XVA: About CVA, DVA, FVA and Other Market Adjustments Preprint of *Opinion and Debates*[*] num. 5, June 2014

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Abstract

Since the crisis, different adjustments are needed to account for counterparty risk and funding costs in the risk management of OTC derivatives, notably credit valuation adjustment (CVA), debt valuation adjustment (DVA) and funding valuation adjustment (FVA). These adjustments, which are, to some extent, interdependent and must be computed jointly, count today among the main P&L centers of investment banks. They touch on many areas: modeling, computation, pricing, risk management, regulation, economics, legal, lobbying, politics, often in conflicting perspectives. Banks have to cope simultaneously with economic risk, accounting P&L and regulatory capital considerations. The current trend of the regulation is to push participants to negotiate centrally via clearing houses or to bring strong guarantees in terms of collateralization. But this evolution poses liquidity and systemic risks issues.

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Introduction

The global credit crisis followed by the European sovereign debt crisis have highlighted the native form of credit (not to say financial) risk, namely counterparty risk. Counterparty risk is the risk of non-payment due to the default of a party in an OTC derivatives transaction. By extension this is also the volatility of the price of this risk, the CVA (credit valuation adjustment). An important aspect of the problem, especially with credit derivatives, is the wrong-way risk, i.e. the risk of adverse dependence between the size of the counterparty risk exposure and the default riskiness of the counterparty. Moreover, as banks themselves have become risky, counterparty risk must now be understood in a bilateral perspective (not only CVA but also DVA, i.e. debt valuation adjustment), where the counterparty risk of the two parties are jointly accounted for in the modeling. In this context, the classical assumption of a risk-free asset that can be used for financing purposes (lending or borrowing as needed) is not sustainable anymore. Hence, another adjustment is needed in the form of an FVA, i.e. funding valuation adjustment, or a more specific LVA (liquidity component of the FVA, net of credit spread, in order to avoid double counting with DVA). As will be seen, there is still another adjustment, called replacement cost (RC), corresponding to the mismatch between the economical value of the contract and its valuation by the liquidator at default time. And the list is not closed since people now talk about KVA for capital valuation adjustment (in reference to the capital cost of CVA volatility), or AVA for additional valuation adjustment (toward "prudent valuation", accounting for model risk and credit spreads including own, recently opposed to fair value by Basel). An acronym XVA was even introduced to generically refer to this increasing list of adjustments (see Carver (2013)). Finally, in August 2007 a new dimension of systemic counterparty risk has appeared, with the emergence of spreads between quantities that were very similar before, such as OIS swap rates and LIBOR rates (again a consequence of banks' counterparty risk, but at a macro level). Through its relation with the concept of discounting, this systemic component of counterparty risk has impacted on all derivatives markets. All the above adjustments, which are interdependent and must be computed jointly, now count among the main P&L centers of investment banks. The current trend of the regulation is to push participants to negotiate centrally via clearing houses or to bring strong guarantees in terms of collateralization. But this evolution poses liquidity and systemic risks issues.

The basic counterparty risk mitigation tool is a credit support annex (CSA, sometimes also-called ISDA agreement) specifying the valuation scheme that will be applied by the liquidator in case of default of a party, including the netting rules applicable to the underlying portfolio. The CSA value process also serves of reference for the determination of margin calls, similar to a margining procedure for futures contracts, except that the collateral posted through a CSA remains the property of the posting party, so that it is remunerated. However, wrong-way risk (see above) and gap risk (slippage of the value of the portfolio between default and liquidation) imply that collateralization cannot be a panacea. It also poses liquidity problems. Therefore counterparty risk cannot be simply mitigated, through collateralization, it also needs to be hedged against default and/or market risk. Eventually, the collateralized and hedged portfolio needs to be funded, which raises the controversial issue of DVA/FVA overlap or double counting.

From the point of view of the organization of the bank, due to netting, counterparty risk and funding costs can only be assessed at the CSA portfolio level (or better, regarding funding costs, at the level of the whole book of the bank). Therefore the trend is to have a central XVA desk in charge of valuing and hedging counterparty risk (funding costs are typically managed by the treasury or ALM of the bank). A “clean” price and hedge ignoring counterparty risk and assuming that all trading strategies are funded at the risk-free rate (or OIS rate, see Box 7) is first computed by the different business trading desks. Then the XVA desk values the counterparty risk of the portfolio and channels this charge back to the various trading desks, after a desallocation and conversion of a global and upfront charge into streams of fixed coupons. The all-inclusive price-and-hedge of a contract are finally obtained as the difference between the clean price-and-hedge and the price-and-hedge adjustment provided by the XVA desk (also accounting for funding costs as provided by the ALM).

Review of the literature Here is a brief review of books about counterparty risk and funding[1]:

- The first book one can mention is the collection of seminal CVA papers in Pykhtin (2005) (now of course a little outdated).

- The book by Gregory (2009, 2012) is technically quite simple but explains basic CVA concepts in a clear way. It is good for managers and finance people who need to get a general grasp of CVA fundamentals with some elements about funding/discounting, without going too technical.

- The book by Cesari, Aquilina, Charpillon, Filipovic, Lee, and Manda (2010) is rather basic from a modeling point of view but it also looks at the IT implications of building a CVA system and tries to solve a number of practical problems that deal with CVA for realistically large portfolios. The focus is on the so-called American (or least square) Monte Carlo technique, first introduced for CVA applications in Brigo and Pallavicini (2007, 2008).

- The book by Kenyon and Stamm (2012), although technically basic, is original, in that it tackles current and relevant problems such as multi-curve modeling and credit valuation adjustments, closeout and especially goodwill (which depends on the creditworthiness of a firm and can therefore be used for hedging the DVA). Funding costs, hints at systemic risk, regulation and Basel III are also considered.

- The book by Brigo, Morini, and Pallavicini (2013) is mostly based on Brigo’s work with several co-authors in the period 2002-2012 and has been written to be widely accessible while being technically advanced. It deals with CVA, DVA and wrong-way risk across different asset classes, gap risk, collateral, closeout, rehypothecation and funding costs. There is also a final part on CVA restructuring through so-called CCDS (“contingent CDS”), CDO tranches type structures, floating rate CVA and margin lending.

- The book by Crépey, Bielecki, and Brigo (2014) focuses on the mathematical dependence structure of the problem, using mainstream stochastic analysis: BSDEs in particular, in line with Crépey (2013), to address the nonlinear recursive nature of the funding issue in a systematic way, and dynamic copulas to reconcile bottom up and top down perspectives in the study of counterparty risk embedded in credit derivatives.

[1] This book review is essentially borrowed from Chapter 1 in Crépey, Bielecki, and Brigo (2014).

In this article, we review counterparty risk and funding costs in their different aspects such as CVA, DVA, FVA, LVA, RC (Sect. 1 through 3), collateral, wrong-way risk (Sect. 4) and central clearing (Sect. 5). On our way, we discuss in box form a number of aside issues such as historical versus risk-neutral valuation ("$\mathbb{P}$ versus $\mathbb{Q}$"), multiple curves and we provide a brief informal introduction to BSDEs. The boxes are also to insist on important messages.

1 Credit Valuation Adjustment

The CVA (credit valuation adjustment) is the price of counterparty risk. By extension, counterparty risk is also the risk of volatility of the CVA. In fact, during the financial crisis, it was said that roughly two-thirds of losses attributed to counterparty were due to CVA losses and only about one-third were due to actual defaults (see Lambe (2011)).

A debt towards a defaulted counterparty is due in full to the liquidator (a nonpaying party would itself be declared in default). Therefore, the counterparty risk exposure is only the positive part of the value of a contract (or portfolio). It follows that for any swapped contract (as opposed to, say, a bond position, which is always the same side of the money, lender or borrower), the loss associated to this risk has an optional feature (see Fig. 1). This is why counterparty risk cannot be simply handled by the application of a simple spread in the discount rate (or this would be an implicit and stochastic spread). Moreover, due to the netting rules that apply at the CSA portfolio level, where a CSA can cover tens of thousands of contracts, often across different classets of assets, in the end a CVA appears not only as a derivative, but as an exceedingly complex one, a kind of giant hybrid option. In fact, it is treated as such by risk control, who imposes market limits and monitors the risks of the CVA desk using VaR and sensitivities.

There are two perspectives on the CVA, an accounting one and a regulatory one. Under last years' advisory and regulatory developments, IFRS 13 allows a bank to account for CVA losses and DVA gains, whereas, since DVA does not act as a buffer against defaults, Basel III only reckons CVA into capital relief calculations. This is stated in Paragraph 75 of Basel Committee on Banking Supervision (2011) as

> "Derecognise in the calculation of Common Equity Tier 1, all unrealised gains and losses that have resulted from changes in the fair value of liabilities that are due to changes in the bank's own credit risk."

(see also Basel Committee on Banking Supervision (2012) for a detailed discussion).

