UNIT – 03 LAWS OF MOTION

TWO MARKS AND THREE MARKS:

01. Explain the concept of inertia. Write two examples each for inertia of motion, inertia of rest and inertia of direction.

This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Examples:

Inertia of Rest :

i) Passengers experience a backward push in a sudden start of bus.
ii) Tightening of seat belts in a car when it stops quickly.

Inertia of Motion:

i) Passengers experience a forward push during a sudden brake in bus.
ii) Ripe fruits fall from the trees in the direction of wind.

Inertia of Direction:

i) A stone moves tangential to Circle.
ii) When a car moves towards left, we turn to the right.

02. State Newton's second law.

The force acting on an object is equal to the rate of change of its momentum. $\vec{F} = \frac{d\vec{p}}{dt}$

03. Define One Newton

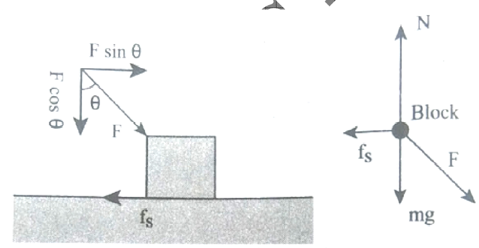
One Newton is defined as the force which acts on 1 kg of mass to give an acceleration 1 m s^{-2} in the direction of the force.

04. Show that impulse is the change of momentum.

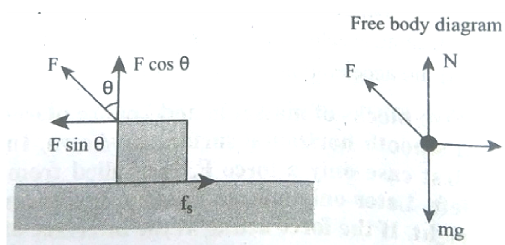
The integral $\int_{t_i}^{t_f} F\,dt = J$ is called the impulse and it is equal to change in momentum of the object.

Proof : If the force is constant over the time interval, then $\int_{t_i}^{t_f} F\,dt = F\int_{t_i}^{t_f} dt = F(t_f - t_i) = F\Delta t = \Delta p$ is called the impulse and it is equal to change in momentum of the object.

05. Using free body diagram, show that it is easy to pull an object than to push it.



$N_{push} = mg + F\cos\theta$



$N_{pull} = mg - F\cos\theta$

06. Explain various types of friction. Suggest a few methods to reduce friction.

Static Friction ($\vec{f_s}$):

01. Static friction is the force which opposes the initiation of motion of an object on the surface.
02. When the object is at rest on the surface, only two forces act on it. They are the downward gravitational force and upward normal force.
03. The resultant of these two forces on the object is zero.

Kinetic Friction ($\overrightarrow{f_k}$):

01. When an object slides, the surface exerts a frictional force called **kinetic friction** ***(fk)***
02. If the external force acting on the object is greater than maximum static friction, the objects begin to slide.
03. The kinetic friction does not depend on velocity.

Rolling Friction:

The force of friction that comes into act when a wheel rolls over a surface.

Methods to reduce friction:

01. By using Lubricants friction 02. By using ball bearings.

07. What is the meaning by 'pseudo force'?

Centrifugal force is called as a 'pseudo force'. A pseudo force has no origin. A pseudo force is an apparent force that acts on all masses whose motion is described using non inertial frame of reference such as a rotating reference frame.

08. State the empirical laws of static and kinetic friction.

i) The magnitude of static frictional force f_s satisfies the following empirical relation.
$0 \leq f_s \leq \mu_s N_s$ where μ_s is the coefficient of static friction.

ii) The force of static friction can take any value from zero to $\mu_s N$.

iii) If the object is at rest and no external force is applied on the object, the static friction acting on the object is zero ($f_s = 0$).

iv) If the object is at rest, and there is an external force applied parallel to the surface, then the force of static friction acting on the object is exactly equal to the external force applied on the object ($f_s = F_{ext}$). But still the static friction f_s is less than $\mu_s N$.

v) When object begins to slide, the static friction (f_s) acting on the object attains maximum.

vi) The static and kinetic frictions depend on the normal force acting on the object.

vii) The static friction does not depend upon the area of contact.

09. State Newton's third law.

For every action there is an equal and opposite reaction.

10. What are inertial frames?

01. If an object is free from all forces, then it moves with constant velocity or remains at rest when seen from inertial frames.
02. Thus, there exists some special set of frames in which if an object experiences no force It moves with constant velocity or remains at rest.

11. Under what condition will a car skid on a leveled circular road?

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid. $\mu < \frac{v^2}{rg}$ (skid)

12. Define impulse.

If a very large force acts on an object for a very short duration, then the force is called impulsive force or impulse.

13. State Newton's First law.

Every object continues to be in the state of rest or of uniform motion unless there is external force acting on it.

14. Define Inertia of rest, motion and direction.

The inability of an object to change its state of rest is called **inertia of rest.**

The inability of an object to change its direction of motion on its own is called **inertia of direction.**

The inability of an object to change its state of uniform speed on its own is called **inertia of motion.**

15. What is free body diagram? What are the steps to be followed for developing free body diagram?

Free body diagram is a simple tool to analyze the motion of the object using Newton's laws.

The following systematic steps are followed for developing the free body diagram:

1. Identify the forces acting on the object. 2. Represent the object as a point.
3. Draw the vectors representing the forces acting on the object.

16. What is the concurrent force?

A collection of forces is said to be concurrent, if the lines of forces act at a common point.

17. State Lami's theorem.

The magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

18. State the law of conservation of total linear momentum.

If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector.

In other words, the total linear momentum of the system is conserved in time.

19. What is the role of air bag in a car?

cars are designed with air bags in such a way that when the car meets with an accident, the momentum of the passengers will reduce slowly so that the average force acting on them will be smaller.

20. Define frictional force.

Which always opposes the relative motion between an object and the surface where it is placed. If the force applied is increased, the object moves after a certain limit.

21. Define Angle of Friction.

The angle of friction is defined as the angle between the normal force (N) and the resultant force (R) of normal force and maximum friction force (f_s^{max})

22. Define Angle of repose.

The same as angle of friction. But the difference is that the angle of repose refers to inclined surfaces and the angle of friction is applicable to any type of surface.

23. What are the applications of angle of repose?

01. The angle of inclination of sand trap is made to be equal to angle of repose.

02. Children are fond of playing on sliding board. Sliding will be easier when the angle of inclination of the board is greater than the angle of repose.

24. How does the rolling wheel's work in suitcase?

01. In rolling motion when a wheel moves on a surface, the point of contact with surface is always at rest.
02. Since the point of contact is at rest, there is no relative motion between the wheel and surface. Hence the frictional force is very less.

25. Where does the friction force act?

Walking is possible because of frictional force. Vehicles (bicycle, car) can move because of the frictional force between the tyre and the road. In the braking system, kinetic friction plays a major role.

26. How did the ball bearing reduce kinetic friction?

If ball bearings are fixed between two surfaces, during the relative motion only the rolling friction comes to effect and not kinetic friction.

27. What is the reason for force changes the velocity of the particle?

01. The magnitude of the velocity can be changed without changing the direction of the velocity.
02. In this case the particle will move in the same direction but with acceleration.
03. The direction of motion alone can be changed without changing the magnitude (speed).
04. Both the direction and magnitude (speed) of velocity can be changed. If this happens non circular motion occurs.

28. Define Centripetal force.

If a particle is in uniform circular motion, there must be centripetal acceleration towards the center of the circle. If there is acceleration then there must be some force acting on it with respect to an inertial frame. This force is called centripetal force.

29. How is the centripetal force act in whirling motion?

In the case of whirling motion of a stone tied to a string, the centripetal force on the particle is provided by the tensional force on the string.

In circular motion in an amusement park, the centripetal force is provided by the tension in the iron ropes.

30. How did the car move on circular track?

When a car is moving on a circular track the centripetal force is given by the frictional force between the road and the tyres. Frictional force = mv^2/r m-mass of the car, v-speed of the car r-radius of curvature of track

Even when the car moves on a curved track, the car experiences the centripetal force which is provided by frictional force between the surface and the tyre of the car.

CONCEPTUAL QUESTIONS:

01. Why it is not possible to push a car from inside?

01. When you push on the car from inside, the reaction force of your pushing is balanced out by our body moving backward.
02. The seat behind you pushes against to bring things to static equilibrium. So, we can't push a car from inside.

02. There is a limit beyond which the polishing of a surface increase frictional resistance rather than decreasing it why?

01. Friction arises due to molecular adhesion. For more polishing the molecules of the surface come closer.
02. They offer greater resistance to surface.

03. Can a single isolated force exist in nature? Explain your answer.

No, a single isolated force cannot exist in nature. Because it will **be violation** of Newton's third law.

