8.1 Understanding Logarithms

R7

(p. 370-379)

The logarithmic function is the inverse of the exponential function. Remember, to find the inverse of a function we switch the x and y values and then solve for y.

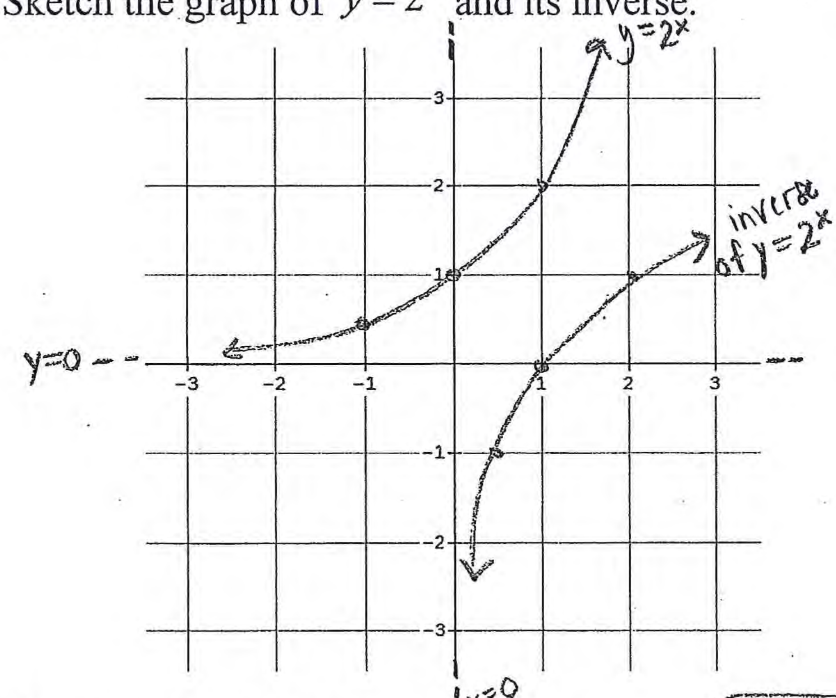
Exponential function	Inverse function
$y = b^x$	$x = b^y$

Notice that the y-value is now an exponent.

In order to isolate and manipulate exponents, we must use something called the logarithm function.

$y = \log_b x$ where $b \rightarrow$ *base of the log*
$y \rightarrow$ *the logarithm (the answer)*
$x \rightarrow$ *the argument*

Ex1: Sketch the graph of $y = 2^x$ and its inverse.



$y = 2^x$

x	y
2	4
1	2
0	1
-1	½
-2	¼

inverse → switch x + y

x	y
4	2
2	1
1	0
½	-1
¼	-2

Note: The equation of the graph of the inverse is $\boxed{y = \log_2 x}$.

exponential $y = b^x$ logarithmic $y = \log_b x$

Ex2: Express $m = 4^n$ in logarithmic form.

$\log_4 m = n$

Ex3: Express $\log_2 8 = 3$ in exponential form.

$2^3 = 8$ we know this to be a true statement

Ex4: Evaluate the following expressions:

need a numeric answer

a) $\log_2 16 = x$ $2^x = 16$ $2^4 = 16 \therefore x = 4$

b) $\log_2\left(\frac{1}{4}\right) \underbrace{= x}_{\text{add in if not there}}$ $2^x = \frac{1}{4}$

$2^x = \frac{1}{2^2} = 2^{-2} \therefore x = -2$

c) $\log_3(-27) = x$ $3^x = \underbrace{-27}_{}$

not possible to get a -ve value.

$\therefore$ no sol'n

Thus, $\log_B A = C$ where $A > 0$, $B > 0$, and $B \neq 1$

Note: The base of a logarithms cannot be negative.
The argument (A) of a logarithm is always positive.

Ex5: Solve the following equations:

a) $\log_x 5 = \dfrac{1}{2}$

$x^{1/2} = 5$

$\left(x^{1/2}\right)^{\frac{2}{1}} = 5^2$

$x = 25$

b) $\log x = -3$

$10^{-3} = x$

$\dfrac{1}{1000} = x$

Note: When the base is not indicated, this means that there is a base of 10.

$\log x = \log_{10} x$

Some Basic Logs to Remember: "Quicksnappers"

a) $\log_c 1 =$ 0

$\log_c 1 = x$

$c^x = 1$

$x = 0$

b) $\log_c c =$ 1

$\log_c c = x$

$c^x = c$

$x = 1$

c) $\log_c c^y =$ y

$\log_c c^y = x$

$c^x = c^y$

$x = y$

d) $c^{\log_c y} =$ y

let $\log_c y = a$

then $c^a = x$

change to log form

$\log_c x = a$

$\log_c x = \log_c y$

$x = y$

Try these; Evaluate

i) $\log_2 16$

$\log_2 2^4$

$= 4$

ii) $5^{\log_5 4}$

$= 5$

iii) $\log 10$

$\log_{10} 10 = 1$

Ex6: Estimate the following value:

$\log_2 30 = x$

$2^x = 30$

Since $2^5 = 32$ and $2^4 = 16$

The value of x is closer to 5

$\therefore \log_2 30 \approx 4.8$

Homework: Page 380 #1-5, 7-10, 13-15

8.2 Transformations of Logarithmic Functions

R9
(p. 383-391)

Ex1: Sketch the graphs of the functions $y = 3^x$ and $y = \log_3 x$.
Note that these two functions are inverses of each other.