1.1 Cash Flows

CVA is not such a well defined quantity. The payoffs of the corresponding "option" are not so clear. Indeed, regarding the CSA value process, ISDA documents use a "replacement" formulation which leaves it open between rather different possibilities, notably default-free versus substitution valuation, where the meaning of "substitution" is not fixed: should substitution mean valuation accounting for the default risk of the surviving party only, or valuation of the contract between the surviving and a new party "similar in every regard" (including credit risk) to the defaulted one? in the latter interpretation, what should be the assessment of the credit risk of the defaulted party: its credit risk right before its default? But what would be the exact meaning of "right before the default"? one day before? or one week? or one month? Another alternative discussed in Brigo, Buescu, and Morini (2012) is

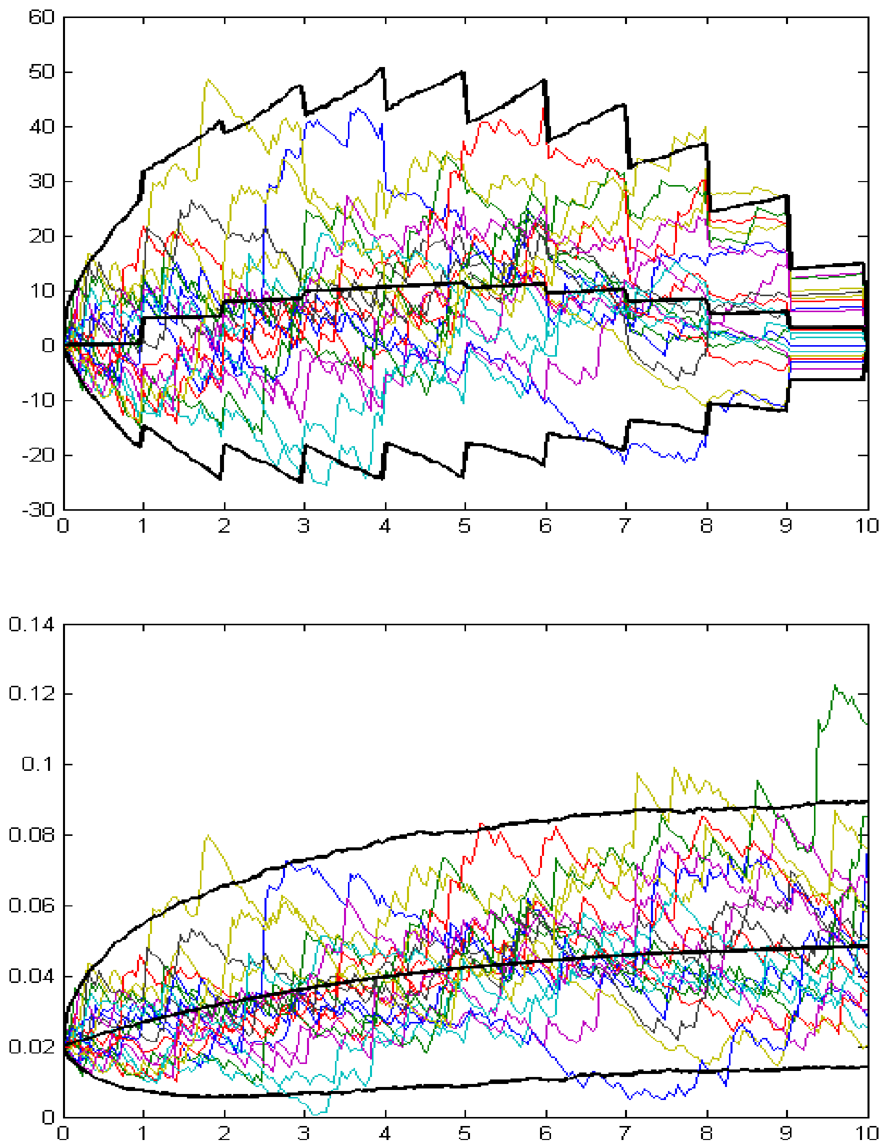


Figure 1: Short rate process corresponding to an increasing term structure of interest rates (bottom) and related mark-to-market process of a payer interest-rate swap (up). Parameters are set so that the fixed leg of the swap is worth € 100 at inception. Each graph shows twenty paths simulated with two hundred time points, along with the mean and 2.5/97.5-percentiles (black curves as function of time) computed over $m = 10^4$ simulated paths (see Crépey, Gerboud, Grbac, and Ngor (2013)). Observe on the upper graph that the mark-to-market of the swap can take positive or negative values depending on time and market scenarios, with a positive trend due to the increasing term structure of interest rates visible on the upper panel.

a CSA value process accounting for a univariate debt valuation adjustment of the surviving party. Such an approach seems to be consistent with the following statement of the ISDA (2009) Close-out Amount Protocol:

> "In determining a Close-out Amount, the Determining Party may consider any relevant information, including, without limitation, one or more of the following types of information: (i) quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that may take into account the creditworthiness of the Determining Party at the time the quotation is provided."

In any case, the discrepancy between the pre-default value of the contract and its CSA value at default time gives rise to another counterparty risk adjustment that we call replacement cost (RC).

A possible fix to these issues would be standardization, but the complexity of the topic makes standardization hardly possible here. In the same line of thought one could think of introducing CVA indices, but these would be difficult to set up and the bespoke CDO tranche experiences of Bear Sterns and Lehman does not push into the direction of a line of indices that would be "mapped" by some ad-hoc devices to account for the variety of situations that occur in practice.

Another way out of CVA (and also DVA), which is in fact the current political and regulatory impetus, is the use of very conservative clearing and collateralization schemes (see Sect. 4 and 5). But, for at least some products, this cannot completely effective (see Tab. 3 in Sect. 4). Moreover, this has a high cost to banks (and therefore ultimately to end-clients).

1.2 Models

In addition to the above ("order zero" and quite uncommon) payoff risk, CVA also has an important model risk.

First, it is not so clear whether CVA should be regarded under the historical or under a risk-neutral pricing measure. Of course, this primarily depends on whether one talks about a pricing adjustment or about a regulatory CVA meant to enter capital relief calculations. But, even in the case of a pricing adjustment, valued under a risk-neutral measure in principle, the situation is not so clear. A risk-neutral modeling approach is only justified in a perfect financial market, without frictions of any kind. Regarding a financial derivative, this approach is only legitimate for a replicable claim, or at least for a liquidly traded derivative. Otherwise risk premia should appear explicitly into the picture. But, in the case of CVA (and even more with DVA in Sect. 2.1), we are in a situation of particularly strong market incompleteness. Advocates of the risk-neutral pricing approach argue that the latter can also be justified on an equilibrium ground, but here again there are some pitfalls (see Box 1).

Box 1 ***"$\mathbb{P}$ versus $\mathbb{Q}$"***[2]
An important and widely open counterparty risk issue regards the choice of a probability measure which should be used in the modeling: historical versus risk-neutral.
There are two possible views on a risk-neutral modeling approach. The first one is based on hedging arguments, mainly considering the risk-neutral measure as an interpretation tool. The corresponding notion of risk-neutral measure is "local" to a given claim (or a few claims jointly considered in a trading strategy). In this understanding, there may be plenty of risk-neutral measures coexisting on different market segments. As own default risk can generally not be hedged in practice, this line of argument does not support the use of a risk-neutral counterparty risk valuation approach.
However, there is another view on a risk-neutral approach, more generally applicable in principle and also global in scope (one measure for the whole market). This alternative approach is based on an equilibrium CAPM kind of analysis (see Cochrane (2005) and the CVA, DVA and FVA discussions in Hull and White (2013a, 2013b) or Burgard and Kjaer (2012)). In a more mathematical finance perspective (focusing on a given claim and its hedge rather than on the economy as a whole), we are now in a world of optimization and indifference prices. But this second approach is better suited for the modeling of market prices as determined by supply-and-demand, rather than for economical valuation, decision-taking by traders and hedging of OTC derivatives. Moreover, the resulting price corresponds to a notion of marginal utility, which also implicitly assumes (as in the first approach) that the unhedged part of the position is "small" with respect to the hedged component. Finally, unless one resorts to dynamic notions of indifference prices (see Musiela and Zariphopoulou (2011) and El Karoui and Mrad (2010)), the resulting risk-neutral probability measure depends on the maturity of the claim, a rather undesirable property.

In the early times of CVA there were some attempts to adopt a hybrid methodology, treating the unhedged risk factors "historically" and the hedged ones "risk-neutrally". Such a hybrid perspective, however, is complicated to implement, and typically results in ad-hoc, not to say inconsistent, solutions.

Second, CVA has a significant sensitivity to model parameters such as correlations (e.g. between interest rates and credit), which are not observed or liquidly priced in the market. Therefore, these parameters cannot be calibrated, but only fitted to some "views" (economic or so). Correlations are also instable quantities, especially during a crisis. This relates to the important wrong-way risk issue (particularly with counterparty risk embedded in credit derivatives), i.e. the possibly adverse dependence between the exposure and the credit risks of the parties (see Sect. 4.1). Right-way risk is in principle not much preferable given that a CVA desk should target a flat P&L.

Third, via the netting agreements that tie all contracts together (for the good sake of counterparty credit risk mitigation), CVA appeals to a consistent modelling across different classes of assets. This modelling needs to be dynamic in view of the optional feature of the CVA. Cross-currency and best-to-deliver optionalities issues are also present when the collateral can be posted in different currencies, especially on foreign-exchange derivatives (see Fujii and Takahashi (2011), Piterbarg (2012) and Sect. 4.1). Finally, it would be advisable to incorporate systemic risk into a CVA model.

All these features require advanced dynamic CVA modeling in terms of processes for all of the risk factors, including the credit risk of the various parties involved. And this is

[2]We are grateful to Nicole El Karoui (université Paris 6) for the discussion with her on this issue.

without even mentioning some tough mathematical issues that arise if one starts questioning the theoretical bottom-lines of the above-sketched financial engineering approach, which points to market incompleteness, utility maximization and imperfect information under random horizons, nonlinear pricing rules, imperfect information, etc..