04. Why does a parachute descend slowly?

A parachute is a device used to slow down on object that is falling towards the ground. When the parachute opens, the air resistance increases. So the person can land safely.

05. When walking on ice one should take short steps. Why?

To **avoid slipping**, take smaller steps. Because these steps causes more normal force and there by **more friction.**

06. When a person walks on a surface, the frictional force exerted by the surface on the person is opposite to the direction of motion. True or false? False.

07. Can the coefficient of friction be more than one?

Yes, $\mu > 1$, friction is stronger than the normal force.

08. Can we predict the direction of motion of a body from the direction of force on it?

In free body diagrams, the force of friction is always parallel to the surface of contact. The force of **kinetic friction** is always opposite the direction of motion.

09. The momentum of a system of particles is always conserved. True or false? True

10. Why is it dangerous to stand near the open door of moving bus?

It is dangerous to stand near the open door (or) steps while travelling in the bus. When the bus takes a sudden turn in a curved road, due to centrifugal force the person is pushed away from the bus. Even though centrifugal force is a pseudo force, its effects are real.

11. When a cricket player catches the ball, he/she pulls his /her hands gradually in the direction of the ball's motion. Why?

01. If he stops his hands soon after catching the ball, the ball comes to rest very quickly.
02. It means that the momentum of the ball is brought to rest very quickly.
03. So the average force acting on the body will be very large.
04. Due to this large average force, the hands will get hurt.
05. To avoid getting hurt, the player brings the ball to rest slowly

12. A man jumping on concrete floor is more dangerous than in sand floor, why?

01. Jumping on a concrete cemented floor is more dangerous than jumping on the sand.
02. Sand brings the body to rest slowly than the concrete floor, so that the average force experienced by the body will be lesser.

FIVE MARKS QUESTIONS:

01. Prove the law of conservation of linear momentum. Use it to find the recoil velocity of a gun when a bullet is fired from it.

i) The force on each particle (Newton's second law) can be written as $\vec{F}_{12}=\frac{d\vec{p}_1}{dt}$ and $\vec{F}_{21}=\frac{d\vec{p}_2}{dt}$

ii) Here $\vec{p}_1$ is the momentum of particle 1which changes due to the force $\vec{F}_{12}$ exerted by particle 2. Further $\vec{p}_2$ is the momentum of particle 2. This changes due to $\vec{F}_{21}$ exerted by particle 1.

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt}\ ;\ \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0\ ;\ \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

iii) It implies that $\vec{p}_1 + \vec{p}_2$ = constant vector (always).

iv) $\vec{p}_1 + \vec{p}_2$ is the total linear momentum of the two particles ($\vec{p}_{tot} = \vec{p}_1 + \vec{p}_2$). It is also called as total linear momentum of the system. Here, the two particles constitute the system.

v) If there are no external forces acting on the system, then the total linear momentum of the system ($\vec{p}_{tot}$) is always a constant vector.

02. What are concurrent forces? State Lami's theorem.

A collection of forces is said to be concurrent, if the lines of forces act at a common point. If they are in the same plane, they are concurrent as well as coplanar forces.

If a system of three concurrent and coplanar forces is in equilibrium, then Lami's theorem states that the magnitude of each force of the system is proportional to sine of the angle between the other two forces. The constant of proportionality is same for all three forces.

$$\frac{|\vec{F}_1|}{\mathrm{Sin}\alpha} = \frac{|\vec{F}_2|}{\mathrm{Sin}\beta} = \frac{|\vec{F}_2|}{\mathrm{Sin}\gamma}$$

Example:

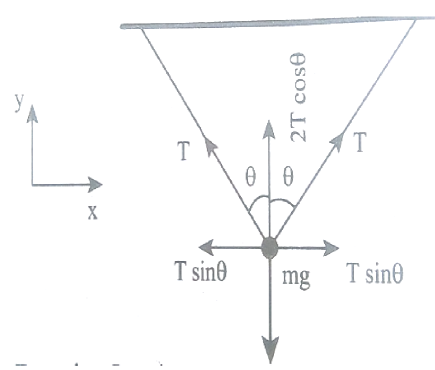
A baby is playing in a swing which is hanging with the help of two identical chains is at rest. Identify the forces acting on the baby. Apply Lami's theorem and find out the tension acting on the chain.

Solution

The baby and the chains are modeled as a particle hung by two strings as shown in the figure. There are three forces acting on the baby.

i) Downward gravitational force along negative y direction (mg)

ii) Tension (T) along the two strings These three forces are coplanar as well as concurrent as shown in the following figure



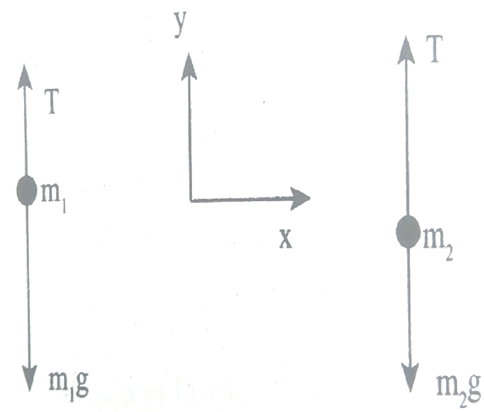
By using Lami's theorem $\frac{T}{\mathrm{Sin}(180-\theta)} = \frac{T}{\mathrm{Sin}(180-\theta)} = \frac{mg}{\mathrm{Sin}(2\theta)}$

Since Sin (180 – θ) = Sin θ and Sin (2θ) = 2sin θcos θ

We get, $\frac{T}{\mathrm{Sin}\theta} = \frac{mg}{2\mathrm{Sin}\theta\mathrm{Cos}\theta)}$;

From this, the tension on each string is $T = \frac{mg}{2\mathrm{Cos}\theta}$

03. Explain the motion of blocks connected by a string in i) Vertical motion ii) Horizontal motion.



Case 1: Vertical motion

i) Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by a light and inextensible string that passes over a pulley.

ii) Let the tension in the string be T and acceleration *a*. When the system is released, both the blocks start moving, m_2 vertically upward and m_1 downward with same acceleration.

The gravitational force m_1g on mass m1 is used in lifting the mass m_2.

Applying Newton's second law for mass m_2,

$$T\hat{j} - m_2g\hat{j} = m_2a\hat{j}$$

iii) The left hand side of the above equation is the total force that acts on m_2 and the right hand side is the product of mass and acceleration of m_2 in y direction.

By comparing the components on both sides, we get

$$T - m_1g = m_2a \text{ ------------- (1)}$$

Similarly, applying Newton's second law for mass m_1

$$T\hat{j} - m_1g\hat{j} = - m_1a\hat{j}$$

As mass m1 moves downward $(-\hat{j})$, its acceleration is along$(-\hat{j})$

iv) By comparing the components on both sides, we get

$$T - m_1g = - m_1a \quad ; m_1g - T = m_1a \text{ ------------- (2)}$$

Adding equations (1) and (2), we get

$$m_1g - m_2g = m_1a + m_2a \,;\, (m_1 - m_2)g = (m_1 + m_2)a \text{ ------- (3)}$$

From equation (3), the acceleration of both the masses is $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$ ------(4)

If both the masses are equal ($m_1 = m_2$), from equation (4) $a = 0$

v) This shows that if the masses are equal, there is no acceleration and the system as a whole will be at rest. To find the tension acting on the string, substitute the acceleration from the equation (4) into the equation (1).

$$T - m_2g = m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right) \;;\; T - m_2g + m_2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \text{ --- (5)}$$

By taking m2g common in the RHS of equation (5)

$$T = m_2g\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) \;;$$

$$T = m_2g\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right)$$

$$\mathbf{T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g}$$

Case 2: Horizontal motion

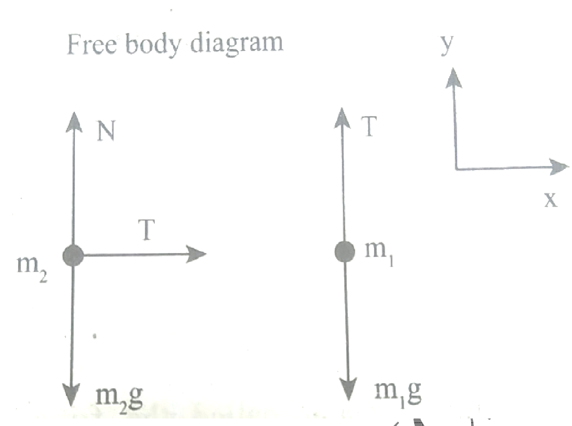
i) In this case, mass m2 is kept on a horizontal table and mass m1 is hanging through a small pulley. Assume that there is no friction on the surface.

ii) As both the blocks are connected to the un-stretchable string, if m_1 moves with an acceleration *a* downward then m_2 also moves with the same acceleration *a* horizontally.