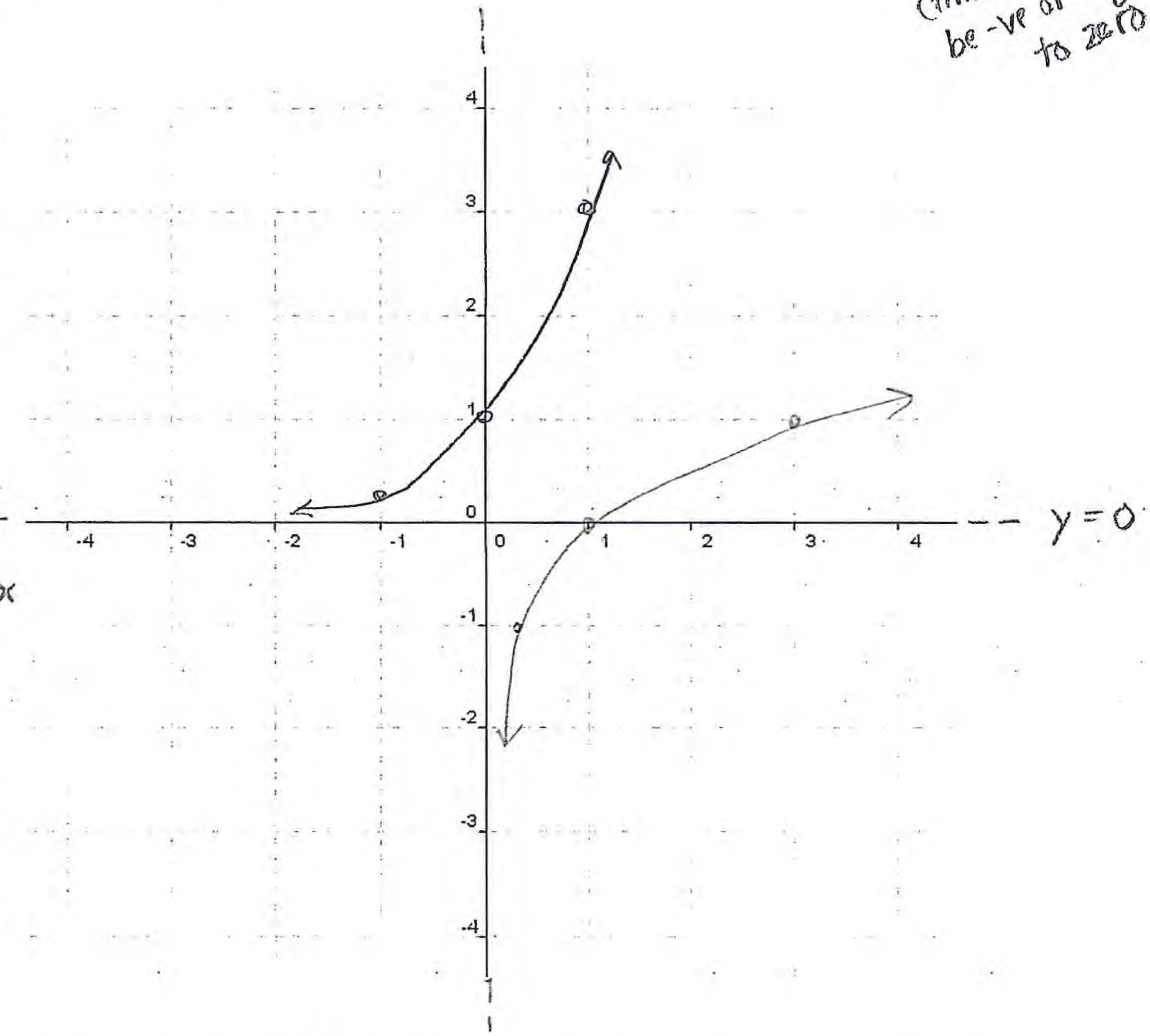
cannot be -ve or equal to zero

$y = 3^x$

x	y
2	9
1	3
0	1
-1	$1/3$
-2	$1/9$

$y = \log_3 x$

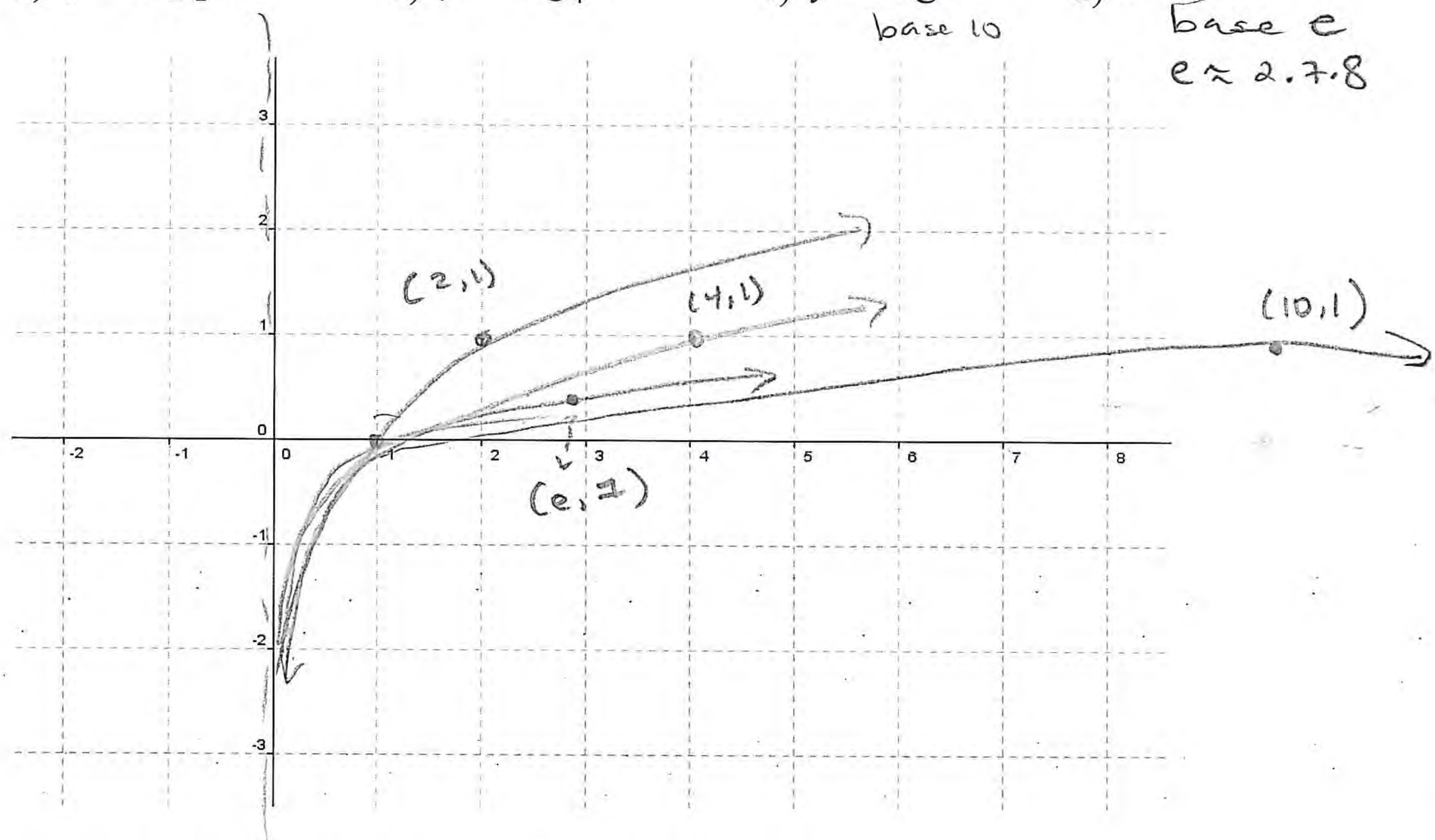
x	y
9	2
3	1
1	0
$1/3$	-1
$1/9$	-2



Note: The graph of $y = \log_3 x$ has a vertical asymptote at $x = 0$
because $x > 0$ is a restriction of the argument.