1.3 Computation

CVA (along with DVA, FVA etc.) computations are, as matters stand, by far the most intensive computational task in the investment bank:

- They entail the overnight valuation of something like one million trades under one thousand scenarios at one hundred time points—and thus one hundred billion contract valuations, times the number of Greeks that are needed for hedging!
- They have to be computed in dynamic global models across different asset classes.
- It's not only about computing risk-neutral expectations for risk-management and accounting purposes, but also about historical quantiles or other risk measures for regulatory purposes, and about consistency between both.
- They bear on very large time horizons, such as twenty to thirty years, over which no "linearization" or kind of scaling is possible. For instance, the standard add-ons, allowing one to compute a (heavier, in principle) ten-day VaR as $\sqrt{10}$ times a (simpler) daily VaR, are prohibited on such time scales, over which it is not only a matter of time horizon, but also of the ageing of a portfolio, with some products expiring or others being converted into different ones. Simulation over such time horizons also precludes the use of standard (notably, martingale lognormal) models, which explode at this time scale, and requires some special techniques to cope with missing calibration data beyond a time horizon of a few years, so that the available data are not used out of their preconditions (e.g. implying behavior at 30 years of interest rates based on $< 10y$ interest rate derivative quotes; as explained in Sokol (2013), one should better use economic theory or "views" instead).

In the end, CVA and valuation of other adjustments to be considered below represent an unprecedented computational challenge for the investment bank. Monte Carlo valuation is unavoidable and the "compound option" feature of the CVA (an option on the mark-to-market of derivatives) added to the high-dimensionality can only be addressed by intensive simulation or simulation/regression ("American Monte Carlo") schemes (see Cesari, Aquilina, Charpillon, Filipovic, Lee, and Manda (2010) and Crépey (2013)). But with current parallel GCP and multi-core technologies (see e.g. Albanese, Bellaj, Gimonet, and Pietronero (2011)), this feat is however actually manageable. Intra-day incremental CVA computations are also necessary for assessing the profitability of a new deal and these can typically be carried out in less than one minute on individual computers.

1.4 Risk management

Hedging of CVA is only possible on counterparties with a liquid CDS market. Moreover, when this is the case, this is essentially only true for the 3y- and 5y- maturity, whereas given the twenty years or more time-horizon of the CVA, one would really need a whole CDS term structure. In addition, CVA hedging instruments must be clean of counterparty risk, meaning collateralized, which can be the case for CDS contracts, but not with the swaptions that one might for instance consider using for hedging the interest rate risk of the CVA.

In the Basel III CVA capital relief formula, credit risk is taken into account based on CDS spreads, not on ratings or fundamental analysis. This pushes banks to dynamic management of CVA through CDS contracts. Intensive CVA hedging by banks can then raise stability and procyclicity issues. In the Euro debt crisis, hedging of CVA by banks was singled out as a big factor of pressure on the sovereign CDS markets.

2 Bilateral Counterparty Risk and Nonlinear Funding Costs

With banks now perceived risky as they are, two additional and partially overlapping terms need be considered, a debt valuation adjustment and a funding valuation adjustment.

2.1 Debt Valuation Adjustment

In a bilateral counterparty risk setup, the unavoidable, logical consequence of CVA is DVA (debt valuation adjustment), which corresponds to the CVA of the bank viewed from the point of view of its counterparty. Following recent years' IFRS I3 recommendations, it has become possible for a bank to account for DVA gains (see the Financial Accounting Standard 157 and the International Accounting Standards 39) and nowadays many banks are actually doing so (see e.g. Moyer and Burne (2011)).

But the DVA raises important points of concern. In particular, hedging one's DVA means selling protection on oneself, which is either impossible (who would buy it), or the last thing to do in view of the related wrong-way-risk. Except for very specific cases (for instance when one can repurchase one's own bonds), all one can do is to hedge the credit spread component of DVA, through peers (as Goldman Sachs did, according to Moyer and Burne (2011)). But, like with any correlation or factor hedge, in adverse scenarios this can be very dangerous (in case of not only deterioration, but actual default of a peer).

One may also argue that DVA is a natural hedge to other risks, being contra-cyclical. But this also only regards its spread risk component, so that in the end, the hedging of DVA (jump-to-default risk in particular) is far from clear, hence it is not clear how to monetize DVA (make the corresponding profit before defaulting). If this cannot be done, then DVA gains reduce to paper money (and are even a direct loss through taxes effectively paid on them). In this perspective the IFRS 13 recommentation regarding DVA is sometimes perceived as an effect of the lobbying of banks, since by times of high and volatile credit spreads of the latter, accounting for DVA gains is of course of considerable stake for them. A contrario it is fair to say that not reckoning the DVA in the accounting result would put a huge pressure on banks by considerably leveraging the volatility of their P&L—a result passing from €600M in one month to €500M in the next one (not unusual with a unilateral CVA accounting rule under which the degradation of the results could not be offset by compensating DVA gains) and the corresponding €100M being treated as a loss is really a killer to a bank. In the end it is worth recalling that IFRS 13 allows but does not force banks to consider DVA gains, and that the spirit of IFRS recommendations is an overall prudential rule which should always predominate. A perverse effect of the system however is that through competitive effects such allowances and tolerances tend to become mandatory.

The above discussion was about hedging, but to the extent that hedging the DVA means selling protection on oneself, pricing the DVA in the first place means buying protection on oneself. This may seem as nonsensical as selling protection on oneself, but it is effectively what happens when one enters a transaction for a price including a DVA. DVA refers to a business model in which one accepts to lose money while one is alive, in the perspective of a compensation at one's default—in a sense, an incentive to default!

Finally, since regulators look at losses, not at gains, Basel III capital charges are myopic to the negative side of the CVA, which is DVA (see the end of the introductory paragraph to Sect. 1). This discrepancy between an accounting CVA/DVA and a unilateral regulatory CVA makes sense in view of two different perspectives, but it makes life difficult for banks, which must also and primarily deal with their economic CVA, i.e. the cost of hedging their counterparty risk—the accounting P&L being probably the most stressful of the three as seen above.

2.2 Funding Valuation Adjustment

Even though there is no regulatory environment in this regard, in order to have a fair appreciation of its P&L, particularly in the long term, it is crucial for a bank to have a fair view of its funding costs, referred henceforth to by FVA, i.e. "funding valuation adjustment" (FVA, or LVA for its liquidity component net of credit spread; see Sect. 3). One of the reasons why there is no regulatory environment for FVA is a fundamental accounting principle to reckon only effectively contracted liabilities (or assets), not foreseen ones. Since a bank typically funds itself short-term for financing longer term investment, an accounting perspective misses the refunding liabilities which will have to be rolled over throughout the whole life of the investment. However, the corresponding funding costs accumulate over years and modify the structure of the P&L.

Now, in a bilateral counterparty risk setup where the bank is also default prone, the classical assumption of a locally risk-free asset which is used by the bank for financing purposes, lending or borrowing as needed, is not sustainable anymore. Mathematically, this funding issue makes matters a big step more complicated, due to recursive and nonlinear features of funding costs (see Box 2 and Crépey (2011,2012a, 2012b) or Pallavicini, Perini, and Brigo (2011, 2012)). In brief, at any time, the funding cash flows are proportional to the funding debt of the bank, which itself depends on the wealth of the bank at that time, including the value of its hedging portfolio (the opposite to the price of the contract if replication holds). Since the time-0 price of the contract is an expectation of its future cash-flows, including funding ones, therefore the time-0 price of the contract depends on its future time-t prices. If the bank is risk-free with the same unsecured borrowing and lending rates, then this dependence is linear, so that one can get rid of it via the introduction of a discount factor at the risk-free rate and obtain an explicit time-0 pricing formula. But, if the bank is default prone (and also due to possible liquidity effects), its unsecured borrowing rate will be greater than (or at least different from) its lending rate (assuming a risk-free "unsecured lender" to the bank for simplicity). As a consequence, the above dependence is nonlinear (see e.g. the example 1.1 in El Karoui, Peng, and Quenez (1997)), so that it is no longer possible to transform the recursive formula into an explicit one by discounting (unless the contract is always the same side of the money, like a bond, so that one can can still do it by resorting to a credit risk-adjusted discount factor). Bilateral counterparty risk and the related funding issue thus lead to recursive (or implicit), nonlinear pricing rules. From a mathematical point of view, the appropriate tool is the theory of BSDEs, i.e. backward stochastic differential equations (see Box 3).

Box 2 ***Nonlinearities***
The CVA is nonlinear in the sense that the corresponding payoff (exposure at default) is given as the positive part of the mark-to-market (assuming no collateralization), so this is a nonlinear payoff. This is why CVA must be viewed as an option that gives to even flow, linear products, a dynamic feature.
The FVA introduces a nonlinearity into the pricing equation itself since, as explained in Sect. 2.2, if we include FVA, the time-0 value of a contract nonlinearly depends on its future time-t values. Then it's not only a nonlinear payoff (boundary condition "at the maturity", or at the default time of a party in the case of CVA), but more broadly a nonlinear equation. If the equation is linear, an even nonlinear payoff can be priced by a standard Monte Carlo loop (as is the case for CVA). However, a nonlinear equation, like with FVA, can only be addressed by more elaborate BSDE schemes, such as simulation/regression (see Cesari, Aquilina, Charpillon, Filipovic, Lee, and Manda (2010), Crépey (2013)), expansions (see Fujii and Takahashi (2012)) or branching particles (see Henry-Labordère (2012)).

