5.7 Division of Polynomials

1. Dividing: $\dfrac{4x^3-8x^2+6x}{2x}=\dfrac{4x^3}{2x}-\dfrac{8x^2}{2x}+\dfrac{6x}{2x}=2x^2-4x+3$

3. Dividing: $\dfrac{10x^4+15x^3-20x^2}{-5x^2}=\dfrac{10x^4}{-5x^2}+\dfrac{15x^3}{-5x^2}-\dfrac{20x^2}{-5x^2}=-2x^2-3x+4$

5. Dividing: $\dfrac{8y^5+10y^3-6y}{4y^3}=\dfrac{8y^5}{4y^3}+\dfrac{10y^3}{4y^3}-\dfrac{6y}{4y^3}=2y^2+\dfrac{5}{2}-\dfrac{3}{2y^2}$

7. Dividing: $\dfrac{5x^3-8x^2-6x}{-2x^2}=\dfrac{5x^3}{-2x^2}-\dfrac{8x^2}{-2x^2}-\dfrac{6x}{-2x^2}=-\dfrac{5}{2}x+4+\dfrac{3}{x}$

9. Dividing: $\dfrac{28a^3b^5+42a^4b^3}{7a^2b^2}=\dfrac{28a^3b^5}{7a^2b^2}+\dfrac{42a^4b^3}{7a^2b^2}=4ab^3+6a^2b$

11. Dividing: $\dfrac{10x^3y^2-20x^2y^3-30x^3y^3}{-10x^2y}=\dfrac{10x^3y^2}{-10x^2y}-\dfrac{20x^2y^3}{-10x^2y}-\dfrac{30x^3y^3}{-10x^2y}=-xy+2y^2+3xy^2$

13. Dividing by factoring: $\dfrac{x^2-x-6}{x-3}=\dfrac{(x-3)(x+2)}{x-3}=x+2$

15. Dividing by factoring: $\dfrac{2a^2-3a-9}{2a+3}=\dfrac{(2a+3)(a-3)}{2a+3}=a-3$

17. Dividing by factoring: $\dfrac{5x^2-14xy-24y^2}{x-4y}=\dfrac{(5x+6y)(x-4y)}{x-4y}=5x+6y$

19. Dividing by factoring: $\dfrac{x^3-y^3}{x-y}=\dfrac{(x-y)(x^2+xy+y^2)}{x-y}=x^2+xy+y^2$

21. Dividing by factoring: $\dfrac{y^4-16}{y-2}=\dfrac{(y^2+4)(y^2-4)}{y-2}=\dfrac{(y^2+4)(y+2)(y-2)}{y-2}=(y^2+4)(y+2)$

23. Dividing by factoring:

$$\begin{aligned}\frac{x^3+2x^2-25x-50}{x-5}&=\frac{x^2(x+2)-25(x+2)}{x-5}\\&=\frac{(x+2)(x^2-25)}{x-5}\\&=\frac{(x+2)(x+5)(x-5)}{x-5}\\&=(x+2)(x+5)\end{aligned}$$

25. Dividing by factoring:

$$\frac{4x^3+12x^2-9x-27}{x+3}=\frac{4x^2(x+3)-9(x+3)}{x+3}$$
$$=\frac{(x+3)(4x^2-9)}{x+3}$$
$$=\frac{(x+3)(2x+3)(2x-3)}{x+3}$$
$$=(2x+3)(2x-3)$$

27. Dividing using long division:

$$\begin{array}{r} x-7 \\ x+2\overline{)\,x^2-5x-7} \\ \underline{x^2+2x} \\ -7x-7 \\ \underline{-7x-14} \\ 7 \end{array}$$

The quotient is $x-7+\frac{7}{x+2}$.

29. Dividing using long division:

$$\begin{array}{r} 2x+5 \\ 3x-4\overline{)\,6x^2+7x-18} \\ \underline{6x^2-8x} \\ 15x-18 \\ \underline{15x-20} \\ 2 \end{array}$$

The quotient is $2x+5+\frac{2}{3x-4}$.