Ex2: Sketch the graphs of the following functions on the same Cartesian plane.

a) $y = \log_2 x$ b) $y = \log_4 x$ c) $y = \log x$ base 10 d) $y = \ln x$ base e $e \approx 2.78$



Note: All the graphs pass through the point (1,0).
The base of the logarithm determines the next point.

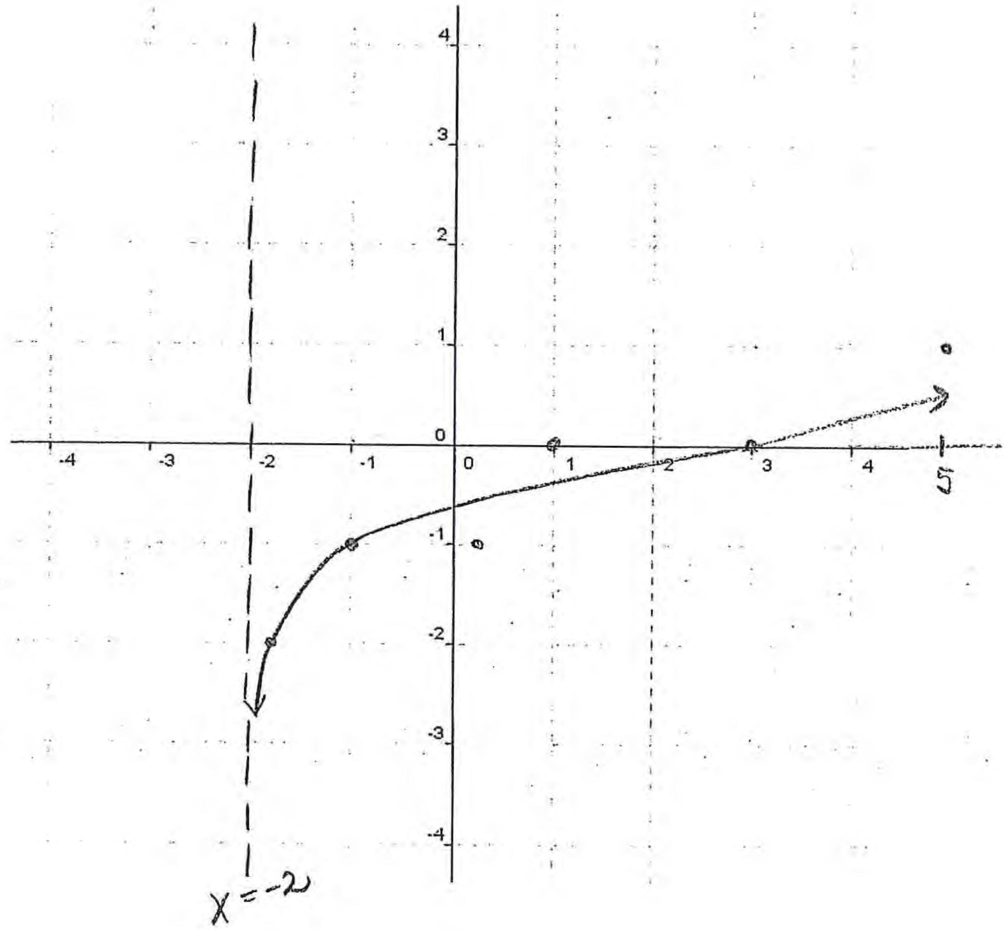
Base 2 $\rightarrow$ (2,1)
Base 4 $\rightarrow$ (4,1)
Base 10 $\rightarrow$ (10,1)