Box 3 ***Backward Stochastic Differential Equations***
Backward stochastic differential equations (BSDEs) are an alternative to partial differential equations (PDEs) for representing prices and Greeks of financial derivatives. BSDEs are a flexible and powerful mathematical tool. They also offer a very efficient pedagogical setup for presenting the financial derivatives pricing and hedging theory. In addition, they are useful for the numerical solution of high-dimensional nonlinear pricing problems, such as those which may appear with XVA computations.
The backward terminology refers to the fact that these equations are stated in terms of a terminal condition (random variable), ξ, at a future maturity time T. BSDEs were first introduced by Bismut (1973) in the case of a linear driver, and then more generally by Pardoux and Peng (1990). They have been extensively studied since then, particularly in relation to mathematical finance (see El Karoui, Peng, and Quenez (1997) for a seminal paper and Crépey (2013) or Delong (2013) for two recent books). The solution to a BSDE consists of a pair of processes (Π, Δ), in which Π corresponds to the price of a financial derivative and Δ to its hedge. In the simplest case, a solution to a backward stochastic differential equation is obtained by invocation of a martingale representation theorem. Yet the theory of backward stochastic differential equations, properly speaking, begins with the presence in the equation of an implicit driver coefficient $g = g_t(\Pi_t, \Delta_t)$ (corresponding to the running cost of a control problem). If g is nonlinear, as with the nonlinear funding issue, Picard iteration and a contraction argument are needed, beyond a martingale representation theorem, to solve a BSDE.

Since BSDEs are in essence a "nonlinear pricing tool", they can also deal with other nonlinearities (other than FVA) that can be present in the problem (see for instance the CSA specification e in (7)). A good point is that the simulation/regression ("American Monte Carlo") schemes that can be used to deal with the large system feature of the CVA are also suitable to deal with BSDEs (see Sect. 3.2 and Box 9).

The difficulty of the FVA problem may contribute to explain why, as of today, there is no standard FVA methodology in banks (see Carver (2012), Cameron (2013)). This being said, before being a technical (mathematical and computational) issue, FVA, as DVA above, raise important financial concerns. In old times a bank used an affordable and essentially risk-free Libor funding rate throughout and channeled the corresponding charge to the client. With Libor rising over OIS rates as we saw in the crisis (also an effect of banks' counterparty risk, but at a macro level; see Figure 2 and Box 4), this additional charge has become problematic.

Box 4 ***The Whys of the LOIS***

The 2007 subprime crisis induced a persistent disharmony between the Libor derivative markets of different tenors and the OIS market. Commonly proposed explanations for the corresponding spreads refer to a combined effect of credit risk and liquidity risk of Libor banks. However, in the literature, the meaning of liquidity is often not stated precisely, or it is simply defined as a residual spread after removal of a credit component. Crépey and Douady (2013) propose an indifference valuation model in which the Libor-OIS spread, named LOIS in Bloomberg, emerges as a consequence of:

- on the one hand, a credit component determined by the skew of the credit curve of a representative Libor panelist (playing the role of the "borrower" in an interbank loan),
- on the other hand, a liquidity component corresponding to the volatility of the spread between the funding rate of a representative Libor panelist (playing the role of the "lender") and the OIS rate.

The credit component is, in fact, a credit skew component (this relates to the refreshment mechanism of the Libor panel). The relevant notion of liquidity appears as the optionality of dynamically adjusting through time the amount of a rolling overnight loan, as opposed to lending a fixed amount up to the tenor horizon on Libor (this optionality is valued by the aforementioned volatility). When the funding rate of the lender and the overnight interbank rate match on average, this results, under diffusive features, in a square root term structure of the LOIS. Specifically, on the euro market considered in the period from mid-2007 to mid-2012 in Crépey and Douady (2013), one observes a square root term structure of the LOIS consistent with this theoretical analysis (see Figure 3), with LOIS found to be explained in a balanced way by the credit and liquidity components through the beginning of 2009, and then dominantly explained by the liquidity component (see Figure 4).

In principle, a higher funding rate compensates for a higher credit risk and/or liquidity costs. Under a credit explanation of the funding spread (between the funding rate and OIS rates) and to the extent that a "benefit at own default" can really be considered as benefit, charging the FVA to the client is not justified, since the FVA is simply the present value to the bank of this benefit in the future. In this last perspective (which, however, is subject to the same paper money issue as DVA regarding benefit at own default), a deal is found valuable as soon as its expected returns exceed the funding cost of the bank net of its credit spread; channeling the totality of the FVA to the client would be double counting. In other words, the relevant funding valuation adjustment charged by the bank should not be the full amount of the funding cost but only its liquidity component, net of the bank's credit spread, i.e. an LVA (funding liquidity valuation adjustment) rather than a full FVA (credit and liquidity). With the paper of Hull and White (2013a) (see also Burgard and Kjaer (2012)), the discussion on this issue has become even more passionate than the 2011-12 DVA debate (see Box 5).

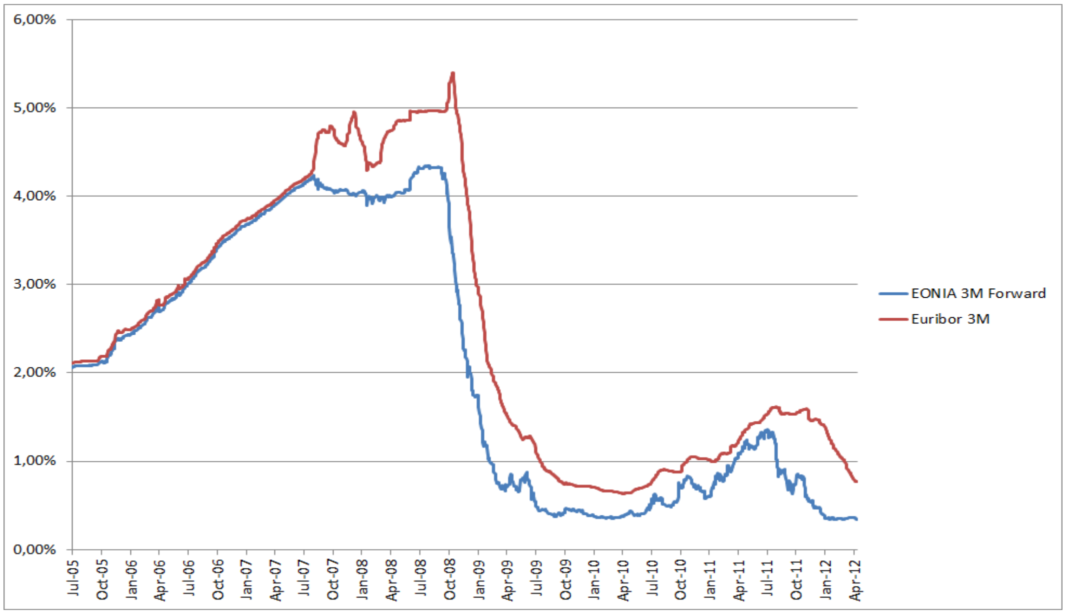


Figure 2: Divergence Euribor / EONIA-swap rates: sudden divergence between the 3m Euribor and the 3m EONIA-swap rate that occurred on Aug 6 2007.

Box 5 ***The FVA Debate***

With the Modigliani-Miller theorem in the background, which states that, under certain conditions, the value of a project does not depend on the way it is funded, Hull and White (2013a) have claimed that banks should not take FVA into account, prompting the so-called "FVA debate". However, Morini (2013) explains how Hull and White (2013a)'s reasoning

> "leads to say that there is no FVA based on three crucial assumptions:
>
> 1. The market has instantaneous efficiency: this is not the case in the reality of funding markets, although we always use indirectly this assumption in pricing
> 2. Funding of a deal happens after the market knows about the deal: this can be true when a project is funded rolling short-term funding, but prudential management includes often part of funding at maturity
> 3. The effect of a new deal on the funding costs of a bank is linear: (...) in fact under rather realistic assumptions the effect is highly non-linear.
>
> (...) Hull and White have the merit of pointing out that FVA is a distortion compared to an efficient market (...) Yet, in the current market situation a dealer following a going concern must take some FVA into account."

By the difficult liquidity times that have been faced during the crisis, funding has sometimes been the main motivation for a deal, allowing a bank to get funded at an OIS rate (the usual remuneration rate of the collateral) in a fully collateralized transaction,

