X-ray Spectroscopy and the Chemistry of Supernova Remnants

Selected Worksheet Keys

- *Calculation Investigation* – Students learn about unit analysis by converting energies to wavelengths to frequencies.
- *Calculate the Energy!* – Students will calculate the energy differences in different energy states of the Bohr atom of Hydrogen.
- *Graphing Spectra, Part 1 and 2* – Practice drawing graphs of spectra, and understanding the different ways spectra can be represented, as well as what each representation can tell us.
- *Satellite Venn Diagram* – Students will organize the information about X-ray observatories into a Venn diagram.

KEY

Solution: Student Worksheet
Calculation Investigation

You are given the following two equations that express the relationships between the speed, the wavelength, the energy and the frequency of light:

$$c = \lambda\nu$$

speed = wavelength × frequency

$$E = h\nu$$

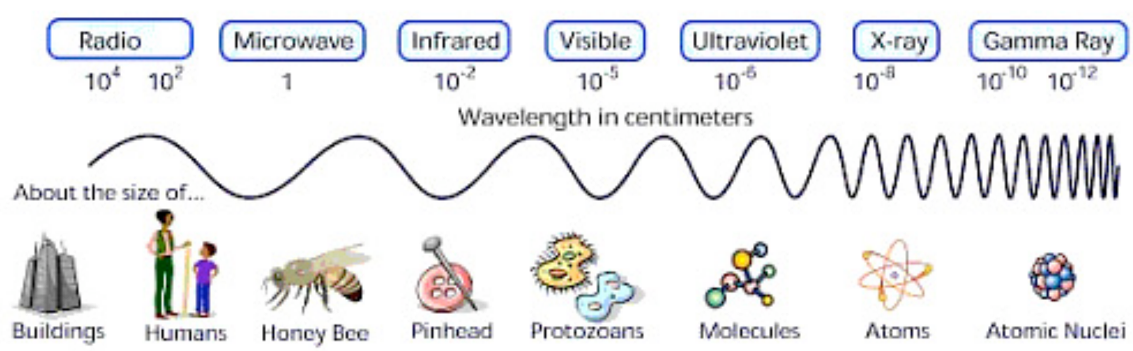
energy = Planck's constant × frequency

Where $h = 6.626 \times 10^{-34}\ m^2\ kg/s$.

Answer This!

1. Check the equations above and show that the units match on each side of the equations.
2. Manipulate both equations to solve for energy (E) as a function of wavelength (l) and fundamental constants. Show each step. Show that the units match on each side of the resulting equations.
3. Given a photon's wavelength, frequency or energy in the chart below, use the above equations to solve for the other two (in the units indicated). Use the useful constants below if you need to. Use the chart of the electromagnetic spectrum (below the table) to fill in the part of the electromagnetic radiation range for each row.

Wavelength (m)	Frequency (Hz)	Energy (J)	Electromagnetic Radiation Range
0.001	3.0×10^{11}	2.0×10^{-22}	microwave
4.3×10^{-6}	7.0×10^{13}	4.6×10^{-20}	infrared
5.0×10^{-7}	6.0×10^{14}	4.0×10^{-19}	visible
1.0×10^{-10}	3.0×10^{18}	2.0×10^{-15}	X-ray
2.5×10^{-14}	1.2×10^{22}	8.0×10^{-12}	gamma ray



Thought Questions

Students should note the inverse relationship between wavelength and frequency: as wavelength increases, frequency decreases or as wavelength decreases, frequency increases. They should note a similar inverse relationship between wavelength and energy. Students should also note the linear, correlated relationship between frequency and energy: as frequency increases, energy increases.

Students might also compare the size of the wavelength of various waves to the sizes of common objects, as illustrated in the above figure. They might also note how small the energies are.

KEY

Solution: Student Worksheet: Calculate the Energy!

Neils Bohr numbered the energy levels (n) of hydrogen, with level 1 (n=1) being the ground state, level 2 being the first excited state, and so on. Remember that there is a maximum energy that each electron can have and still be part of its atom. Beyond that energy, the electron is no longer bound to the nucleus of the atom and it is ionized. In that case n approaches infinity.

The equation for determining the energy of any state (the nth) is as follows:

$E = -13.6/n^2$ eV

Because the energy is so small, the energy is measured in electron-volts, designated as "eV".

1 eV = 1.6×10^{-19} J.