The forces acting on mass m_2 are (i) Downward gravitational force (m_2g)

(ii) Upward normal force (N) exerted by the surface (iii) Horizontal tension (T) exerted by the string

The forces acting on mass m_1 are
(i) Downward gravitational force (m_1g)
(ii) Tension (T) acting upwards
The free body diagrams for both the masses



Applying Newton's second law for m_1

$T\hat{j} - m_1g\hat{j} = m_1a\hat{j}$

By comparing the components on both sides of the above equation,

$T - m_1g = -m_1a$ ------------- (1)

Applying Newton's second law for m_2

$T\hat{i} - m_2a\hat{i}$, By comparing the components on both sides of above equation,

$T = m_2a$ --------------(2)

There is no acceleration along y direction for m_2.

$N\hat{j} - m_2g\hat{j} = 0$, By comparing the components on both sides of the above equation

$N - m_2g = 0$; $N = m_2g$ -------------- (3)

By substituting equation (2) in equation (1), we can find the tension T

$m_2a - m_1g = - m_1a$; $m_2a + m_1a = m_1g$; $a = \frac{m_1}{m_1+m_2}g$ -----------(4)

Tension in the string can be obtained by substituting equation (4) in equation (2)

$$\mathbf{T = \frac{m_1m_2}{m_1+m_2}g}$$

Comparing motion in both cases, it is clear that the tension in the string for horizontal motion is half of the tension for vertical motion for same set of masses and strings.

04. Briefly explain the origin of friction. Show that in an inclined plane, angle of friction is equal to angle of repose.

i) If a very gentle force in the horizontal direction is given to an object at rest on the table, it does not move.

ii) It is because of the opposing force exerted by the surface on the object which resists its motion.

iii) This force is called the frictional force which always opposes the relative motion between an object and the surface where it is placed.

iv) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ . For small angles of θ , the object may not slide down.

v) As θ is increased, for a particular value of θ , the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

vi) Consider the various forces in action here. The gravitational force mg is resolved into components parallel ($mg \sin\theta$) and perpendicular ($mg \cos\theta$) to the inclined plane.

vii) The component of force parallel to the inclined plane ($mg \sin\theta$) tries to move the object down. The component of force perpendicular to the inclined plane ($mg \cos\theta$) is balanced by the Normal force (N).

$N = mg \cos\theta$ --------------(1)

When the object just begins to move, the static friction attains its maximum value,

$f_s = f_s^{max} = \mu_s N$. This friction also satisfies the relation

$f_s^{max} = \mu_s\ mg \sin\theta$ ------------- (2)

Equating the right hand side of equations (1) and (2), we get

$(f_s^{max}) / N = \sin\theta / \cos\theta$

From the definition of angle of friction, we also know that $\tan\theta = \mu_s$

in which **θ** is the angle of friction

05. State Newton's three laws and discuss their significance.

Newton's First Law

i) Every object continues to be in the state of rest or of uniform motion (constant velocity) unless there is external force acting on it.

ii) This inability of objects to move on its own or change its state of motion is called inertia. Inertia means resistance to change its state.

Newton's Second Law

i) The force acting on an object is equal to the rate of change of its momentum $\vec{F} = \frac{d\vec{p}}{dt}$

ii) In simple words, whenever the momentum of the body changes, there must be a force acting on it. The momentum of the object is defined as $\vec{p} = m\vec{v}$. In most cases, the mass of the object remains constant during the motion. In such cases, the above equation gets modified into a simpler form $\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$. $\vec{F} = m\vec{a}$

Newton's third law

i) Newton's third law assures that the forces occur as equal and opposite pairs. An isolated force or a single force cannot exist in nature.

ii) Newton's third law states that for every action there is an equal and opposite reaction.

iii) Here, action and reaction pair of forces do not act on the same body but on two different bodies.

iv) Any one of the forces can be called as an action force and the other the reaction force. Newton's third law is valid in both inertial and non-inertial frames.

v) These action-reaction forces are not cause and effect forces. It means that when the object 1 exerts force on the object 2, the object 2 exerts equal and opposite force on the body 1 at the same instant.

06. Explain the similarities and differences of centripetal and centrifugal forces.

Centripetal force	Centrifugal force
It is a real force which is exerted on the body by the external agencies like gravitational force, tension in the string, normal force etc.	It is a pseudo force or fictitious force which cannot arise from gravitational force, tension force, normal force etc.
Acts in both inertial and non-inertial frames	Acts only in rotating frames (non-inertial frame)
It acts towards the axis of rotation or center of the circle in circular motion	It acts outwards from the axis of rotation or radially outwards from the center of the circular motion
$\lvert F_{cp} \rvert = m\omega^2 r = \frac{mv^2}{r}$	$\lvert F_{cf} \rvert = m\omega^2 r = \frac{mv^2}{r}$
Real force and has real effects.	Pseudo force but has real effects
Origin of centripetal force is interaction between two objects	Origin of centrifugal force is inertia. It does not arise from interaction.
In inertial frames centripetal force has to be included when free body diagrams are drawn.	In an inertial frame the object's inertial motion appears as centrifugal force in the rotating frame. In inertial frames there is no centrifugal force. In rotating frames, both centripetal and centrifugal force have to be included when free body diagrams are drawn.

07. Briefly explain 'centrifugal force' with suitable examples.

i) Consider the case of a whirling motion of a stone tied to a string. Assume that the stone has angular velocity ω in the inertial frame (at rest).

ii) If the motion of the stone is observed from a frame which is also rotating along with the stone with same angular velocity ω then, the stone appears to be at rest.

iii) This implies that in addition to the inward centripetal force $-m\omega^2 r$ there must be an equal and opposite force that acts on the stone outward with value $+m\omega^2 r$.

iv) So the total force acting on the stone in a rotating frame is equal to zero ($-m\omega^2 r + m\omega^2 r = 0$).

v) This outward force $+m\omega^2 r$ is called the centrifugal force.

08. Briefly explain 'Rolling Friction'.

i) One of the important applications is suitcases with rolling on coasters. Rolling wheels makes it easier than carrying luggage.

ii) When an object moves on a surface, essentially it is sliding on it. But wheels move on the surface through rolling motion.

iii) **In rolling motion** when a wheel moves on a surface, **the point of contact with surface is always at rest.**

iv) **Since the point of contact is at rest**, there is no relative motion between the wheel and surface. Hence the **frictional force is very less**. At the same time if an object moves **without a wheel**, there is a relative motion between the object and the surface.

v) As a result **frictional force is larger**. This makes it **difficult to move the object**.

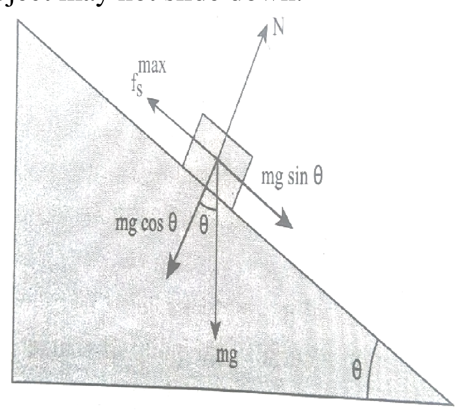
vi) Ideally in pure rolling, motion of the point of contact with the surface should be at rest, but in practice it is not so.

vii) Due to the elastic nature of the surface at the point of contact there will be some deformation on the object at this point on the wheel or surface.

viii) **Due to this deformation, there will be minimal friction between wheel and surface. It is called 'rolling friction'**. In fact, 'rolling friction' is much smaller than kinetic friction.