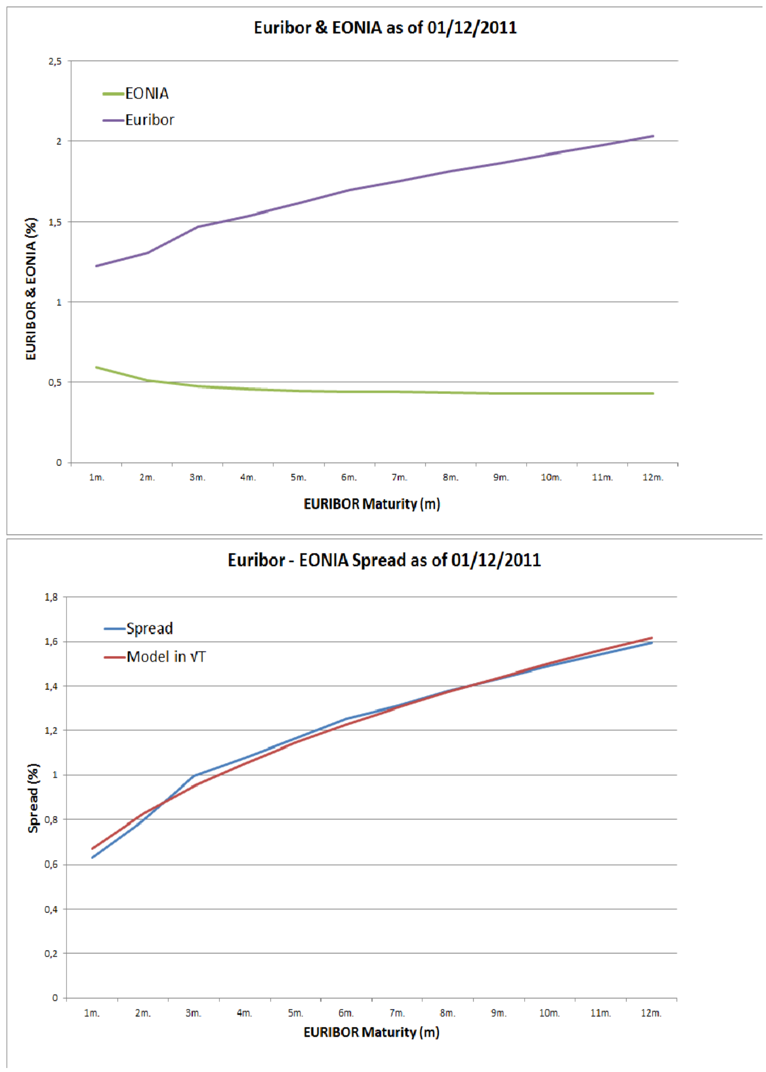


Figure 3: 16 April 2012. *Top:* term structure of Euribor vs EONIA-swap rates ($T = 1$m to 12m). *Bottom:* square root fit of the LOIS.

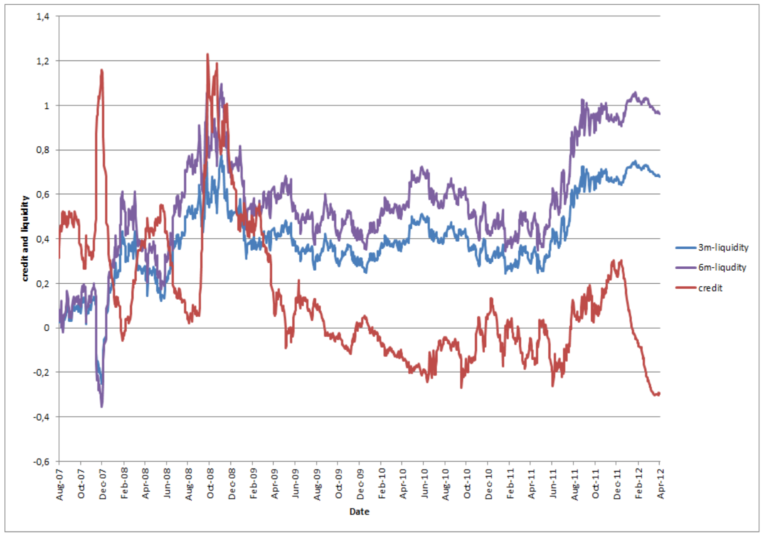


Figure 4: Time series of the credit component of the LOIS (red) and of the liquidity component of the 3m- (blue) and 6m- (purple) LOIS (all in %), over the period from 15/08/2007 to 16/04/2012 (see Crépey and Douady (2013)).

rather than at a much higher rate in the context of an unsecured transaction. Funding considerations should not be the main motivation of a bank in a transaction, however one can say that funding should be considered for not going into a trade that could look worthwhile without it. Again, in the long run, funding may damage the P&L of a trade which looks worthwhile on a shorter time horizon.

3 CVA, DVA, LVA, RC: The Four Wings of the TVA

We now reformulate in mathematical terms the developments of the previous sections and we illustrate them numerically. We consider a simplified situation where a risky bank faces a single risky counterparty. In reality a bank has to deal with hundreds to thousands of netting sets and counterparties and all the credits should be modeled jointly since, in particular, the funding cash-flows are netted at the level of the whole book of the bank.

3.1 TVA Equation

As explained above, different interdependent valuation adjustments, or XVAs, must be computed on top of a clean price (mark-to-market) P in order to account for counterparty risk and funding costs. We refer to Part III of Crépey, Bielecki, and Brigo (2014) for a detailed presentation. We only recall that the aggregated adjustment, which we call TVA for total valuation adjustment, can be viewed as the price of an option on the clean price P at the first-to-default time τ of a party. Moreover, this option pays dividends that correspond to the funding costs (in excess over the OIS rate r_t). Specifically, for a CSA with time horizon T, the TVA equation is of the following form, relatively to a risk-neutral (pricing) measure $\mathbb{Q}$:

$$\Theta_t = \mathbb{E}_t\left(\int_t^T f_s(\Theta_s)ds\right), \quad t \in [0,T], \tag{1}$$

where Θ_t and $f_t(\vartheta)$ respectively represent the TVA process one is looking for and the coefficient that generates this TVA. Note that (1) is a backward stochastic differential equation (BSDE) for the TVA process Θ (see Box 3). The all-inclusive price of the contract for the bank (cost of the corresponding hedge, including the counterparty and funding risks) is

$$\Pi = P - \Theta.$$

The coefficient f of the BSDE (1) is given, for every real ϑ (representing the TVA Θ_t that one is looking for in the probabilistic interpretation), by:

$$\begin{aligned} f_t(\vartheta) + r_t\vartheta = \; & \underbrace{\gamma_t^c\,(1-R_c)(Q_t-\Gamma_t)^+}_{\text{CVA coefficient } (\mathrm{cva}_t)} \\ & - \underbrace{\gamma_t^b(1-R_b)(Q_t-\Gamma_t)^-}_{\text{DVA coefficient } (\mathrm{dva}_t)} \\ & + \underbrace{\bar{b}_t\Gamma_t^+ - b_t\Gamma_t^- + \widetilde{\lambda}_t\big(P_t-\vartheta-\Gamma_t\big)^+ - \lambda_t\big(P_t-\vartheta-\Gamma_t\big)^-}_{\text{LVA coefficient } (\mathrm{lva}_t)} \\ & + \underbrace{\gamma_t\,(P_t-\vartheta-Q_t)}_{\text{RC coefficient } (\mathrm{rc}_t)} \end{aligned} \tag{2}$$

where:

- γ_t^b, γ_t^c and γ_t are the default intensities of the bank, of its counterparty and their first-to-default intensity (in a model where the bank and the counterparty can default together, γ_t is less than $\gamma_t^b + \gamma_t^c$),
- R_b and R_c are the recovery rates of the bank towards the counterparty and vice versa,
- Q_t is the value of the contract according to the scheme used by the liquidator in case of a default at time $t < T$, e.g. $Q_t = P_t$ (used henceforth unless otherwise stated) or $Q_t = P_t - \Theta_t$,
- $\Gamma_t = \Gamma_t^+ - \Gamma_t^-$, where Γ_t^+ (respectively Γ_t^-) represents the value of the collateral posted by the counterparty to the bank (respectively by the bank to the counterparty), e.g. $\Gamma_t = 0$ (used henceforth unless otherwise stated) or $\Gamma_t = Q_t$,
- $\bar{b}_t$ and b_t are the spreads over the OIS (risk-free) short rate r_t for the remuneration of the collateral Γ_t^+ and Γ_t^- posted by the counterparty and the bank to each other,
- λ_t (respectively $\widetilde{\lambda}_t$) is the liquidity funding spread over the OIS short rate r_t corresponding to the remuneration of the external funding loan (respectively debt) of the bank. By liquidity funding spreads we mean that these are free from credit risk, i.e.

$$\widetilde{\lambda}_t = \bar{\lambda}_t - \gamma_t^b(1-\bar{R}_b), \tag{3}$$

where $\bar{\lambda}_t$ is the all-inclusive funding borrowing spread of the bank and where $\bar{R}_b$ stands for a recovery rate of the bank to its unsecured lender. Recall that the unsecured lender is assumed to be risk-free so that in the case of λ_t there is no credit risk involved anyway.

The data Q_t, Γ_t, b_t and $\bar{b}_t$ are specified in the CSA contracted between the two parties.

Note that the above presentation corresponds to "reduced-form" (or "pre-default") modeling approach, under the immersion hypothesis of a "reference filtration" $\mathbb{F}$ into the full model filtration $\mathbb{G}$. For more details the reader is referred to the remark 2.1 in Crépey (2012b) and to Crépey and Song (2014). In this article, we only work with pre-default values (which is why the default times of the parties are only represented by the intensities γ_t^c, γ_t^b and γ_t in the equations above).

We now discuss in box form important consequences of the structure (2) for the TVA coefficient f in (1), regarding the decomposition of the all-inclusive TVA Θ_t (Box 6), the case of full collateralization (Box 7) and an asymetrical TVA approach that can be used for solving the puzzle of the benefit at own default (Box 8).

