mutually exclusive** of each other are called **complementary events**.

The **sum of the probabilities** of two **complementary events** is always equal to one.

An **event** having **zero probability** of occurrence is called an **impossible event**.

An **event** having a **probability** of 1 is called a **sure or certain event**.

An **event** E is a number P(*E*), such that $0 \leq P(E) \leq 1$.

The **theoretical or classical probability** of an event *E* is denoted by **P(*E*),** where

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$