09. Describe the method of measuring Angle of Repose.

i) Consider an inclined plane on which an object is placed. Let the angle which this plane makes with the horizontal be θ. For small angles of θ , the object may not slide down.



ii) As θ is increased, for a particular value of θ , the object begins to slide down. This value is called angle of repose. Hence, the angle of repose is the angle of inclined plane with the horizontal such that an object placed on it begins to slide.

iii) Consider the various forces in action here. The gravitational force mg is resolved into components parallel (mg sin θ) and perpendicular (mg cos θ) to the inclined plane.

iv) The component of force parallel to the inclined plane (mg sin θ) tries to move the object down. The component of force perpendicular to the inclined plane (mg cos θ) is balanced by the Normal force (N).

$$N = mg \cos\theta \quad \text{--------------(1)}$$

When the object just begins to move, the static friction attains its maximum value,

$f_s = f_s^{max} = \mu_s N$. This friction also satisfies the relation

$$f_s^{max} = \mu_s \, mg \sin\theta \quad \text{------------- (2)}$$

Equating the right hand side of equations (1) and (2), we get

$$(f_s^{max}) / N = \sin\theta / \cos\theta$$

From the definition of angle of friction, we also know that $\mathbf{\tan\theta = \mu_s}$

10. Explain the need for banking of tracks.

i) In a leveled circular road, skidding mainly depends on the coefficient of static friction μ_s The coefficient of static friction depends on the nature of the surface which has a maximum limiting value.

ii) To avoid this problem, usually the **outer edge of the road is slightly raised compared to inner edge**

iii) This is **called banking of roads or tracks**. This introduces an inclination, and the angle is called banking angle.

iv) Let the surface of the road make angle θ with horizontal surface. Then the normal force makes the same angle θ with the vertical.

v) When the car takes a turn, there are two forces acting on the car:

a) Gravitational force mg (downwards)

b) Normal force N (perpendicular to surface)

vi) We can resolve the normal force into two components. $N\cos\theta$ and $N\sin\theta$

vii) The component $N\cos\theta$ balances the downward gravitational force 'mg' and component $N\sin\theta$ will provide the necessary centripetal acceleration. By using Newton second law

$$N\cos\theta = mg \; ; \; N\sin\theta = \frac{mv^2}{r}$$

By dividing the equations we get, $\tan\theta = \frac{v^2}{rg}$

$$\mathbf{v = \sqrt{rg\tan\theta}}$$

Need Banking of tracks:

1) The banking angle θ and radius of curvature of the road or track determines the Safe speed of the car at the turning. If the speed of car exceeds this safe speed, then it starts to skid outward but frictional force comes into effect and provides an additional centripetal force to prevent the outward skidding.

2) At the same time, if the speed of the car is little lesser than safe speed, it starts to skid inward and frictional force comes into effect, which reduces centripetal force to prevent inward skidding.

3) However if the speed of the vehicle is sufficiently greater than the correct speed, then frictional force cannot stop the car from skidding.

11. Calculate the centripetal acceleration of Moon towards the Earth?

i) The centripetal acceleration is given by $a = \frac{v^2}{r}$. This expression explicitly depends on Moon's speed which is non trivial. We can work with the formula $\omega^2 R_m = a_m$

ii) a_m is centripetal acceleration of the Moon due to Earth's gravity.

ω is angular velocity.

R_m is the distance between Earth and the Moon, which is 60 times the radius of the Earth.

$$R_m = 60R = 60 \times 6.4 \times 10^6 = 384 \times 10^6 \text{m}$$

As we know the angular velocity $\omega = \frac{2\pi}{T}$ and T = 27.3 days = 27.3 x 24 × 60 × 60 second = 2.358×10^6 sec

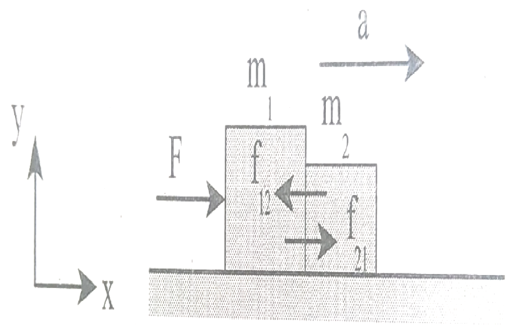
By substituting these values in the formula for acceleration

$a_m = \frac{(4\pi^2)(384\text{x}10^6)}{(2.358\text{x}10^6)^2}$ $\mathbf{a_m = 0.00272ms^{-1}}$

The centripetal acceleration of Moon towards the Earth is $\mathbf{a_m = 0.00272ms^{-1}}$

12. How will you confirm Newton's third law by the way of two bodies in contact on a horizontal surface?

i) Consider two blocks of masses m_1 and m_2 ($m_1 > m_2$) kept in contact with each other on a smooth, horizontal frictionless surface as shown



ii) By the application of a horizontal force F, both the blocks are set into motion with acceleration 'a' simultaneously in the direction of the force F.

iii) To find the acceleration $\vec{a}$, Newton's second law has to be applied to the system (combined mass $m = m_1 + m_2$)

$\vec{F} = m\vec{a}$, If we choose the motion of the two masses along the positive x direction, $F\hat{i} = ma\hat{i}$

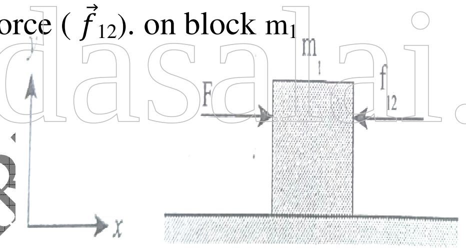
By comparing components on both sides of the above equation

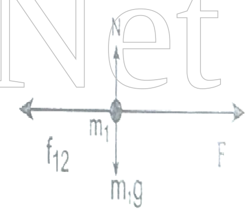
$F = ma$ where $m = m_1 + m_2$

The acceleration of the system is given by $a = \frac{F}{m_1+m_2}$ --------- (1)

iv) The force exerted by the block m_1 on m_2 due to its motion is called force of contact ($\vec{f}_{21}$). According to Newton's third law, the block m_2 will exert an equivalent opposite reaction force ($\vec{f}_{12}$). on block m_1

Free body diagram for mass m





$$F\hat{i} - f_{12}\hat{i} = m_1 a\hat{i}$$

By comparing the components on both sides of the above equation, we get

$$F - f_{12} = m_1 a \ ; \ f_{12} = F - m_1 a \quad \text{-------------(2)}$$

Substituting the value of acceleration from equation (1) in (2) we get,

$$f_{12} = F - m_1\left(\frac{F}{m_1+m_2}\right) ; \ f_{12} = F\left[1 - \frac{m_1}{m_1+m_2}\right]$$

$$f_{12} = \frac{\mathbf{Fm_2}}{\mathbf{m_1+m_2}} \quad \textbf{-------------(3)}$$

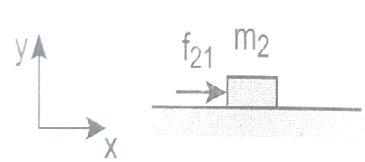
Equation (3) shows that the magnitude of contact force depends on mass m_2 which provides the reaction force. Note that this force is acting along the negative x direction.

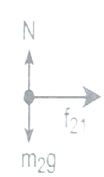
In vector notation, the reaction force on mass m_1 is given by $\vec{f}_{12} = \frac{\mathbf{Fm_2}}{\mathbf{m_1+m_2}}\hat{i}$

v) For mass m_2 there is only one force acting on it in the x direction and it is denoted by $\vec{f}_{21}$. This force is exerted by mass m_1.

Free body diagram for mass m_2

The free body diagram for mass m_2.





Applying Newton's second law for mass m_2.

$$f_{21}\hat{i} = m_2 a\hat{i}$$

By comparing the components on both sides of the above equation

$$f_{21} = m_2 a \quad \text{----------- (4)}$$

Substituting for acceleration from equation (1) in equation (4), we get $\mathbf{f_{21}} = \frac{\mathbf{Fm_2}}{\mathbf{m_1+m_2}}$

vi) In this case the magnitude of the contact force is $\mathbf{f_{21}} = \frac{\mathbf{Fm_2}}{\mathbf{m_1+m_2}}$.

The direction of this force is along the positive x direction.

vii) In vector notation, the force acting on mass m_2 exerted by mass m_1 is $\vec{f}_{21} = \frac{\mathbf{Fm_2}}{\mathbf{m_1+m_2}}\hat{i}$

Note : $\vec{f}_{12} = -\vec{f}_{21}$ which confirms Newton's third law

13. Write the salient features of Static and Kinetic friction.

Static friction	Kinetic friction
It opposes the starting of motion	It opposes the relative motion of the object with respect to the surface
Independent of surface of contact	Independent of surface of contact
μ_s depends on the nature of materials in mutual contact	μ_k depends on nature of materials and temperature of the surface
Depends on the magnitude of applied force	Independent of magnitude of applied force
It can take values from zero to $\mu_s N$	It can never be zero and always equals to $\mu_k N$ whatever be the speed (true <10 ms^{-1})
$f_s^{max} > f_k$	It is less than maximal value of static friction
$\mu_s > \mu_k$	Coefficient of kinetic friction is less than coefficient of static friction

14. Briefly explain what are all the forces act on a moving vehicle on a leveled circular road?

i) When a vehicle travels in a curved path, there must be a centripetal force acting on it. This centripetal force is provided by the frictional force between tyre and surface of the road.

ii) Consider a vehicle of mass 'm' moving at a speed 'v' in the circular track of radius 'r'. There are three forces acting on the vehicle when it moves

1. Gravitational force (mg) acting downwards
2. Normal force (mg) acting upwards
3. Frictional force (F_s) acting horizontally inwards along the road

iii) Suppose the road is horizontal then the normal force and gravitational force are exactly equal and opposite. The centripetal force is provided by the force of static friction F_s between the tyre and surface of the road which acts towards the center of the circular track, $\frac{mv^2}{r} = F_s$,

the static friction can increase from zero to a maximum value $F_s \leq \mu_s mg$

iv) The static friction would be able to provide necessary centripetal force to bend the car on the road. So the coefficient of static friction between the tyre and the surface of the road determines what maximum speed the car can have for safe turn.