Answer the following questions:

1. **Using the above expression, calculate the energy of the first excited state. Your answer will be negative. This signifies that the electron is bound to the atom (as opposed to being a free electron).**

   For the first excited state, n=2. Using this in the above equation gives E = -3.40 eV

2. **Use the above expression to find the energy of the photon released when an electron around a hydrogen atom moves from the 4th to the 2nd level.**

   The energy of the photon is found by computing the difference in the energies of the fourth (n=4) and second (n=2) levels

   $E = -13.6/4^2 – (-13.6/2^2)$
   $E = -0.85 + 3.40$
   $E = 2.55$ eV

3. **Now use the above expression to find the energy of the photon released when a free electron is captured to the 2nd level.**

   We represent a free electron by assigning it an infinite n. Hence, its energy is zero.

   The energy of the photon emitted by a free electron captured to the n=2 level is thus

   $E = 0 – (-13.6/22) = 3.4$ eV

4. **Use the relationship between a photon's energy and its wavelength to calculate the wavelength of the photon emitted in question 2.**

From the Calculation Investigation, we learned that energy and wavelength are related through E = h c / l.

We can solve this for the wavelength, l = h c / E. where h = 6.626×10^{-34} J-s, and c = 3×10^{8} m/s

We convert our energy E= 2.55 eV into Joules using 1 eV = 1.6×10^{-19} J. This gives an energy of E = 4.08×10^{-19} J

We then find a wavelength of

l = $((6.626 \times 10^{-34}) \times (3 \times 10^{8})) / (4.08 \times 10^{-19})$

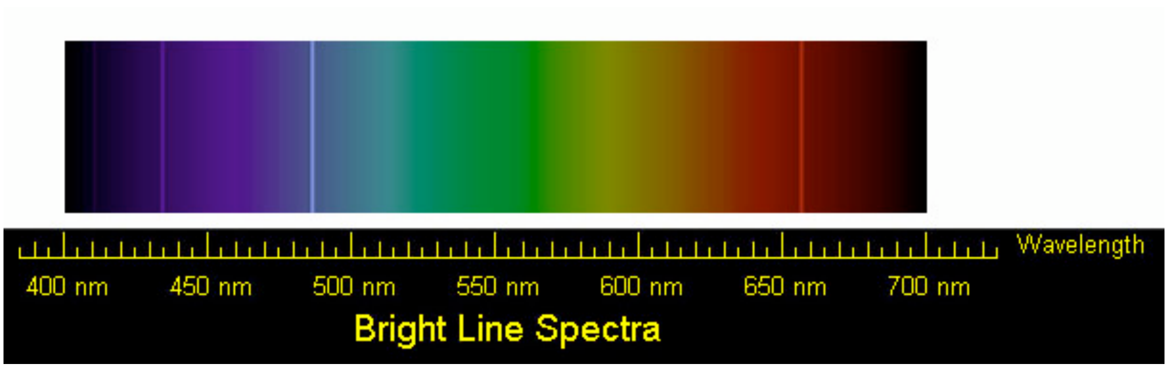
l = 4.87×10^{-7} m

Or, using 1 nm = 1×10^{-9} m,

l = 487 nm.

5. **Compare the wavelength for this transition with the lab spectrum of hydrogen below.**

The transition is the bright blue line, just to the left of the center.



KEY

Solution: Student Worksheet:
Graphing Spectra Part 1

Below are the answers to the "Think About" questions.

1. As you move along the wavelength axis from 300 Ångstroms to 350 Ångstroms, what will happen to the amount of energy emitted by the source? Explain why.
   The energy decreases. This is because the energy is inversely proportional to the wavelength: E = hc/l

2. In the second spectrum, explain why the emission lines are at different heights.
   The varying heights represents the different intensities of the lines. The lines in the left-most portion of the spectrum are brighter than any of the others.

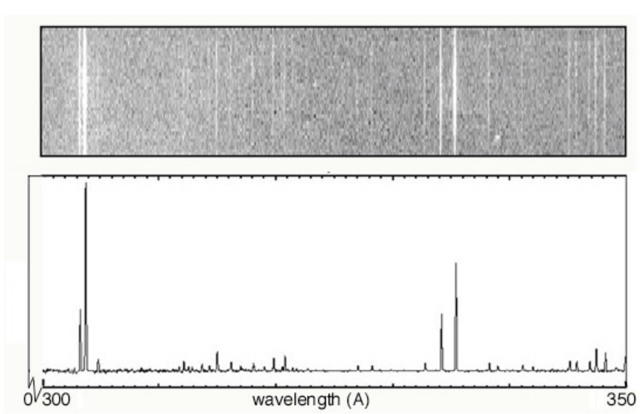
3. In order for bottom plot to include more "quantitative" data, what variable should go along the y-axis?
   The y-axis should be labeled as "Intensity".

4. How is this variable illustrated in both graphs?
   In the top image, it is represented by the brightness of the line. In the bottom plot, it is represented by the height of the line.