$$y = \log_2 x \rightarrow 2^y = x \rightarrow 2^1 = 2 \rightarrow (2,1)$$

Ex3: Sketch the graph of the function $y = \log_5(x+2) - 1$.

base graph

left 2, down 1



State the domain: $\{x > -2\}$ or $(-2, \infty)$

Note: $\log_5 \underbrace{(x+2)}$ -1 must be +ve

$\therefore x+2 > 0$

$x > -2$.

Determine the y-intercept:

$y = \log_5(0+2) - 1$

$y = \log_5 2 - 1$

$y = 0.43 - 1$

$y = -0.57$

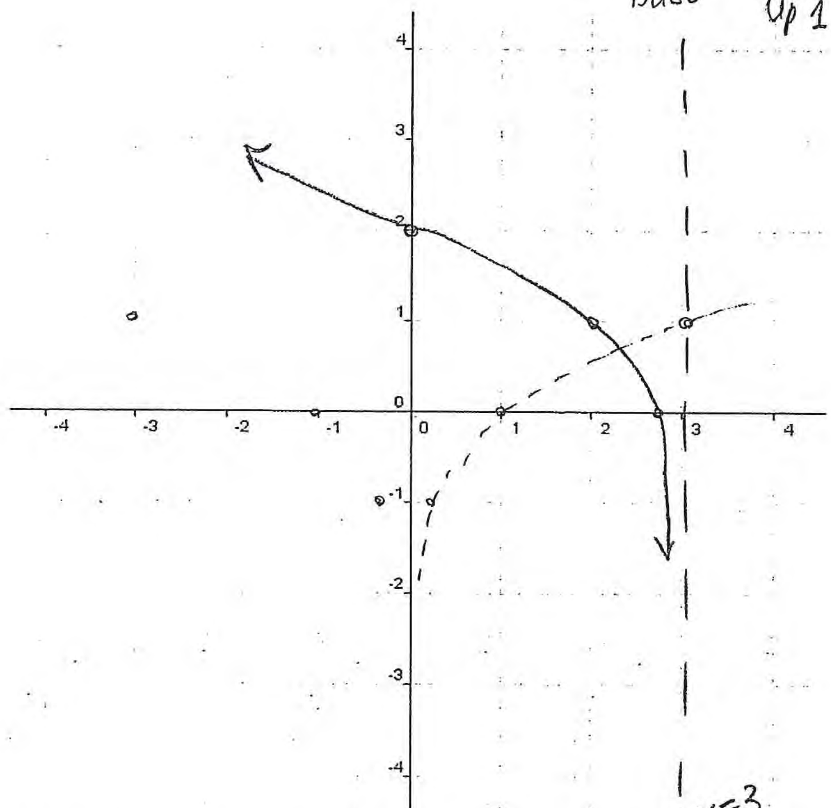
OBB's law → change of base theorem

$\log_5 2 = \frac{\log 2}{\log 5} = 0.43$.

$(-x+3) \rightarrow (-(x-3))$

Ex4: Sketch the graph of the function $y = \log_3(3-x)+1$

base   up 1   reflect over y-axis   right 3



Determine the following characteristics of the graph:

Domain: $\{x \in \mathbb{R} \mid x < 3\}$ or $(-\infty, 3)$

Range: $\{y \in \mathbb{R}\}$ OR $(-\infty, \infty)$

x-intercept: $0 = \log_3(3-x)+1$

$-1 = \log_3(3-x)$

$3^{-1} = 3-x$

$3^{-1} - 3 = -x$

$\frac{1}{3} - 3 = -x$

$-2^{2/3} = -x$

$x = -2^{2/3}$

y-intercept :

y-intercept

$y = \log_3(3-0)+1$

$y = \underbrace{\log_3 3}_{1} + 1$

$y = 2$

Equation of the asymptote:

$x = 3$

Homework: Page 389 #1, 3-9, 15

Homework: Graph each of the following and find the intercepts: * the x int must always be included

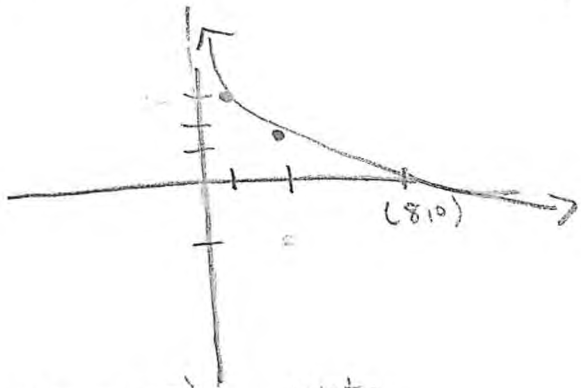
1) $y = -\log_2(x) + 3$

2) $y = 2\log(x+1) - 1$ → $(x, y) \to (x-1, 2y-$
$(1, 0) \to (0, -1)$
$(10, 1) \to (9, 1)$
$x = 0$ $\quad x = -1$