If the static friction is not able to provide enough centripetal force to turn, the vehicle will start to skid.

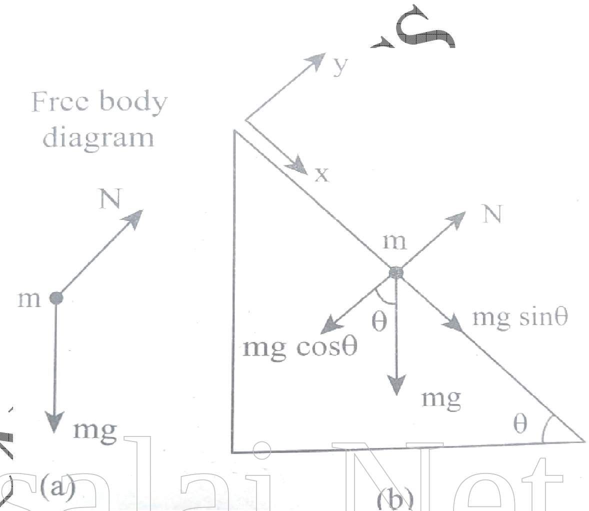
if $\frac{mv^2}{r} > \mu_s$ mg , of $\mu_s < \frac{v^2}{rg}$ (skid)

15. Find i) acceleration ii) speed of the sliding object using free body diagram.

i) To draw the free body diagram, the block is assumed to be a point mass Since the motion is on the inclined surface, we have to choose the coordinate system parallel to the inclined surface as shown in Figure (b).

ii) The gravitational force mg is resolved in to parallel component *mg* sinθ along the inclined plane and perpendicular component *mg* cosθ perpendicular to the inclined surface Figure b.

iii) Note that the angle made by the gravitational force (mg) with the perpendicular to the surface is equal to the angle of inclination θ



iv) There is no motion (acceleration) along the y axis. Applying Newton's second law in they direction

$-\text{mg}\cos\theta\hat{j} + N\hat{j} = 0$(No acceleration)

By comparing the components on both sides, $N - mg\cos\theta = 0$

$N = mg\cos\theta$

v) The magnitude of normal force (N) exerted by the surface is equivalent to $mg\cos\theta$.
The object slides (with an acceleration) along the x direction. Applying Newton's second law in the x direction $mg\sin\theta\,\hat{\imath} = ma\,\hat{\imath}$

By comparing the components on both sides, we can equate $mg\sin\theta = ma$

The acceleration of the sliding object is a = g sinθ

vi) Note that the acceleration depends on the angle of inclination θ . If the angle θ is 90 degree, the block will move vertically with acceleration $a = g$. Newton's kinematic equation is used to find the speed of the object when it reaches the bottom. The acceleration is constant throughout the motion.

$v^2 = u^2 + 2as$ along the x direction

vii) The acceleration a is equal to g sin θ. The initial speed (u) is equal to zero as it starts from rest. Here s is the length of the inclined surface.

The speed (v) when it reaches the bottom is $v = \sqrt{2sg\sin\theta}$

UNIT – 04 WORK, ENERGY AND POWER

TWO MARKS AND THREE MARKS:

01. Explain how the definition of work in physics is different from general perception.

1. Generally any activity can be called as work
2. But in physics, work is said to be done by the force when the force applied on a body displaces it.

02. Write the various types of potential energy. Explain the formulae.

1. The energy possessed by the body due to gravitational force gives rise to gravitational potential energy U = mgh
2. The energy due to spring force and other similar forces give rise to elastic potential energy. $U = \frac{1}{2} Kx^2$
3. The energy due to electrostatic force on charges gives rise to electrostatic potential energy. U = – E. dr

03. Write the differences between conservative and Non-conservative forces. Give two examples each.

Conservative forces	Non-conservative forces
Work done is independent of the path	Work done depends upon the path
Work done in a round trip is zero	Work done in a round trip is not zero
Total energy remains constant	Energy is dissipated as heat energy
Work done is completely recoverable	Work done is not completely recoverable
Force is the negative gradient of potential energy	No such relation exists.

04. Explain the characteristics of elastic and inelastic collision.

Elastic Collision	Inelastic Collision
Total momentum is conserved	Total momentum is conserved
Total kinetic energy is conserved	Total kinetic energy is not conserved
Forces involved are conservative forces	Forces involved are non-conservative forces
Mechanical energy is not dissipated	Mechanical energy is dissipated into heat, light, sound etc.

05. Define the following

a) Coefficient of restitution b) Power c) Law of conservation of energy
d) Loss of kinetic energy in inelastic collision.

a) Coefficient of restitution

It is defined as the ratio of velocity of separation (relative velocity) after collision to the velocity of approach (relative velocity) before collision, i.e.,

$$e = \frac{\text{Velocity of separation (after collision)}}{\text{Velocity of approach (before collision)}} \; ; \; \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

b) Power

The rate of work done or energy delivered.

$$\text{Power (P)} = \frac{\text{Workdone (W)}}{\text{Time taken (t)}}$$

c) Law of conservation of energy

Energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

d) Loss of kinetic energy in inelastic collision

In perfectly inelastic collision, the loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, heat, light etc. Let KE_i be the total kinetic energy before collision and KE_f be the total kinetic energy after collision. Total kinetic energy before collision,

$$KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \text{ -----------(1)}$$

Total kinetic energy after collision, $KE_f = \frac{1}{2}(m_1 + m_2)v^2$ ------------ (2)

Then the loss of kinetic energy is Loss of KE, $\Delta Q = KE_f - KE_i$

$$= \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \text{ -------- (3)}$$

Substituting equation $v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$ in equation (3), and on simplifying (expand v by using the algebra $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$$\textbf{Loss of KE, } \Delta Q = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

06. Define unit of power:

One watt is defined as the power when one joule of work is done in one second. $1W = 1Js^{-1}$

07. Explain Work done.

i) Work is said to be done by the force when the force applied on a body displaces it.
ii) work done is a scalar quantity. It has only magnitude and no direction.
iii) In SI system, unit of work done is N m (or) joule (J). Its dimensional formula is ML^2T^{-2}

08. When does work done becomes zero?

i) When the force is zero (F = 0). ii) When the displacement is zero (dr = 0).
iii) When the force and displacement are perpendicular ($\theta = 90^0$) to each other.

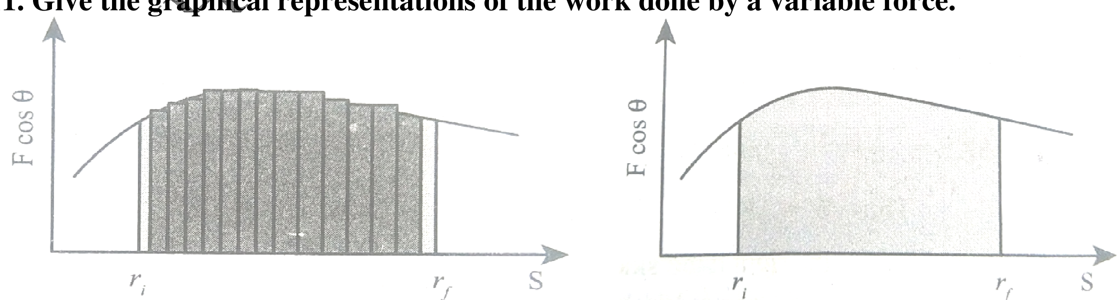
09. Define Work done by a constant force

When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, $dW = (F \cos\theta)\, dr$

10. Define Work done by a variable force

When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation $dW = (F \cos\theta)\, dr$
[F cos θ is the component of the variable force F]

11. Give the graphical representations of the work done by a variable force.



12. Define Energy, Kinetic energy and Potential Energy

Energy: The capacity to do work , Dimension : ML^2T^{-2} , SI Unit : Nm or joule .
Kinetic energy : The energy possessed by a body due to its motion.
Dimension : ML^2T^{-2} , SI Unit : Nm or joule .
Potential Energy: The energy possessed by the body by virtue of its position
Dimension : ML^2T^{-2} , SI Unit : Nm or joule .

13. Write the significance of kinetic energy in the work – kinetic energy theorem.

1. If the work done by the force on the body is **positive** then its **kinetic energy increases.**
2. If the work done by the force on the body is **negative** then its **kinetic energy decreases**.
3. If there is **no work done** by the force on the body then there is **no change** in its kinetic energy

14. Define Work – kinetic energy theorem.

The work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.