5. Describe how the second spectrum would look if it was a function of energy (instead of wavelength).
   Keeping the usual sense of values increase from left to right, the order the emission lines would be flipped left-to-right. That is, the brightest lines would be on the right.

6. What types of information are gathered from both spectra?
   From the spectra, we can identify the emission lines. With knowledge of the characteristic emission lines of various elements, we could then identify the elements giving rise to this spectrum.

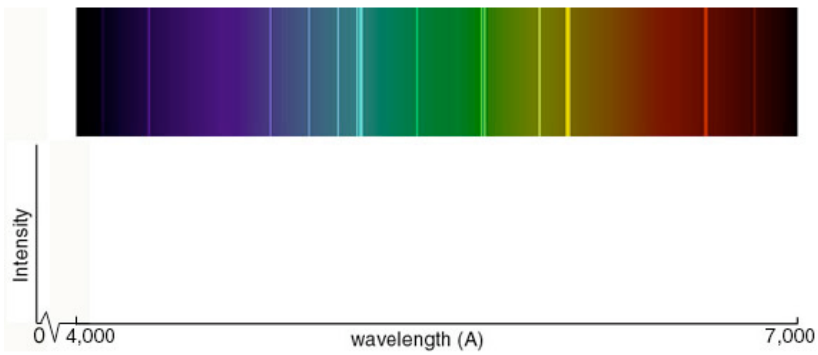
Solar UV Spectra



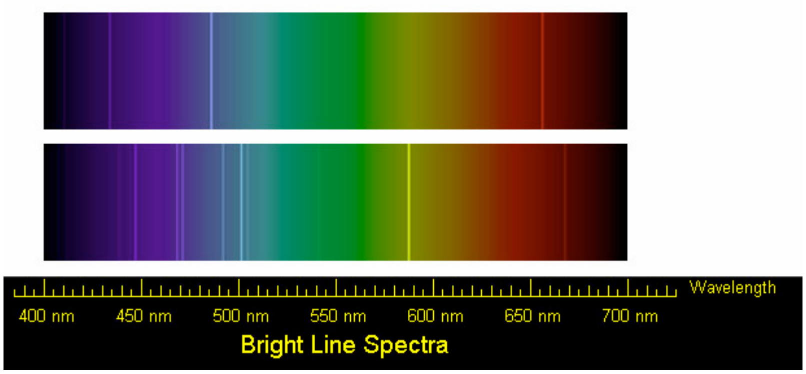
KEY

Solution: Student Worksheet: Graphing Spectra Part 2

The graphical representation should include all visible lines shown in the color spectrum. The continuum should rise gradually from 4000 Ångstroms, and remain fairly constant through blue, and decrease slightly in green portion of the spectrum. It should increase again, reach a maximum near yellow, and then decline again in the red.



Below are the solutions for the identifying the lines in the spectra of hydrogen and helium.



Hydrogen

We can identify three bright lines for hydrogen in the top spectrum. Measuring from the scale, the wavelengths are 435 nm (purple), 486 nm (blue) and 657 nm (red). Recall (e.g. from the Calculation Investigation) that the frequency is given by $\nu = c/\lambda$, and the energy is given by $E = h\nu$ (where $h = 6.626 \times 10^{-34}$ J-s, and $c = 3 \times 10^{8}$ m/s). In the table below we summarize the frequency and energy results for these lines. (We include the color to aid in identifying the line in the spectrum.)

Wavelength (nm)	Color	Frequency (Hz)	Energy (J)
435	purple	6.90×10^{14}	4.57×10^{-19}
486	blue	6.17×10^{14}	4.09×10^{-19}
657	red	4.57×10^{14}	3.03×10^{-19}