31. Dividing using long division:

$$\begin{array}{r} 2x^2-5x+1 \\ x+1\overline{)\,2x^3-3x^2-4x+5} \\ \underline{2x^3+2x^2} \\ -5x^2-4x \\ \underline{-5x^2-5x} \\ x+5 \\ \underline{x+1} \\ 4 \end{array}$$

The quotient is $2x^2-5x+1+\frac{4}{x+1}$.

33. Dividing using long division:

$$\begin{array}{r} y^2-3y-13 \\ 2y-3\overline{)\,2y^3-9y^2-17y+39} \\ \underline{2y^3-3y^2} \\ -6y^2-17y \\ \underline{-6y^2+\ 9y} \\ -26y+39 \\ \underline{-26y+39} \\ 0 \end{array}$$

The quotient is $y^2-3y-13$.

35. Dividing using long division:

$$\begin{array}{r} x-3 \\ 2x^2-3x+2\overline{)\,2x^3-9x^2+11x-6} \\ \underline{2x^3-3x^2+\ 2x} \\ -6x^2+\ 9x-6 \\ \underline{-6x^2+\ 9x-6} \\ 0 \end{array}$$

The quotient is $x-3$.

37. Dividing using long division:

$$\begin{array}{r} 3y^2+6y+8 \\ 2y-4\overline{)\,6y^3+\ 0y^2-8y+5} \\ \underline{6y^3-12y^2} \\ 12y^2-8y \\ \underline{12y^2-24y} \\ 16y+5 \\ \underline{16y-32} \\ 37 \end{array}$$

The quotient is $3y^2+6y+8+\dfrac{37}{2y-4}$.

39. Dividing using long division:

$$\begin{array}{r} a^3+2a^2+4a+6 \\ a-2\overline{)\,a^4+0a^3+0a^2-2a+5} \\ \underline{a^4-2a^3} \\ 2a^3+0a^2 \\ \underline{2a^3-4a^2} \\ 4a^2-2a \\ \underline{4a^2-8a} \\ 6a+5 \\ \underline{6a-12} \\ 17 \end{array}$$

The quotient is $a^3+2a^2+4a+6+\dfrac{17}{a-2}$.

41. Dividing using long division:

$$\begin{array}{r} y^3+2y^2+4y+8 \\ y-2\overline{)\,y^4+0y^3+0y^2+0y-16} \\ \underline{y^4-2y^3} \\ 2y^3+0y^2 \\ \underline{2y^3-4y^2} \\ 4y^2+0y \\ \underline{4y^2-8y} \\ 8y-16 \\ \underline{8y-16} \\ 0 \end{array}$$

The quotient is y^3+2y^2+4y+8.

43. Dividing using long division:

$$\begin{array}{r} x^2-2x+1 \\ x^2+3x+2\overline{)\,x^4+\ x^3-3x^2-\ x+2} \\ \underline{x^4+3x^3+2x^2} \\ -2x^3-5x^2-\ x \\ \underline{-2x^3-6x^2-4x} \\ x^2+3x+2 \\ \underline{x^2+3x+2} \\ 0 \end{array}$$

The quotient is x^2-2x+1.

45. First use long division to find the remaining factors:

$$\begin{array}{r} x^2+3x+2 \\ x+3\overline{)\,x^3+6x^2+11x+6} \\ \underline{x^3+3x^2} \\ 3x^2+11x \\ \underline{3x^2+\ 9x} \\ 2x+6 \\ \underline{2x+6} \\ 0 \end{array}$$

Thus $x^3+6x^2+11x+6=(x+3)\left(x^2+3x+2\right)=(x+3)(x+2)(x+1)$.

47. First use long division to find the remaining factors:

$$\begin{array}{r} x^2+2x-8 \\ x+3\overline{)\,x^3+5x^2-2x-24} \\ \underline{x^3+3x^2} \\ 2x^2-2x \\ \underline{2x^2+6x} \\ -8x-24 \\ \underline{-8x-24} \\ 0 \end{array}$$

Thus $x^3+5x^2-2x-24=(x+3)\left(x^2+2x-8\right)=(x+3)(x+4)(x-2)$.

49. Yes, both answers are identical.

51. $P(-2)=(-2)^2-5(-2)-7=4+10-7=7$

The answer is the same.