Box 6 ***The "four wings" of the TVA***

As it follows from the equations (1) and (2), the time-0 TVA can be represented as

$$\begin{aligned}\Theta_0 =&\underbrace{\mathbb{E}\Big[\int_0^T \beta_t(1-R_c)\gamma_t^c(Q_t-\Gamma_t)^+dt\Big]}_{\mathrm{CVA}_0}\\ &\underbrace{-\mathbb{E}\Big[\int_0^T \beta_t(1-R_b)\gamma_t^b(Q_t-\Gamma_t)^-dt\Big]}_{\mathrm{DVA}_0}\\ &\underbrace{+\mathbb{E}\Big[\int_0^T \beta_t\big(\bar{b}_t\Gamma_t^+ - b_t\Gamma_t^- + \widetilde{\lambda}_t\big(P_t-\Theta_t-\Gamma_t\big)^+ - \lambda_t\big(P_t-\Theta_t-\Gamma_t\big)^-\big)dt\Big]}_{\mathrm{LVA}_0}\\ &\underbrace{+\mathbb{E}\Big[\int_0^T \beta_t\gamma_t(P_t-\Theta_t-Q_t)dt\Big]}_{\mathrm{RC}_0}.\end{aligned} \tag{4}$$

By the four wings of the TVA, we mean its CVA, DVA, LVA and RC components. These terms have clear and distinct financial interpretations so that it is worthwhile to consider them separately. However, it should be emphasized that they are interdependent and must be computed jointly, as visible in the explicit dependence of LVA_0 and RC_0 on Θ_t in (4). In fact, the neatest decomposition is local, at the level of the instantaneous coefficient $f_t(\vartheta)$ in (2). The positive (resp. negative) TVA terms, e.g. the CVA (resp. the DVA), are "deal adverse" (resp. "deal friendly") as they increase (resp. decrease) the TVA Θ and therefore decrease (resp. increase) the price $\Pi = P - \Theta$—with, depending on the sign of Π, a "less positive" Π interpreted as a lower buyer price by the bank or a "more negative" Π interpreted as a higher seller price by the bank. Note that such "buyer and seller" prices only reflect funding costs and not the issue of different pricing measures that can coexist in an incomplete market (see El Karoui and Quenez (1995) or Eberlein, Madan, Pistorius, and Yor (2013), as well as Box 1).

Box 7 ***Full Collateralization CSA***
In case $\Gamma = Q = \Pi$, where $\Pi = P - \Theta$, (2) yields

$$f_t(\vartheta) + r_t\vartheta = \bar{b}_t(P_t - \vartheta)^+ - b_t(P_t - \vartheta)^-.$$

The coefficient reduces to the remuneration of the collateral, which is exchanged between the two parties. Therefore in this case the bank's price of the contract, Π, is also the counterparty's price (cost of its hedge), say $\bar{\Pi}$, so that $\Gamma = Q = \Pi = \bar{\Pi}$. In this sense, this specification deserves the name of full collateralization. Note however that for b or $\bar{b} \neq 0$, this specification still entails a TVA. It is only in the special case $b = \bar{b} = 0$ (case where r_t represents a symetrical interest rate on the posted collateral, typically the OIS rate) that we have $\Theta = 0$ and $\Pi = P = \Gamma = Q = \bar{\Pi}$. There is then no need for pricing-and-hedging a (null) TVA. The problem reduces to the computation of a clean price P and a related hedge.

Box 8 ***Best practice: Asymmetrical TVA approach?***
In practice the bank can hardly hedge its jump-to-default and, therefore, cannot monetize ("benefit" from) its default unless and before it actually happens. To be consistent with this view, one can avoid to reckon any benefit of the bank from its own default by setting $R_b = \bar{R}_b = 1$, $R_b = \bar{R}_b = 1$. Indeed, in this case, (2) reduces to

$$\begin{aligned} f_t(P_t - \vartheta) + r_t\vartheta =& \gamma_t^c(1 - R_c)(Q_t - \Gamma_t)^+ \\ &+ \bar{b}_t\Gamma_t^+ - b_t\Gamma_t^- + \bar{\lambda}_t\,(P_t - \vartheta - \Gamma_t)^+ - \lambda_t\,(P_t - \vartheta - \Gamma_t)^- \\ &+ \gamma_t\,(P_t - \vartheta - Q_t)\,, \end{aligned} \tag{5}$$

where the DVA coefficient disappears (for $R_b = 1$) and where the borrowing funding basis $\bar{\lambda}_t$ is interpreted as a pure liquidity funding cost. Such an asymmetrical (but still bilateral) TVA approach allows one to avoid the difficulties and the paradox related to the benefit of the bank at its own default time and the puzzle for the bank of having to hedge its own jump-to-default in order to monetize the corresponding "benefit" prior to its default (see Sect. 2). This approach is also justified with regard to the fact that the benefits at own defaults are in effect cashflows to senior bondholders, whereas only the interest of the shareholders should be considered in the optimization (or hedging) process of the bank (see Albanese, Brigo, and Oertel (2013) and Albanese and Iabichino (2013)).

3.2 Numerical Solution

To solve the BSDE (1) numerically, the first step ("forwardation") is to generate, forward in time by an Euler scheme, a stochastic grid with n time steps and $m = 10^4$ scenarios for an underlying factor process X_t (so that every involved process $f = f_t(\omega)$ can be rewritten as $f(t, X_t(\omega))$ for a suitable function denoted by the same letter as the process) and for the clean value process $P_t = P(t, X_t)$ of the contract.

The second step ("backwardation") is to compute the TVA process Θ_t, backward in time, by nonlinear regression on the time-space grid generated in the previous step. We thus approximate by $\widehat{\Theta}_i^j$ on the grid the solution $\Theta_t(\omega)$ to (1), where the time-index i runs from 1 to n and the space-index j runs from 1 to m. Denoting by $\widehat{\Theta}_i = (\widehat{\Theta}_i^j)_{1\leq j\leq m}$ the vector of TVA values on the space grid at time i, we have $\widehat{\Theta}_n = 0$ and then, for every

$i = n-1, \cdots, 0$ and $j = 1, \cdots, m$ (assuming a uniform time-step $h = \frac{T}{n}$):

$$\widehat{\Theta}_i^j = \widehat{\mathbb{E}}_i^j \left(\widehat{\Theta}_{i+1} + f_{i+1}(\widehat{X}_{i+1}, \widehat{\Theta}_{i+1}) h \right), \tag{6}$$

where $\widehat{\mathbb{E}}_i^j$ is an estimate for the conditional expectation given $X_i = X_i^j$ (see e.g. Sect. 6.10.2 in Crépey (2013)).

Note that in case of an exotic contract with no explicit formula for $P(t, x)$ in the first step, an approximate computation of the process P_t can be done jointly with that of Θ_t in the second step (see Box 9).

Box 9 ***Nonlinear regressions***
Simulation/regression schemes always require imaginative thinking to find appropriate regressors depending on the markets and products under consideration. In this regard, it is important to distinguish between:

- computing by simulation/regression the exposure (mark-to-market and collateral) at all points of a simulated grid of the factor processes, for which low-dimensional regressions can be performed independently for the different contracts of a CSA portfolio, using the one to three (say) more relevant risk factors for each product,
- solving an intrinsically high-dimensional nonlinear TVA (including FVA) equation at the portfolio all-factors level. Then, except for low-dimensional toy examples (cf. Fig. 6), simulation/regression schemes are no longer appropriate. Purely forward branching particles schemes can be used instead but in high-dimension they may be subject to variance issues. One can must then use approximations, such as the expansions of Fujii and Takahashi (2012).

3.3 Toy example

Following the methodology of Sect. 3.2, we compute the TVA of the bank regarding the interest-rate swap of Figure 1. We use $n = 100$ uniform time steps, $m = 10^4$ scenarios and (essentially[3]) $X_t = r_t$. The outputs of the first step are visible in Fig. 1. For the second step, we use the TVA parameters

$$\gamma^b = 5\%,\ \gamma^c = 7\%,\ \gamma = 10\%, b = \bar{b} = \lambda = 1.5\%, \bar{\lambda} = 4.5\%$$

and we consider five possible CSA specifications (a with DVA benefit at default, b collateralized, c with DVA and funding benefits at default, d without benefit at default, e without benefit at default and with pre-default close-out valuation):

$$\begin{array}{llll}
\mathsf{a.} & (\bar{R}_b, R_b, R_c) = (100, 40, 40)\%, & Q = P, & \Gamma = 0 \\
\mathsf{b.} & (\bar{R}_b, R_b, R_c) = (100, 40, 40)\%, & Q = P, & \Gamma = Q = P \\
\mathsf{c.} & (\bar{R}_b, R_b, R_c) = (40, 40, 40)\%, & Q = P, & \Gamma = 0 \\
\mathsf{d.} & (\bar{R}_b, R_b, R_c) = (100, 100, 40)\%, & Q = P, & \Gamma = 0 \\
\mathsf{e.} & (\bar{R}_b, R_b, R_c) = (100, 100, 40)\%, & Q = P - \Theta, & \Gamma = 0.
\end{array} \tag{7}$$

The clause $Q = P - \Theta$ in case e represents the situation (admittedly rather artificial) of a bank that would be in a "dominant" position, able to impose the value of the contract

[3] Up to a mild path-dependence of a swap reflecting the payments in arrears (see Sect. 5.1 in Crépey, Gerboud, Grbac, and Ngor (2013)).

from its own perspective ("cost of its own hedge"), i.e. Π, for the CSA close-out valuation process Q. Fig. 5 shows, in the same format as the clean price process of the upper graph in Figure 1, the TVA process obtained in cases a (upper panel) and b (lower panel), using a nearest neighbors average estimate for $\widehat{\mathbb{E}}_i^j$ in (6) (see Hastie, Tibshirani, and Friedman (2009)). Note the different scales of the y-axis (smaller values in the case b that corresponds to full collateralization, i.e. no CVA/DVA).