Helium

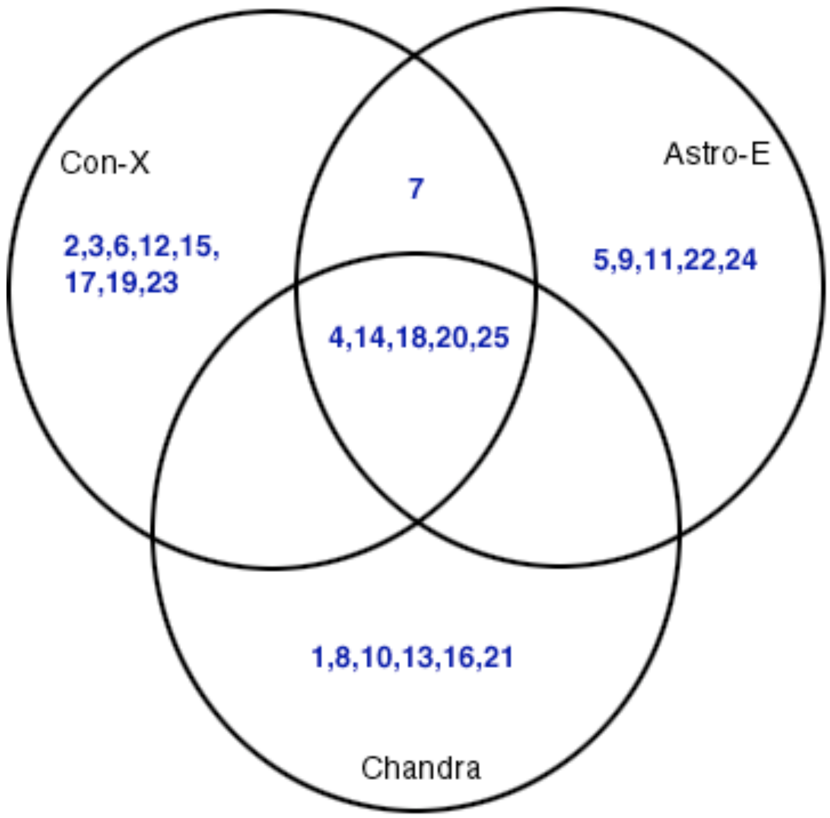
We can identify a number of lines in the spectrum of Helium. The bright lines are listed in the table below, along with their frequencies and energies. Students may identify any two of these.

Wavelength (nm)	Color	Frequency (Hz)	Energy (J)
447	purple	6.71×10^{14}	4.45×10^{-19}
469	blue	6.40×10^{14}	4.24×10^{-19}
472	blue	6.36×10^{14}	4.21×10^{-19}
493	blue-green	6.09×10^{14}	4.03×10^{-19}
501	blue-green	5.99×10^{14}	3.97×10^{-19}
505	blue-green	5.94×10^{14}	3.94×10^{-19}
587	yellow	5.11×10^{14}	3.39×10^{-19}
669	red	4.48×10^{14}	2.97×10^{-19}

KEY

Solution: Student Handout
Satellite Venn Diagram

The finished Venn diagram should look like this.



In the listing below, the correct answer is indicated in parentheses.

1. launched in 1999 (Chandra)
2. will require several rocket missions to launch the entire observatory (Con-X)
3. consists of four individual satellites (Con-X)
4. perform detailed studies of blackholes, supernovas, dark matter, origin, evolution, and destiny of the universe (Chandra, Astro-E, and Con-X)
5. launched in 2000 (Astro-E)
6. more quantitative data on abundance, velocity, temperature of gas (Con-X)
7. superior ability to discriminate amongst different x-rays wavelengths (Con-X and Astro-E)
8. flies more than 1/3 of the way to the moon (Chandra)
9. an array of 32 individual microcalorimeters (Astro-E)
10. exquisitely shaped for pairs of mirrors (Chandra)
11. incorporates a three stage cooling system capable of operating the array at 60 mK for about two years (Astro-E)

12. will be placed 1.5 million miles from Earth (Con-X)
13. images are 25x sharper than previous x-ray telescopes (Chandra)
14. designed to study the universe in x-rays (Chandra, Astro-E, and Con-X)
15. detects broadest range of x-ray wavelengths(Con-X)
16. focusing power equivalent to the ability to read a newspaper a half a mile away (Chandra)
17. focus on smaller areas which will exclude picking up signals from external medium of hot gas (Con-X)
18. X-ray telescopes are one way to observe extremely hot matter with temperature of millions of degrees (Chandra, Astro-E, and Con-X)
19. data collected in hours instead days (Con-X)
20. observatory must be placed high above Earth's surface because Earth's atmosphere absorbs X-rays (Chandra, Astro-E, and Con-X)
21. deployment of observatory commanded by woman (Chandra)
22. 10X higher spectral resolution for detecting emission from Iron (Astro-E)
23. collecting areas 3 square meters which will detect x-ray sources 100x fainter (Con-X)
24. a high resolution X-ray spectrometer based on a microcalorimeter array, four CCD X-ray cameras, and a hard X-ray telescope (Astro-E)
25. detects and images X-ray sources billions of light years away (Chandra, Astro-E, and Con-X)

Thought Questions

Each satellite represents an improvement over previous satellites. Chandra provides better imaging capabilities than previous satellites, and Astro-E provides better spectral resolution. In addition, Con-X improves spectral resolution beyond that of Astro-E.

The satellites are complementary because Chandra provides superior images, while Astro-E and Con-X provide superior spectra.

All the satellites must be placed above the Earth's atmosphere in order to study the universe in X-rays.