3) $y = \ln(x-1)$

4) $y = 2^{x-1} + 3$

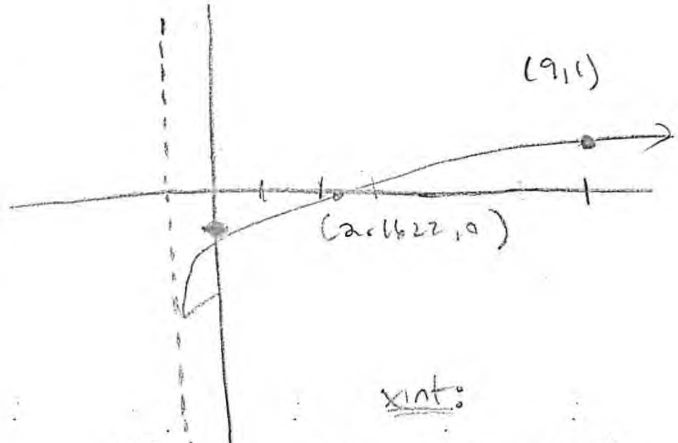
1)



(x, y) $\quad (x, -y+3)$
$(1, 0)$ $\quad (1, 3)$
$(2, 1)$ $\quad (2, 2)$
$x = 0$ $\quad x = 0$

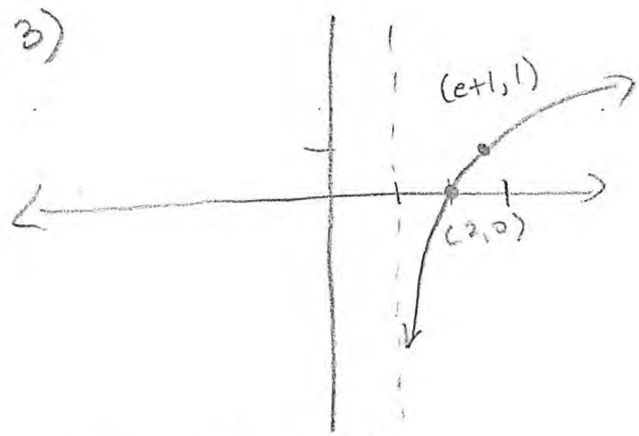
xint:
$0 = -\log_2(x) + 3$
$-3 = -\log_2 x$
$3 = \log_2 x$
$2^3 = x$
$x = 8$

2)



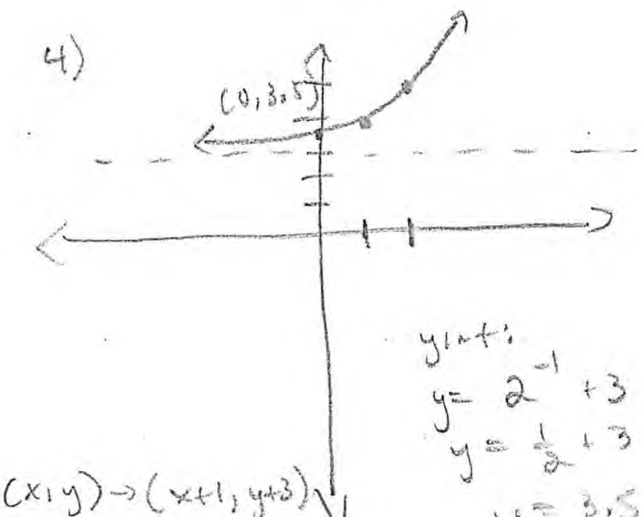
xint:
$0 = 2\log(x+1) - 1$
$\frac{1}{2} = \log(x+1)$
$10^{1/2} = x+1$
$\sqrt{10} - 1 = \cancel{x}$
$2.1622 = x$

3)



$(x, y) \to (x+1, y)$
$(1, 0) \to (2, 0)$
$(e, 1) \to (e+1, 1)$
$x = 0$ $\quad x = 1$

4)



yint:
$y = 2^{-1} + 3$
$y = \frac{1}{2} + 3$
$y = 3.5$

$(x, y) \to (x+1, y+3)$
$(0, 1) \to (1, 4)$
$(1, 2) \to (2, 5)$
$y = 0$ $\quad y = 3$

8.3 Laws of Logarithms

1. $\boxed{\log_a MN = \log_a M + \log_a N}$

   ex: $\log_2 6 = \log_2 (2)(3) = \log_2 2 + \log_2 3$

2. $\boxed{\log_a \frac{M}{N} = \log_a M - \log_a N}$

   ex: $\log_x \frac{8}{3} = \log_x 8 - \log_x 3$

3. $\boxed{\log_a M^x = x \log_a M}$

   ex: $\log_3 \sqrt{6} = \log_3 6^{1/3} = \frac{1}{3} \log_3 6$

4. $\boxed{\log_a x = \frac{\log x}{\log a}}$

   ex: $\log_2 7 = \frac{\log 7}{\log 2} = 2.807$

   OBB's law
   Old base bottom

The $\boxed{\log}$ button on a calculator is of base 10. Therefore, this button can only be used to find solutions to questions with base 10.

For example, $\log_2 7$ cannot be entered into the calculator.
We must use the 'change of base' law to convert this expression to base 10 and then a calculator can be used to solve it.

$$\log_2 7 = \frac{\log 7}{\log 2}$$

Ex1: Simplify the expression $2\log x-(\log y-\frac{1}{3}\log z)$.