53. **a.** Using long division:

$$x-2\overline{)x^3-3x^2+5x-6}$$

$$\begin{array}{r} x^2-x+3 \\ x-2\overline{)x^3-3x^2+5x-6} \\ \underline{x^3-2x^2} \\ -x^2+5x \\ \underline{-x^2+2x} \\ 3x-6 \\ \underline{3x-6} \\ 0 \end{array}$$

Since the remainder is 0, $x-2$ is a factor of x^3-3x^2+5x-6.

Also note that: $P(2)=(2)^3-3(2)^2+5(2)-6=8-12+10-6=0$

b. Using long division:

$$\begin{array}{r} x^3-x+1 \\ x-5\overline{)x^4-5x^3-x^2+6x-5} \\ \underline{x^4-5x^3} \\ -x^2+6x \\ \underline{-x^2+5x} \\ x-5 \\ \underline{x-5} \\ 0 \end{array}$$

Since the remainder is 0, $x-5$ is a factor of $x^4-5x^3-x^2+6x-5$.

Also note that: $P(5)=(5)^4-5(5)^3-(5)^2+6(5)-5=625-625-25+30-5=0$

55. **a.** Completing the table:

x	1	5	10	15	20
$C(x)$	2.15	2.75	3.50	4.25	5.00

b. The average cost function is $\overline{C}(x)=\frac{2}{x}+0.15$.

c. Completing the table:

x	1	5	10	15	20
$\overline{C}(x)$	2.15	0.55	0.35	0.28	0.25

d. The average cost function decreases.

e. For $y=C(x)$, the domain is $\{x \mid 1\le x\le 20\}$ and the range is $\{y \mid 2.15\le y\le 5.00\}$.

For $y=\overline{C}(x)$, the domain is $\{x \mid 1\le x\le 20\}$ and the range is $\{y \mid 0.25\le y\le 2.15\}$.

57. **a**. Substituting the values:

$$T(100) = 4.95 + 0.07(100) = \$11.95$$

$$T(400) = 4.95 + 0.07(400) = \$32.95$$

$$T(500) = 4.95 + 0.07(500) = \$39.95$$

b. The average cost function is $\bar{T}(m) = \dfrac{4.95}{m} + 0.07$.

c. Substituting the values:

$$\bar{T}(100) = \frac{4.95}{100} + 0.07 = \$0.1195 \text{ per minute}$$

$$\bar{T}(400) = \frac{4.95}{400} + 0.07 = \$0.0824 \text{ per minute}$$

$$\bar{T}(500) = \frac{4.95}{500} + 0.07 = \$0.0799 \text{ per minute}$$

59. Performing the operations: $\dfrac{2a+10}{a^3} \bullet \dfrac{a^2}{3a+15} = \dfrac{2(a+5)}{a^3} \bullet \dfrac{a^2}{3(a+5)} = \dfrac{2}{3a}$

61. Performing the operations: $(x^2-9)\left(\dfrac{x+2}{x+3}\right) = (x+3)(x-3)\left(\dfrac{x+2}{x+3}\right) = (x-3)(x+2)$

63. Performing the operations: $\dfrac{2x-7}{x-2} - \dfrac{x-5}{x-2} = \dfrac{2x-7-x+5}{x-2} = \dfrac{x-2}{x-2} = 1$

65. Simplifying the expression: $\dfrac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}+\frac{1}{3}} = \dfrac{\left(\frac{1}{x}-\frac{1}{3}\right)\bullet 3x}{\left(\frac{1}{x}+\frac{1}{3}\right)\bullet 3x} = \dfrac{3-x}{3+x}$

67. Solving the equation:

$$\frac{x}{x-3} + \frac{3}{2} = \frac{3}{x-3}$$

$$2(x-3)\left(\frac{x}{x-3} + \frac{3}{2}\right) = 2(x-3)\left(\frac{3}{x-3}\right)$$

$$2x + 3(x-3) = 6$$

$$2x + 3x - 9 = 6$$

$$5x = 15$$

$$x = 3 \quad (\text{does not check})$$

There is no solution (3 does not check).