Tab. 1 shows the time-0 TVA of the bank and its decomposition into time-0 CVA, DVA, LVA and RC for the swap of Fig. 1 and 5, for each CSA specification a to e (recall that the fixed leg of the swap is worth € 100 at inception). The numerical results are consistent

	Θ_0	CVA_0	DVA_0	LVA_0	RC_0
a	3.17	3.28	-0.64	2.41	-1.92
b	0.51	0.00	0.00	0.81	-0.31
c	2.08	3.28	-0.64	0.66	-1.25
d	3.59	3.28	0.00	2.38	-2.11
e	4.80	2.49	0.00	2.26	0.00

Table 1: Time-0 TVA and CVA, DVA, LVA and RC components corresponding to a long position in the swap of Fig. 1 and 6.

with the CVA/DVA/LVA/RC interpretation of the four components in the right-hand side of (2). In case a, the "highly positive" CVA of 3.28 reflects the fact that the bank is, on average over the lifetime of the swap, in-the-money, due to the increasing term structure of interest rates. The DVA, on the contrary, is moderate (-0.64). The LVA is quite important (2.41). A RC of (-1.92) is not negligible with respect to the other three components. Next, passing from:

a→ b: the CVA and DVA vanish; the dominant effect is the cancellation of the previously highly positive CVA, resulting in a lower TVA, whence a higher buyer price (cost of the hedge) for the bank;

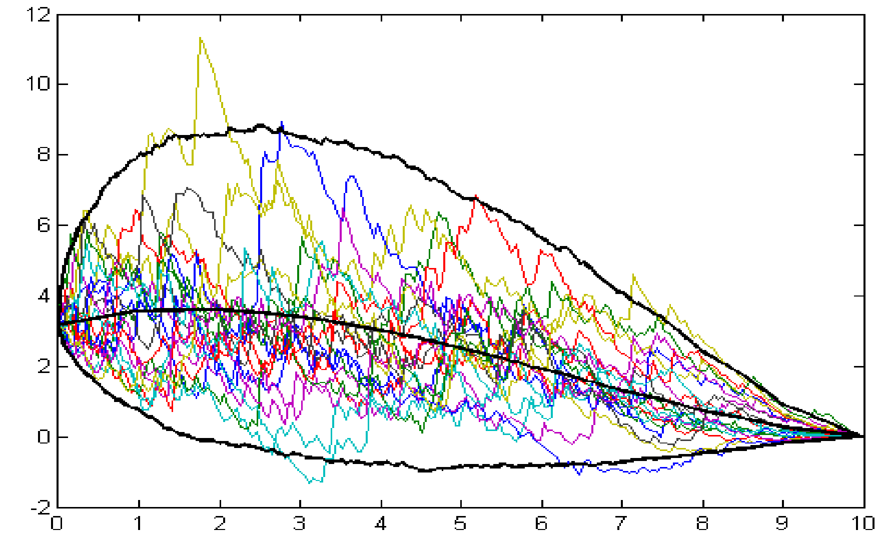
a→ c: a funding benefit at own default is acknowledged by the bank, resulting in lower LVA and TVA, whence a higher buyer price;

a→ d: the DVA is ignored by the bank as fake benefit, resulting in a higher TVA, whence a lower buyer price;

d→ e: the RC vanishes and the CVA also changes.

For these data the LVA is significant and positive in cases a, d and e. Note that all these numbers could be much higher (in absolute value) in a model accounting for wrong-way risk dependence effects between interest rates and credit risk (see e.g. Brigo, Morini, and Pallavicini (2013)).

Case of a Short Position Tab. 2 and Fig. 6 display the results analogous to the previous ones in case of a bank with a short position in the swap of Figure 1. Observe that, because of the nonlinear, nonsymetrical funding data, the numbers in Tab. 2 are not simply the opposite of those in Tab. 1. In case a, the "highly negative" DVA of (-2.34) reflects the fact



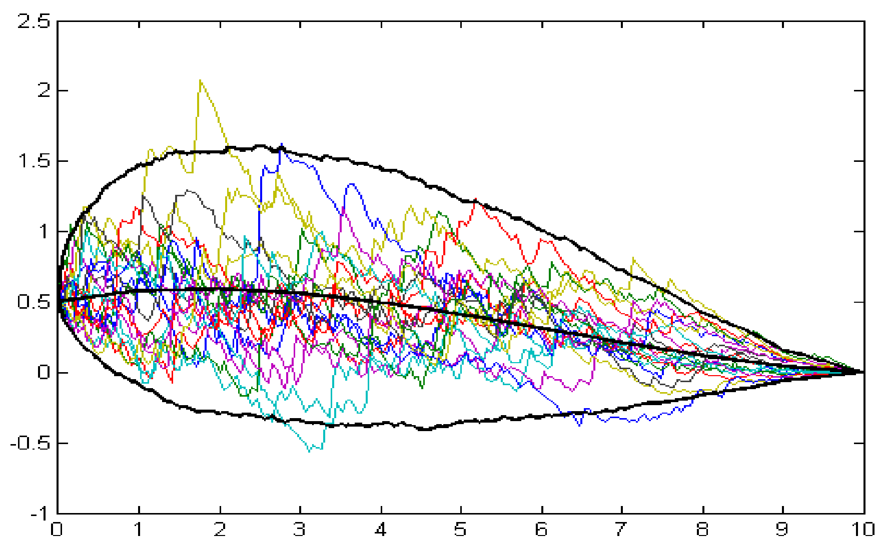


Figure 5: TVA process of the bank on the swap of Figure 1, computed by simulation/regression under the CSA specifications a (top) and b (bottom). Each graph shows twenty paths of the TVA process at two hundred time points, along with the process mean and 2.5/97.5-percentiles as a function of time, all computed using $m = 10^4$ simulated paths of r_t.

that the bank shortening the swap is, on average over the lifetime of the swap, out-of-the-money, due to the increasing term structure of interest rates. The CVA, on the contrary, is a moderate 0.90.

	Θ_0	CVA_0	DVA_0	LVA_0	RC_0
a	-0.93	0.90	-2.34	-0.15	0.68
b	-0.44	0.00	0.00	-0.72	0.29
c	-1.34	0.90	-2.34	-0.72	0.85
d	0.45	0.90	0.00	-0.32	-0.12
e	0.43	0.76	0.00	-0.32	0.00

Table 2: Analog of Tab. 1 in case of a short swap position of the bank in the swap of Figure 1.

This time, passing from:

a→ b: the CVA and DVA vanish; the dominant effect is the cancellation of the highly negative DVA of case a, resulting in a higher TVA (in spite of a decrease of the LVA), whence a lower buyer price for the bank;

a→ c: a funding benefit at own default is acknowledged by the bank, resulting in lower LVA and TVA, whence a higher buyer price;

a→ d: the DVA is ignored by the bank as fake benefit, resulting in a higher TVA, whence a lower buyer price;

d→ e: the RC vanishes and the CVA also changes.

For these data, the LVA is significant negative in cases c (due to the acknowledgement of a funding benefit at own default) and b.

In conclusion, the interpretation and choice of the parameters, own default benefit parameters R_b and $\bar{R}_b$ in particular, has tangible consequences in terms of, for instance, the relevance of entering in a deal or not.

4 Collateral

4.1 Wrong-Way and Gap Risks

The variation margin regularly updated by the two parties to mitigate the counterparty risk arising from changes in the mark-to-market of the portfolio cannot suffice to guarantee a perfect collateralization. The margin period of risk between the last margin call before the default and the close-out of the position induces gap risk, i.e. risk of mismatch between the position and its collateral. On certain classes of assets, notably credit derivatives (since credit cash flows are to some extent unpredictable; think of Lehman collapsing over a weekend), finding an efficient collateralization scheme is particularly difficult (see Tab. 3).

For this reason and also in view of the previously under-estimated systemic risk, Basel III, Dodd-Franck in the US and EMIR in Europe push dealers to clear as many trades as possible. Central clearing (more specifically dealt with in Sect. 5) will even become mandatory for flow products that are traded on an exchange. On top of the traditional variation