15. Define elastic potential energy

The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy.

16. Define Conservative force

A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.

17. Define Non-conservative force

A force is said to be non-conservative if the work done by or against the force in moving a body depends upon the path between the initial and final positions. This means that the value of work done is different in different paths.

18. Define A*verage power*

The average power (P_{av}) is defined as the ratio of the total work done to the total time taken. $P_{av} = \frac{\text{Total work done}}{\text{Total time taken}}$

19. Define Instantaneous power

The instantaneous power (P_{inst}) is defined as the power delivered at an instant (as time interval approaches zero), $P_{inst} = \frac{dw}{dt}$

20. What is meant by collision?

Collision is a common phenomenon that happens around us every now and then. For example, carom, billiards, marbles, etc.,. Collisions can happen between two bodies with or without physical contacts.

21. What is Elastic Collision?

In a collision, the total initial kinetic energy of the bodies (before collision) is equal to the total final kinetic energy of the bodies (after collision) then, it is called as elastic collision. i.e., Total kinetic energy before collision = Total kinetic energy after collision

22. What is Inelastic Collision?

In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e., Total kinetic energy before collision ≠ Total kinetic energy after collision

CONCEPTUAL QUESTIONS

01. Which is conserved in inelastic collision? Total energy or Kinetic energy?

The total energy of the system is conserved in inelastic collision. The kinetic energy is not conserved because it is carried by the moving objects or it is transformed into other form of energy.

02. Is there any net work done by external forces on a car moving with a constant speed along a straight road?

At a constant speed, a car can be carrying around corners or driving in a curved path. That would cause all sorts of acceleration. But an object travelling at a constant speed, in a straight line and by Newton's first law of motion. If has no external force acting on it.

03. A charged particle moves towards another charged particle. Under what conditions the total momentum and the total energy of the system conserved?

1. If positive and negative charged particles moves towards another
2. After collision the charged particles should stick together permanent.
3. So, they should move with common velocity under this situation the total momentum and total energy of the system is conserved.

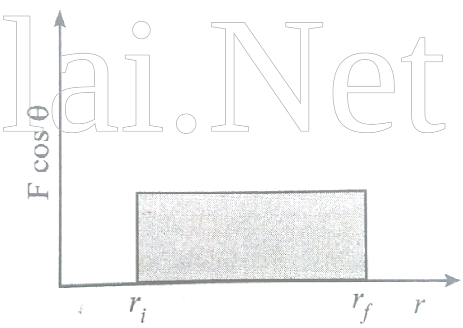
FIVE MARKS QUESTIONS

01. Explain with graphs the difference between work done by a constant force and by a variable force.

i) When a constant force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation, dW= (F cosθ) dr

ii) The total work done in producing a displacement from initial position $\mathbf{r_i}$ to final position $\mathbf{r_f}$ is, $W = \int_{r_i}^{r_f} dW$;

$W = \int_{r_i}^{r_f} (F\cos\theta)dr = (F\cos\theta)\int_{r_i}^{r_f} dr = (F\cos\theta)(r_f - r_i)$



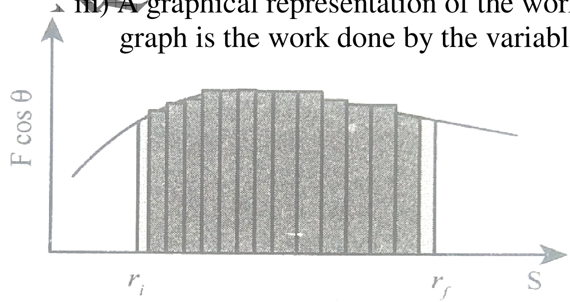
iii) The graphical representation of the work done by a constant force .The area under the graph shows the work done by the constant force.

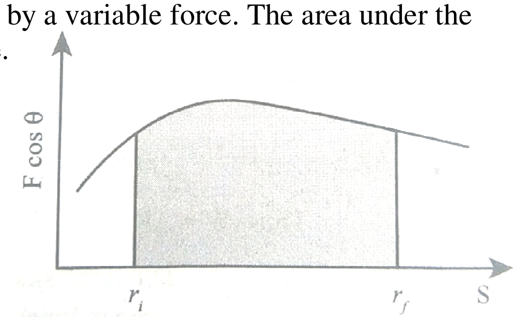
Work done by a variable force

i) When the component of a variable force F acts on a body, the small work done (dW) by the force in producing a small displacement dr is given by the relation dW = (Fcos θ) dr [F cos θ is the component of the variable force F] where, F and θ are variables.

ii) The total work done for a displacement from initial position r_i to final position r_f is given by the relation, $W = \int_{r_i}^{r_f} dW; \; = \int_{r_i}^{r_f} (F\cos\theta)dr$

iii) A graphical representation of the work done by a variable force. The area under the graph is the work done by the variable force.





02. State and explain work energy principle. Mention any three examples for it.

1) It states that work done by the force acting on a body is equal to the change produced in the kinetic energy of the body.
2) Consider a body of mass m at rest on a frictionless horizontal surface.
3) The work (W) done by the constant force (F) for a displacement (s) in the same direction is, W = Fs ------------- (1)

The constant force is given by the equation, F = ma ---------- (2)

The third equation of motion can be written as, $v^2 = u^2 + 2as$

$a = \frac{v^2-u^2}{2s}$ ------------------- (3)

Substituting for a in equation (2), $F = m\left(\frac{v^2-u^2}{2s}\right)$ ------------- (4)

Substituting equation (4) in (1), $W = m\left(\frac{v^2}{2s}s\right) - m\left(\frac{u^2}{2s}s\right)$

$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ --------------------- (5)

The expression for kinetic energy:

i) The term ½ (mv^2) in the above equation is the kinetic energy of the body of mass (m) moving with velocity (v). $KE = \frac{1}{2} mv^2$ ------------------ (6)

ii) Kinetic energy of the body is always positive. From equations (5) and (6)
$\Delta KE = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ -----------------(7) thus, W = ΔKE

iii) The expression on the right hand side (RHS) of equation (7) is the change in kinetic energy (ΔKE) of the body.

iv) This implies that the work done by the force on the body changes the kinetic energy of the body. This is called work-kinetic energy theorem.

03. Arrive at an expression for power and velocity. Give some examples for the same.

i) The work done by a force $\vec{F}$ for a displacement $d\vec{r}$ is $W = \int \vec{F}.d\vec{r}$ ------- (1)

Left hand side of the equation (1) can be written as

$W = \int dW = \int \frac{dW}{dt}dt$ (multiplied and divided by dt) -------------- (2)

ii) Since, velocity $\vec{v} = \frac{d\vec{r}}{dt}$; $d\vec{r} = \vec{v}dt$. Right hand side of the equation (1) can be written as $\int \vec{F}.d\vec{r} = \int\left(\vec{F}.\frac{d\vec{r}}{dt}\right)dt = \int\left(\vec{F}.\vec{v}\right)dt$ $\left[\vec{v} = \frac{d\vec{r}}{dt}\right]$ ---------------- (30

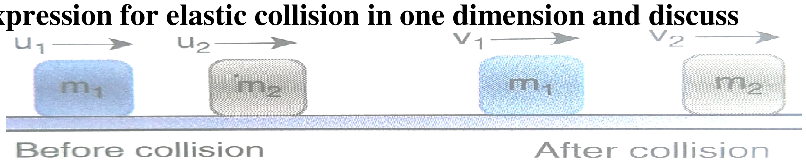
Substituting equation (2) and equation (3) in equation (1), we get

$\int \frac{dW}{dt}dt = \int\left(\vec{F}.\vec{v}\right)dt$; $\int\left(\frac{dW}{dt} - \vec{F}.\vec{v}\right)dt = 0$

iii) This relation is true for any arbitrary value of dt. This implies that the term within the bracket must be equal to zero, i.e.,

$\frac{dW}{dt} - \vec{F}.\vec{v} = 0$ (or) $\frac{dW}{dt} = \vec{F}.\vec{v}$

04. Arrive at an expression for elastic collision in one dimension and discuss various cases.



Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface.

i) In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e., $\mu_1 > \mu_2$ For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

From the law of conservation of linear momentum, Total momentum before collision (*pi*) = Total momentum after collision (*pf*)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ -------------- (1) (or)

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ------------------- (2)

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$ ------------------ (3)

After simplifying and rearranging the terms,

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

Using the formula, $a^2 - b^2 = (a + b)(a - b)$, we can rewrite the above equation as

$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2)$ ------------ (4)

Dividing equation (4) by (2) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)} = \frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$u_1 + v_1 = v_2 + u_2$, Re-arranging $u_1 - u_2 = v_2 - v_1$ -------------- (5)