$2\log x - \log y + \frac{1}{3}\log z$

$\log x^2 - \log y + \log z^{1/3}$

$\log \frac{x^2 \sqrt[3]{z}}{y}$

Ex2: Expand the expression $\log_a\left(\frac{A\sqrt{C}}{B^2D}\right)$.

$\log_a A + \frac{1}{2}\log_a C - 2\log_a B - \log_a D$

Ex3: Simplify and evaluate the following expressions using laws of logarithms.

a) $\log_6 8+\log_6 9-\log_6 2$

$\log_6 \left(\frac{8(9)}{2}\right)$

$\log_6 36$

$\log_6 6^2$

$2\log_6 6$

$2(1)$

2

b) $2\log_2 12-\left(\log_2 6+\frac{1}{3}\log_2 27\right)$

$\log_2 12^2 - \log_2 6 - \log_2 27^{1/3}$

$\log_2 \frac{12^2}{6(27^{1/3})}$

$\log_2 \frac{144}{6(3)}$

$\log_2 8$

$\log_2 2^3$

$=3$

Ex4: If $\log_a 2 = 0.30$, $\log_a 3 = 0.48$ and $\log_a 5 = 0.70$, evaluate

$\log_a 150$ 3 $15 \cdot 10$

$\log_a 3 \cdot 5 \cdot 2 \cdot 5$

$\log_a 3(5)^2(2)$

$\log_a 3 + 4\log_a 5^2 + \log_a 2$

$\log_a 3 + 2\log_a 5 + \log_a 2$

$0.48 + 2(0.7) + 0.3$

2.18

Homework: Page 400 #1-3, 5-10

CASE 1: log ~ + log ~ = log ~

Ex1: Solve: $\log_2(x-2)-\log_2 x=\log_2 3$

$$\log_2\frac{(x-2)}{x}=\log_2 3$$

$$(x)\ \frac{x-2}{x}=3\ (x)$$

$$x-2=3x$$
$$-x \qquad -x$$
$$\frac{-2}{2}=\frac{2x}{2}$$
$$-1=x$$

① Simplify where possible.

② once equ'n becomes log__ = log__ then you can drop the logs.

③ Solve for x.

④ verify sol'n

* Remember → the arguement must be +ve & greater than 1

plug in to verify.

$\log_2(-1-2)=\log_2(-3)$
↳ arguement $\neq$ -ve

∴ no sol'n

OR check domain
$x-2>0$
$x>2$
$-1 \not> 2$ ∴ no sol'n

Ex2: Solve: $\log_4(11-x)+\log_4(x+6)=2$

CASE 2: log ~ + log ~ = #

$$\log_4(11-x)(x+6)=2$$
$$4^2=(11-x)(x+6)$$
$$16=11x+66-x^2-6x$$
$$x^2-5x-50=0$$
$$(x-10)(x+5)=0$$
$$\boxed{x=10}\quad\boxed{x=-5}$$

① Simplify logs where possible

② Once you get log ~ = #, change to exponential form

③ Solve for x

④ verify.

Check: $11-10=1$ ✓ $\quad 11-(-5)=16$ ✓
$10+6=16$ ✓ $\quad -5+6=1$ ✓

Your turn

Solve: $2\log x - \log(x+2) = \log(2x-3)$

$\log x^2 - \log(x+2) = \log(2x-3)$

$\log \frac{x^2}{x+2} = \log(2x-3)$

$(x+2)\ \frac{x^2}{\cancel{x+2}} = 2x-3\ (x+2)$

$x^2 = 2x^2 + 4x - 3x - 6$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3 \quad x = 2$

this will give a -ve arguement

∴ not a sol'n

$x > 0$

$x+2 > 0$

$x > -2$

$2x-3 > 0$

$2x > 3$

$x > \frac{3}{2}$

∴ $x > -2$

$\underbrace{2,\ 2+2=4,\ 2(2)-3=1}$

all +ve arguements

∴ $\boxed{x=2}$ is a sol'n.

Solve: $\log_6(x+3) = 1 - \log_6(x+4)$

$x+3 > 0$

$x > -3$

$x+4 > 0$

$x > -4$

$\log_6(x+3) + \log_6(x+4) = 1$

$\log_6(x+3)(x+4) = 1$

$6^1 = x^2 + 7x + 12$

$0 = x^2 + 7x + 6$

$0 = (x+1)(x+7)$

$\boxed{x=-1} \quad x = -7$

not in domain

∴ not a sol'n

OR will give a -ve arguement

$-7+3 = -4$ ✗

CASE 3 :

Solving Exponential Equations With Different Bases

When the bases of exponents cannot be changed to the same value we must use logarithms to solve the equation.

Ex1: Solve : $3^x = 8$

Note: Although we do not know the value of x, we can estimate that $x \approx 1.9$.