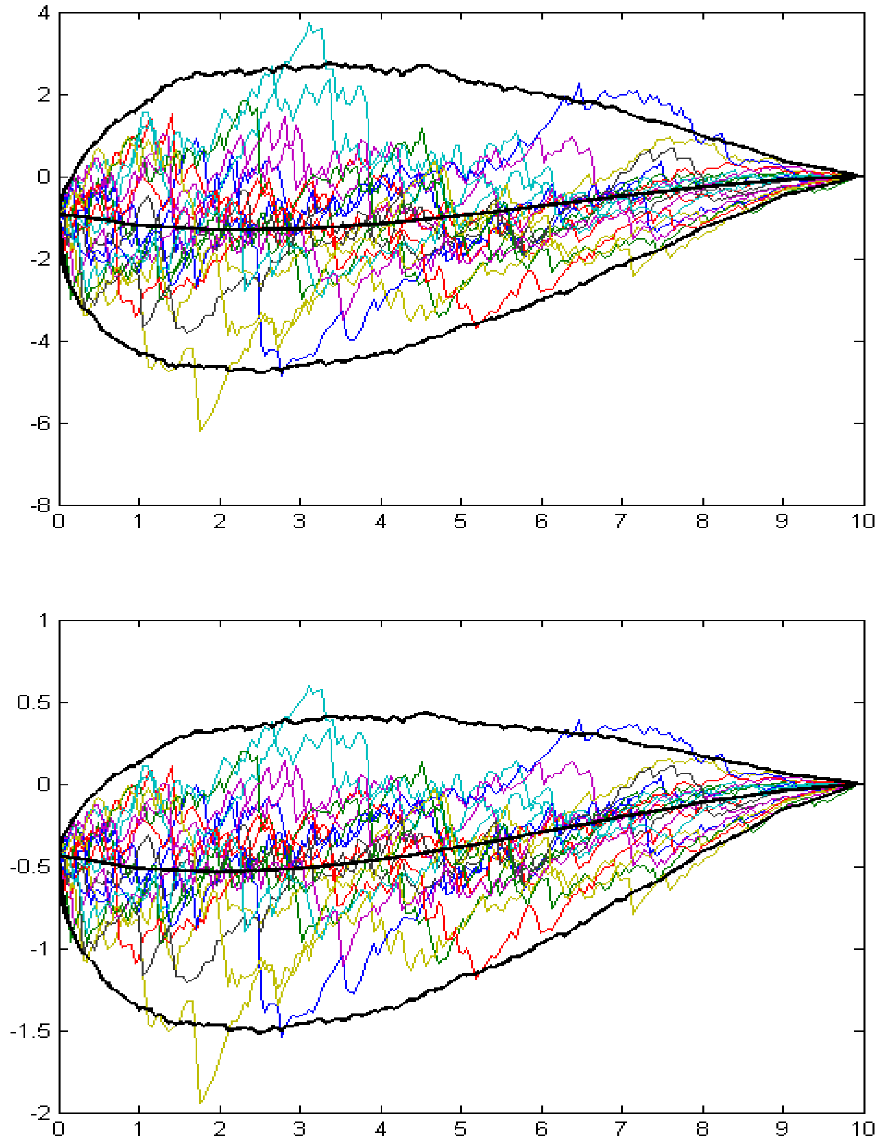


Figure 6: Analog of Figure 5 in case of a short position of the bank in the swap of Figure 1.

margin, a clearing house mitigates gap risk by asking to all its clients an additional layer of collateralization, the initial margins, which are segregated. In addition, starting January 2015, an even bilateral (non cleared) transaction initiated between two dealers (not with end-clients, for which this would put an excessive liquidity pressure) will have to be collateralized under a so-called sCSA, i.e. standard CSA, including not only variation but also initial margins (called in this context independent amount) and excluding "exotic" CSA clauses such as collateral optionalities. The determination of the margin calls procedure, including the possibility or not to compute them at aggregated levels across different classes of assets, has been the topic of intensive discussion and debate between banks and regulators (see Basel Committee on Banking Supervision and Board of the International Organization of Securities Commissions (2012, 2013)).

Current estimates are that about half of the business will become centrally traded and the rest (most of the exotic business in particular) will stay bilateral. Anyway in the future CVA will still be there via in particular shadow-banking (OTC trading between banks and hedge funds), which is not regulated.

	Naked			Collateralized		
Tranche	0-5%	5-35%	35+%	0-5%	5-35%	35+%
CVA	4.78	2.96	2.44	3.41	2.73	2.26
σ	0.08	0.24	0.20	0.05	0.16	0.14
$\%\sigma$	1.6	8.1	8.2	1.4	6.0	6.0

Table 3: Naked (uncollateralized) versus continuously collateralized CVA on CDO tranches in the common-shock model used in Crépey and Rahal (2014). Each of the one hundred obligors has a notional of 100, so that the maximal loss on the the tranche (a, b) is $(b-a) \times 10^4$, e.g. $5\% \times 10^4 = 500$ for the equity tranche. In the common-shock model (see Part IV of Crépey, Bielecki, and Brigo (2014), or see Bielecki, Cousin, Crépey, and Herbertsson (2013) for an informal introduction), the impact of the collateralization is very limited, especially for higher tranches. So, for the equity tranche 0-5%, the naked CVA is 4.78, whereas the collateralized CVA is 3.41. For the $35+\%$ tranche these numbers respectively become 2.44 and 2.26. That's because in this model the main source of counterparty risk is joint defaults, which can hardly be collateralized (at least by the variation margin that is used here, hence the need for initial margins).

4.2 Liquidity

Extensive collateralization requires a huge amount of cash or liquid assets (see Singh and Aitken (2009), Singh (2010), Levels and Capel (2012)), so that the current regulatory trend puts a high pressure on liquidity. This can be particularly tough for corporates, which is why end-clients are not commited to dealers' obligations of clearing trades that are proposed on an exchange.

Some people claim that huge reserves of liquidity are not exploited by the system, stuck in custodians (e.g. Bank of New York), which follow the "Roman law" of keeping deposits without reinvesting them, and have been very popular during the crisis (even at negative deposit rates!). Another perspective could be a securitization of the CVA, whereby the payment of the margin calls would be transferred to investors (see Albanese, Brigo, and Oertel (2013)). A similar possibility regarding FVA was recently considered in Albanese and

Iabichino (2013). Perhaps the introduction of financial devices making such risk transfers possible could indeed be a solution to the CVA, DVA, FVA and other XVA spiral. However, it is far from clear that the regulators will buy such ideas after the 2008 subprime debacle of securitization of assets as simple as bonds.

Finally, an excessive collateralization can damage recovery rates (see Kenyon and Stamm (2012)) So collateralization does not only raise liquidity and systemic risk concerns, it can also worsen the severity of defaults as they occur.

5 Clearinghouses and Centrally Cleared Trading

The current trend of the regulation is to push dealers to negotiate via CCPs, i.e. central counterparties (or clearinghouses). In the case of centrally cleared trading, the counterparty risk is transferred from the counterparties themselves to the clearing house and from contracts (or portfolios) to margin calls. A clearinghouse can deal with risk differently, on a mutualization basis. A CCP asks for a double layer of collateralization to its clients: variation and initial margin. Cleared transactions were found to be better managed in the crisis than bilateral transactions, but at that time the most toxic assets were not cleared. Centrally cleared trading differs from bilateral trading by several distinguising features, notably the margining procedure and the default/auction procedures (see Sect. 5.1). Differences between the margining procedures of bilaterally and centrally cleared trades have pricing implications that are analyzed in Cont, Mondescu, and Yu (2011) (see also Pallavicini and Brigo (2013)). So, quoting Morini (2013) (see also Risk Magazine July 2013): "It is well known, through the mathematics of convexity adjustments, that the price of a futures contract has to be greater than the price of the corresponding forward contract. However, the reverse relation has been observed in July 2013, and market praticipants say the reason is FRAs have now been moved to CCPs, and initial margin affects pricing."

A clearinghouse can set up better management for collateral, by calling for variation margins five to six times per day, versus daily (at best) for a bank. Initial margins can be updated too, up to a daily basis. However, there are a number of issues with clearinghouses. The first one is fragmentation. Clearing has to go by asset classes, since otherwise, in case of a default, holders of more liquid assets (e.g. interest rate swaps) are treated much faster and better than holders of less liquid ones (e.g. CDS). But this implies a large number of clearinghouses, whereas Duffie and Zhu (2011) have argued that a large number of clearinghouses fragment the market and make it inefficient in the end. A contrario Cont and Kokholm (2012) claim that this conclusion only holds under irrealistic homogeneity assumptions on the financial network. Maybe it would be better to restrict clearing to some asset classes.

The liquidation strategy of a defaulted member by the clearinghouse is another important issue, considered in Avellaneda and Cont (2013). But the solution they propose requires a rather liquid market, whereas in the case of CDS clearly, but even of interest rate swaps, the market is concentrated among very few major players.

The generalization of central clearing and collateralization poses severe liquidity, systemic and concentration issues (see Duffie (2010), Cont, Santos, and Moussa (2013)), along with the danger of creating "too big to fail" (or " too connected to fail") clearinghouses.

5.1 Margining and default processes [4]

In the case of centrally cleared trading with double level of margins (variation and initial) updated at regular time intervals, the main modeling issue concerns the margining schemes (since the residual counterparty risk after mitigation by the margins becomes negligeable, the issue of hedging becomes of secondary importance). The variation margin tracks the mark-to-market of the portfolio up to some thresholds ("free credit lines" of the clients) and minimal transfer amounts (to avoid transfers of little use). As for the initial margins to be provided by the clients of a CCP, the standard procedure uses quantiles of the portfolio loss distribution, at a time horizon determined both by the frequency of the variation margins call (which for centrally cleared transactions can be very short, e.g. a few hours) and by a cure period (time of liquidating the portfolio) usually estimated as five days. Regarding the determination of the quantiles, Gaussian VaR models are generally banned since the crisis and people typically focus on either Pareto laws or on historical VaR (sometimes bootstrapped to make it a bit richer). With the crisis, the focus has shifted from the cores of the distributions, dominated by volatility effects, to their queues, dominated by scenarios of crisis and default events. A "good" model (see Lopez, Harris, Hurlin, and Pérignon (2013) for a tentative axiomatisation) is one in which the margins increase sufficiently fast with the volatility of the market, without decreasing too quickly when the market gets more quiet. So, a certain asymetry of the margining process matters. Important points of concern are: procyclicity, as margins adjust to market volatility (especially via haircuts which increase with the distress of the posting party); liquidity, given the generalization of centrally cleared trading and collateralization. This being said, one can say that the whole margining process worked well on interest rate swaps during the crisis (at that time there was no clearing of CDS).

The default process refers to what happens in case of a default of a member. The liquidation of the defaulted name is achieved by a team of traders of the members. Their first commitment is to hedge the book of the defaulted name in order to avoid speculative trading. Then the members are asked to provide bids on the assets of the defaulted names. This procedure, called the auction process, is based on voluntary bids by the members. In case of residual losses beyond the margins (variation plus initial margin) of the defaulted name, the clearinghouse uses its default fund (or guarantee fund, contributed by the members based on higher-order, "rarer events" quantiles than those guaranteed through their variation and initial margins).

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