Equation (5) can be rewritten as $(u_1 - u_2 = -(v_1 - v_2)$

ii) This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2 ,

$v_1 = v_2 + u_2 - u_1$ ------------- (6) **or** $v_2 = u_1 + v_1 - u_2$ ----------------(7)

To find the final velocities v_1 and v_2:

Substituting equation (7) in equation (2) gives the velocity of m_1 as

$m_1(u_1 - v_1) = m_2 (u_1 + v_1 - u_2 - u_2)$

$m_1(u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2) u_1 + 2m_2u_2 = (m_1 + m_2)v_1$ **(or)**

$$v_1 = \left(\frac{m_1 - m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right) u_2 \text{ ----------(8)}$$

Similarly, by substituting (6) in equation (2) or substituting equation (8) in equation (7), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right) u_1 + \left(\frac{m_2 - m_1}{m_1+m_2}\right)u_2 \text{ ----------- (9)}$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

Equation (8) ⟶ $v_1 = (0)u_1 + \left(\frac{2m_2}{2m_2}\right) u_2$; $v_1 = u_2$ ------------- (10)

Equation (9) ⟶ $v_2 = \left(\frac{2m_1}{2m_1}\right) u_1 + (0) u_2$; $v_2 = u_1$ ------------- (11)

The equations (10) and (11) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and second body (usually called target) is at rest ($u_2 = 0$),

By substituting $m_1 = m_2$ and $u_2 = 0$ in equations (8) and equations (9) we get,

From equation (8) ⟶ $v_1 = 0$ ---------------(12)

From equation (9) ⟶ $v_2 = u_1$ --------------- (13)

Equations (12) and (13) show that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: The first body is very much lighter than the second body

$\left(m_1 \ll m_2, \frac{m_1}{m_2} \ll 1\right)$ then the ratio $\frac{m_1}{m_2} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_2, we get

$$v_1 = \left(\frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2}+1}\right) u_1 + \left(\frac{2}{\frac{m_1}{m_2}+1}\right)(0);\ v_1 = \left(\frac{0-1}{0+1}\right)u_1;\ v_1 = -u_1 \text{ ------------(14)}$$

Similarly, Dividing numerator and denominator of equation (9) by m_2, we get

$$v_2 = \left(\frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0)\ ;\ v_2 = (o)u_1 + \left(\frac{1-\frac{m_1}{m_2}}{\frac{m_1}{m_2}+1}\right)(0)\ ;\ v_2 = 0 \text{ ----- } (15)$$

The equation (14) implies that the first body which is lighter returns back rebounds) in the opposite direction with the same initial velocity as it has a negative sign.

The equation (15) implies that the second body which is heavier in mass continues to remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back from the wall with the same velocity with which it was thrown but in opposite direction.

Case 4: The second body is very much lighter than the first body

$\left(m_2 \ll m_1, \frac{m_2}{m_1} \ll 1\right)$ then the ratio $\frac{m_2}{m_1} \approx 0$. And also if the target is at rest (u_2=0)

Dividing numerator and denominator of equation (8) by m_1, we get

$$v_1 = \left(\frac{1-\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{2\frac{m_2}{m_1}}{1+\frac{m_2}{m_1}}\right)(0)\ ;$$

$$v_1 = \left(\frac{0-1}{0+1}\right)u_1 + \left(\frac{0}{1+0}\right)(0)\ ;$$

$$v_1 = u_1 \text{ ------------------- } (16)$$

Similarly, Dividing numerator and denominator of equation (14) by m_1, we get

$$v_1 = \left(\frac{2}{1+\frac{m_2}{m_1}}\right)u_1 + \left(\frac{\frac{m_2}{m_1}-1}{1+\frac{m_2}{m_1}}\right)(0)\ ;$$

$$v_2 = \left(\frac{2}{1+0}\right)u_1\ ;\ v_2 = 2u_1 \text{ ------ } (17)$$

The equation (16) implies that the first body which is heavier continues to move with the same initial velocity.

The equation (17) suggests that the second body which is lighter will move with twice the initial velocity of the first body.

It means that the lighter body is thrown away from the point of collision.

05. What is inelastic collision? In which way it is different from elastic collision. Mention few examples in day to day life for inelastic collision.

1) In a collision, the total initial kinetic energy of the bodies (before collision) is not equal to the total final kinetic energy of the bodies (after collision) then, it is called as inelastic collision. i.e.,
2) Momentum is conserved. Kinetic energy is not conserved in elastic collision. Mechanical energy is dissipated into heat, light, sound etc. When a light body collides against any massive body at rest it sticks to it.
3) Total kinetic energy before collision $\neq$ Total kinetic energy after collision

$$\begin{bmatrix}\text{Total kinetic energy}\\ \text{after collision}\end{bmatrix} - \begin{bmatrix}\text{Total kinetic energy}\\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Loss in energy during}\\ \text{collision}\end{bmatrix}$$

4) Even though kinetic energy is not conserved but the total energy is conserved.
5) loss in kinetic energy during collision is transformed to another form of energy like sound, thermal, etc.
6) if the two colliding bodies stick together after collision such collisions are known as completely inelastic collision or perfectly inelastic collision.
7) For example when a clay putty is thrown on a moving vehicle, the clay putty (or Bubblegum) sticks to the moving vehicle and they move together with the same velocity.

06. Deduce the relation between momentum and kinetic energy.

i) Consider an object of mass m moving with a velocity $\vec{v}$. Then its linear momentum is $\vec{p}$ = m$\vec{v}$ and its kinetic energy, KE = ½ mv^2

KE = ½ mv^2 ; = ½ m($\vec{v}.\vec{v}$) --------(1)

ii) Multiplying both the numerator and denominator of equation (1) by mass, m

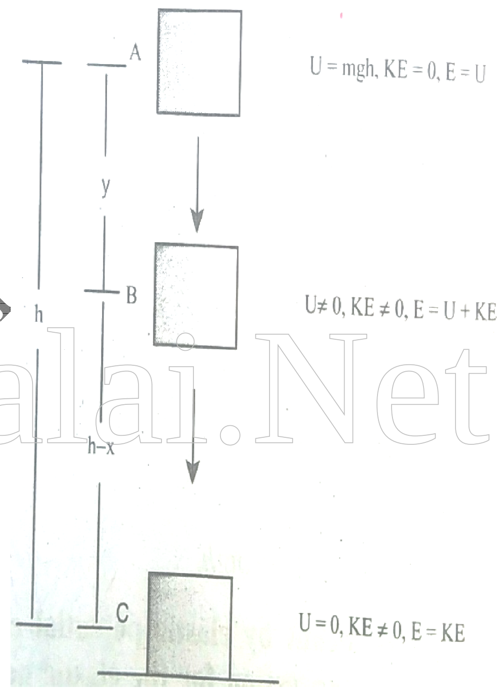
$$KE = \frac{1}{2}\frac{m^2(\vec{v}.\vec{v})}{m} \;;\; = \frac{1}{2}\frac{(m\vec{v}).(m\vec{v})}{m} \;\; [\vec{p} = m\vec{v}] \;;\; = \frac{1}{2}\frac{(\vec{p}).(\vec{p})}{m}$$

$$= \frac{\vec{p}^2}{2m} \;;\; KE = \frac{p^2}{2m}$$

iii) Where $|\vec{p}|$is the magnitude of the momentum. The magnitude of the linear momentum can be obtained by $|\vec{p}| = p = \sqrt{2m(KE)}$

iv) Note that if kinetic energy and mass are given, only the magnitude of the momentum can be calculated but not the direction of momentum. It is because the kinetic energy and mass are scalars.

07. State and prove the law of conservation of energy.



i) When an object is thrown upwards its kinetic energy goes on decreasing and consequently its potential energy keeps increasing (neglecting air resistance).

ii) When it reaches the highest point its energy is completely potential. Similarly, when the object falls back from a height its kinetic energy increases whereas its potential energy decreases.

iii) When it touches the ground its energy is completely kinetic. At the intermediate points the energy is both kinetic and potential.

iv) When the body reaches the ground the kinetic energy is completely dissipated into some other form of energy like sound, heat, light and deformation of the body etc.

v) In this example the energy transformation takes place at every point. The sum of kinetic energy and potential energy i.e., the total mechanical energy always remains constant, implying that the total energy is conserved. This is stated as the law of conservation of energy.

vi) The law of conservation of energy states that energy can neither be created nor destroyed. It may be transformed from one form to another but the total energy of an isolated system remains constant.

vii) The figure illustrates that, if an object starts from rest at height *h*, the total energy is purely potential energy (U=mgh) and the kinetic energy (KE) is zero at *h*. When the object falls at some distance *y*, the potential energy and the kinetic energy are not zero whereas, the total energy remains same as measured at height *h*. When the object is about to touch the ground, the potential energy is zero and total energy is purely kinetic.