To access the exponent (the variable), take the "log" of both sides. This will allow the exponent to drop down in front of the "log" and become accessible.

$$3^x = 8$$

$$\log(3^x) = \log(8)$$

Now isolate "x".

$$\frac{x\log 3}{\log 3} = \frac{\log 8}{\log 3}$$

$$x = \frac{\log 8}{\log 3}$$

Now use calculator to evaluate x.

$$\boxed{x = 1.893}$$

Ex2: Solve the following equation below
Express the answer correct to 3 decimal places.

a) $19^{x-5} = 3^{2x+1}$

$$\log 19^{x-5} = \log 3^{2x+1}$$
$$(x-5)\log 19 = (2x+1)\log 3$$
$$x\log 19 - 5\log 19 = 2x\log 3 + \log 3$$
$$x\log 19 - 2x\log 3 = \log 3 + 5\log 19$$
$$\frac{x(\log 19 - 2\log 3)}{\log 19 - 2\log 3} = \frac{\log 3 + 5\log 19}{\log 19 - 2\log 3}$$
$$x = \frac{\log 3 + 5\log 19}{\log 19 - 2\log 3} \qquad \boxed{x = 21.173}$$

STEPS

1. Log both sides
2. Drop down exponents
   * if binomial, be sure to use brackets!
3. Distribute / Expand
4. Collect x's on one side, #'s on the other side.
5. Isolate x by factoring
6. Evaluate x using the calculator.

b) $2(3)^x = 6^{3x-1}$

Note: $2(3)^x \neq 6^x$

* use log laws to expand!

$$\log 2(3)^x = \log 6^{3x-1}$$
$$\log 2 + \log 3^x = \log 6^{3x-1}$$
$$\log 2 + x\log 3 = (3x-1)\log 6$$
$$\log 2 + x\log 3 = 3x\log 6 - \log 6$$
$$x\log 3 - 3x\log 6 = -\log 6 - \log 2$$
$$x(\log 3 - 3\log 6) = -\log 6 - \log 2$$
$$x = \frac{-\log 6 - \log 2}{\log 3 - 3\log 6}$$
$$\boxed{x = 0.581}$$

Homework: Page 412 #1-6, 7(a & d), 8(a & b), 20b), C1, C4b)

8.4 Applications of Logarithmic & Exponential Functions Part 1

Ex1: The Richter magnitude, M, of an earthquake is defined as $M = \log\left(\frac{A}{A_0}\right)$ where A is the amplitude of the ground motion and A_0 is the amplitude associated with a "standard" earthquake.

a) In 1946, in Haida Gwaii, British Columbia, an earthquake with an amplitude measuring $10^{7.7}$ times A_0 struck. Determine the magnitude of this earthquake on the Richter scale.

$$M = \log\left(\frac{10^{7.7}\cancel{A_0}}{\cancel{A_0}}\right)$$

$$M = \log 10^{7.7}$$

$$M = 7.7 \log 10$$

$$M = 7.7$$

b) The strongest recorded earthquake in Haida Gwaii was in 1949 and had a magnitude of 8.1 on the Richter scale. Determine how many times stronger this earthquake was than the one in 1946.

1949

$$8.1 = \log\left(\frac{A_1}{A_0}\right)$$

$$10^{8.1} = \frac{A_1}{A_0}$$

$$A_0 10^{8.1} = A_1$$

1946

$$7.7 = \log\left(\frac{A_2}{A_0}\right)$$

$$10^{7.7} = \frac{A_2}{A_0}$$

$$A_0 10^{7.7} = A_2$$

$$\frac{A_1}{A_2} = \frac{\cancel{A_0} 10^{8.1}}{\cancel{A_0} 10^{7.7}}$$

$$\frac{A_1}{A_2} = 10^{8.1-7.7}$$

$$\frac{A_1}{A_2} = 10^{0.4}$$

The 1949 quake was 2.512 times greater

Short cut

$$10^{8.1-7.7}$$

$$10^{0.4}$$

$$2.512$$

The 1949 quake was 2.512 times stronger

Ex2: The pH scale is used to measure the acidity or alkalinity of a solution. it is defined as $pH = -\log(H^+)$ where H^+ is the concentration of hydrogen ions measured in moles per litre (mol/L). A neutral solution, such as pure water, has a pH of 7. The closer the solution is to 0, the more acidic the solution. The closer the solution is to 14, the more alkaline the solution.

a) A cola drink has a pH of 2.5 whereas milk has a pH of 6.6. How many times as acidic as milk is a cola drink? This is calculated by comparing the number of ions in each substance.

Cola: $\frac{2.5}{-1} = \frac{-\log(H^+)}{-1}$

$-2.5 = \log(H^+)$

$10^{-2.5} = H^+$

Milk: $6.6 = -\log(H^+)$

$-6.6 = \log(H^+)$

$10^{-6.6} = H^+$

Comparison: $\frac{10^{-2.5}}{10^{-6.6}} = 12\,589$ times more acidic.

b) An apple is 5 times as acidic as a pear. If a pear has a pH of 3.8, determine the pH of an apple.

Pear: $3.8 = -\log(H^+)$

$-3.8 = \log(H^+)$

$10^{-3.8} = H^+$

Apple: $5 \times 10^{-3.8} = H^+$

$pH = -\log(5 \times 10^{-3.8})$

$pH = 3.101$

Ex3: The human ear is able to detect sounds of different intensities. Sound intensity, β, in decibels, is defined as $\beta = 10\log\frac{I}{I_0}$ where I is the intensity of the sound measured in watts per square metre (W/m²), and I_o is 10^{-12} W/m² which is the threshold of hearing.

a) It is recommended a person wears protective ear gear when the sound intensity is 85dB or greater. The MTS Centre measures 110dB when the Jets score a goal. How many times louder is the MTS Centre than the recommended maximum sound intensity?