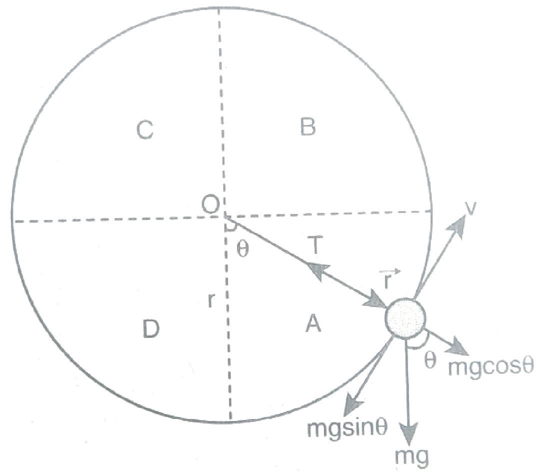
08. Derive an expression for the velocity of the body moving in a vertical circle and also find a tension at the bottom and the top of the circle.

1) A body of mass (m) attached to one end of a mass less and inextensible string executes circular motion in a vertical plane with the other end of the string fixed. The length of the string becomes the radius ($\vec{r}$) of the circular path.

2) The motion of the body by taking the free body diagram (FBD) at a position where the position vector ($\vec{r}$) makes an angle θ with the vertically downward direction and the instantaneous velocity .

3) There are two forces acting on the mass.

1. Gravitational force which acts downward
2. Tension along the string.



Applying Newton's second law on the mass,

In the tangential direction,

$mg \sin\theta = ma_t$;

$mg \sin\theta = -m\left(\frac{dv}{dt}\right)$

where , $a_t = -\left(\frac{dv}{dt}\right)$ is tangential retardation in the radial direction,

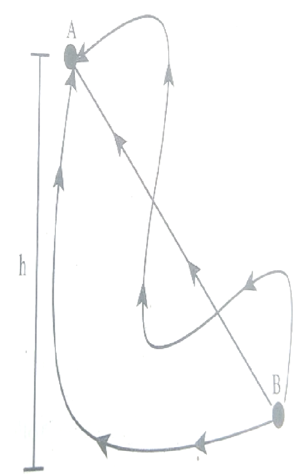
$T - mg\cos\theta = m\,a_r$;

$T - mg\cos\theta = \frac{mv^2}{r}$

where , $a_r = \frac{v^2}{r}$ is the centripetal acceleration.

09. What is conservative force? State how it is determined from potential energy?

i) A force is said to be a conservative force if the work done by or against the force in moving the body depends only on the initial and final positions of the body and not on the nature of the path followed between the initial and final positions.



ii) Consider an object at point A on the Earth. It can be taken to another point B at a height h above the surface of the Earth by three paths.

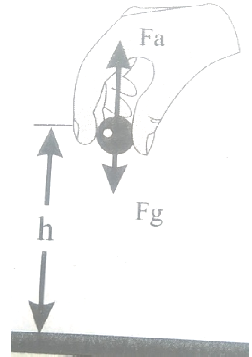
iii) Whatever may be the path, the work done against the gravitational force is the same as long as the initial and final positions are the same.

iv) This is the reason why gravitational force is a conservative force.

v) Conservative force is equal to the negative gradient of the potential energy. In one dimensional case, $F_x = -\frac{dU}{dx}$

vi) Examples for conservative forces are elastic spring force, electrostatic force, magnetic force, gravitational force, etc.

10. Derive an expression for the potential energy of a body near the surface of the Earth.



1) The gravitational potential energy (U) at some height is equal to the amount of work required to take the object from ground to that height with constant velocity.
2) Consider a body of mass being moved from ground to the height h against the gravitational force.
3) The gravitational force $\vec{F}_g$ acting on the body is, $\vec{F}_g = -mg\hat{j}$ (as the force is in y direction, unit vector is used). Here, negative sign implies that the force is acting vertically downwards. In order to move the body without acceleration (or with constant velocity), an external applied force $\vec{F}_a$ equal in magnitude but opposite to that of gravitational force $\vec{F}_g$ has to be applied on the body i.e., $\vec{F}_a - \vec{F}_g$ This implies that $\vec{F}_a = -mg\hat{j}$
4) The positive sign implies that the applied force is in vertically upward direction. Hence, when the body is lifted up its velocity remains unchanged and thus its kinetic energy also remains constant.
5) The gravitational potential energy (U) at some height *h* is equal to the amount of work required to take the object from the ground to that height *h*.

$U = \int \vec{F}_a . d\vec{r}$

$= \int_0^h |\vec{F}_a| \, |d\vec{r}| \cos\theta$

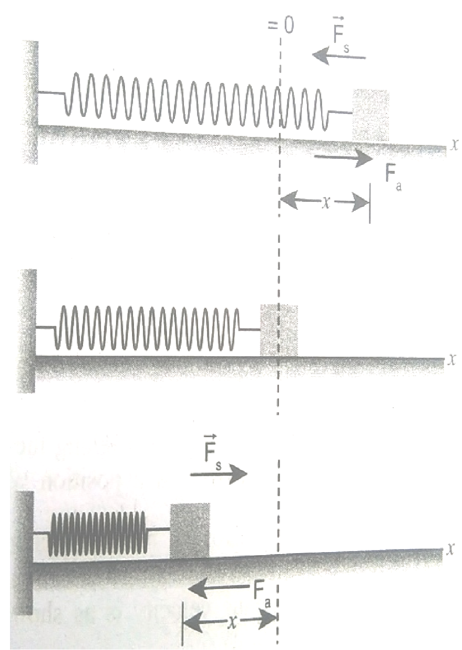
6) Since the displacement and the applied force are in the same upward direction, the angle between them, $\theta = 0$.

Hence Cos 0 = 1 and $|\vec{F}_a|$ = mg and $|d\vec{r}|$ = dr

$U = mg \int_0^h dr$;

$U = mg\, [r]_0^h$; **U = mgh**

11. What is meant by elastic potential energy? Derive an expression for the elastic potential energy of the spring?



1) The potential energy possessed by a spring due to a deforming force which stretches or compresses the spring is termed as elastic potential energy. The work done by the applied force against the restoring force of the spring is stored as the elastic potential energy in the spring.

2) Consider a spring-mass system. Let us assume a mass, lying on a smooth *m* horizontal table. Here, *x* = 0 is the equilibrium position. One end of the spring is attached to a rigid wall and the other end to the mass.

3) As long as the spring remains in equilibrium position, its potential energy is zero. Now an external force $\vec{F}_a$ is applied so that it is stretched by a distance (*x*) in the direction of the force.

4) There is a restoring force called spring force $\vec{F}_s$ developed in the spring which tries to bring the mass back to its original position. This applied force and the spring force are equal in magnitude but opposite in direction i.e., $\vec{F}_a = -\vec{F}_s$. According Hooke's law, the restoring force developed in the spring is $\vec{F}_s = -k\vec{x}$

5) The negative sign in the above expression implies that the spring force is always opposite to that of displacement $\vec{x}$ and k is the force constant. Therefore applied force.
is $\vec{F}_a = +k\vec{x}$. The positive sign implies that the applied force is in the direction of displacement $\vec{x}$. The spring force is an example of variable force as it depends on the displacement $\vec{x}$. Let the spring be stretched to a small distance $d\vec{x}$. The work done by the applied force on the spring to stretch it by a displacement $\vec{x}$ is stored as elastic potential energy

$$U = \int \vec{F}_a . d\vec{r}$$
$$= \int_0^x |\vec{F}_a|\, |d\vec{r}| \cos\theta \;;$$
$$= \int_0^x F_a dx \cos\theta$$

6) The applied force $\vec{F}_a$ and the displacement $d\vec{r}$(i.e., here dx) are in the same direction. As, the initial position is taken as the equilibrium position or mean position, x=0 is the lower limit of integration.

$$U = \int_0^x kx dx \;;$$
$$U = k\left[\frac{x^2}{2}\right]_0^x \;;$$
$$U = \tfrac{1}{2} kx^2 \text{ ---------------(1)}$$

7) If the initial position is not zero, and if the mass is changed from position x_i to x_f , then the elastic potential energy is

$$U = \tfrac{1}{2} k\,(x_f^2 - x_i^2) \text{ --------------(2)}$$

From equations (1) and (2), we observe that the potential energy of the stretched spring depends on the force constant k and elongation or compression x.

PREPARED BY
RAJENDRAN M, M.Sc., B.Ed., C.C.A.,
P. G. ASSISTANT IN PHYSICS,
DEPARTMENT OF PHYSICS,
SRM HIGHER SECONDARY SCHOOL,
KAVERIYAMPOONDI,
THIRUVANNAMALAI DISTRICT.
For your suggestion: mrrkphysics@gmail.com, murasabiphysics@gmail.com