β of recommended

β of MTS Centre

Recommended:

$$\frac{85}{10} = \frac{10\log\left(\frac{I}{I_0}\right)}{10}$$

$$8.5 = \log\left(\frac{I}{I_0}\right)$$

$$10^{8.5} = \frac{I}{I_0}$$

$$10^{8.5}(I_0) = I$$

MTS Centre:

$$\frac{110}{10} = \frac{10\log\left(\frac{I}{I_0}\right)}{10}$$

$$11 = \log\left(\frac{I}{I_0}\right)$$

$$10^{11} = \frac{I}{I_0}$$

$$10^{11}(I_0) = I$$

Comparison:

$$\frac{10^{11}(\cancel{I_0})}{10^{8.5}(\cancel{I_0})}$$

$$= 10^{11-8.5}$$

$$= 10^{2.5}$$

$\simeq$ 316 stronger than the recommended level.

b) A truck emits a sound intensity of 0.001 W/m². Determine its decibel level.

$I = 0.001$ W/m² $I_0 = 10^{-12}$ W/m² (given)

$\beta = ?$

$$\beta = 10\log\frac{0.001}{10^{-12}}$$

$$= 10\log\frac{10^{-3}}{10^{-12}}$$

$$= 10\log 10^{-3-(-12)}$$

$$= 10\underbrace{\log 10^9}_{\log_{10}10 = 1}$$

$$= 10\log 10^9$$

$$= 10\cdot 9\underbrace{\log 10}_{1}$$

$$= 10\cdot 9$$

$$= 90 \text{ dB}$$

Homework: Page 381 #17 & 19, Page 391 #13, Page 401 #13(b & c), 14,16

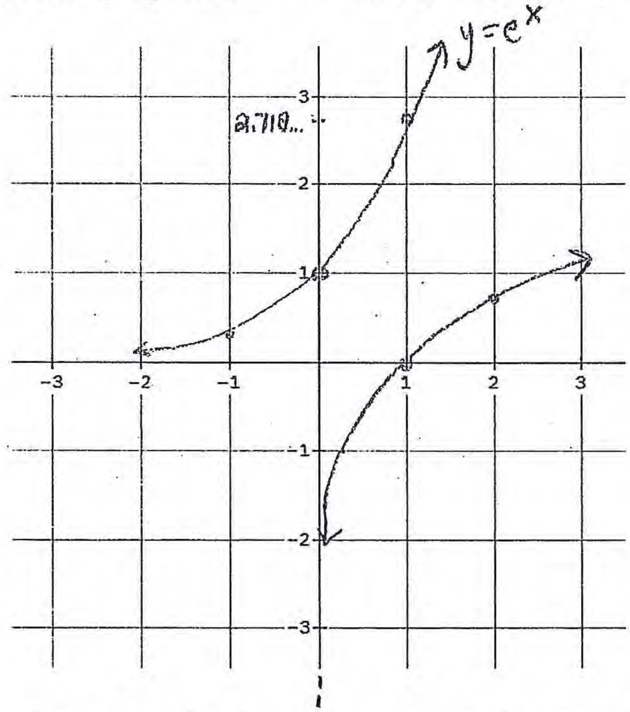
The value of *e* and the Natural Logarithm

Like π, *e* is a symbol used to represent the number 2.718281828… .

$e = 2.718281828....$

It has its own special logarithm called the natural logarithm.
« ln » is used to represent the natural logarithm and thus has a base of « *e* »
"l" "n" (not i/I/1)

Ex1: Sketch the graph of $y = e^x$ and its inverse $y = \ln x$.



$y = e^x = 2.718...^x$

$y = \ln e$ is the same as

$y = \underbrace{\log_e e}_{1}$

$\therefore \ln e = 1$

Note: « ln » has exactly the same properties as « log ».

Ex1: Evaluate the following expressions:

a) $\ln\frac{5}{3} = \ln 5 - \ln 3$
(use calculator)
$= \boxed{0.511}$

b) $\ln e^{3} = 3 \ln e$
$\log_e e = 1$
$= 3(1)$
$= \boxed{3}$

SAme bases!

c) $10^{\log 5} = x$

quick snappu!

$x = 5$

d) $e^{\ln 7} = x$

$x = 7$

Ex2: Solve the following equations.

$\ln e = 1$

a) $e^{2t-1} = 5$

$\ln e^{2t-1} = \ln 5$

$2t - 1 = \ln 5$

$2t = \ln 5 + 1$

$t = \dfrac{\ln 5 + 1}{2}$

SAme base

b) $e^{\ln(2y-1)} = 5$

$2y - 1 = 5$

$2y = 6$

$y = 3$

c) $e^x = 2^{x+1}$

$\ln e^x = \ln 2^{x+1}$

$x = (x+1)\ln 2$

$x = x\ln 2 + \ln 2$

$x - x\ln 2 = \ln 2$

$x(1 - \ln 2) = \ln 2$

$x = \dfrac{\ln 2}{1 - \ln 2}$

d) $\ln(x+1) = 1 + \ln x$

$\ln(x+1) - \ln x = 1$

$\ln\left(\dfrac{x+1}{x}\right) = 1$

$e^1 = \dfrac{x+1}{x}$

$ex = x + 1$

$ex - x = 1$

$x(e - 1) = 1$

$x = \dfrac{1}{e-1}$

The value of e is often used to solve exponential application problems.

Ex3: There are 500 mice found in a field on June 1. On June 20, 800 mice are counted. If the population of mice continues to increase at the same rate, determine how many mice there will be on June 28.

Use $A = Pe^{rt}$ where P = *initial value*
A = *final value*
t = *time, in days*
r = *rate of increase or decrease*

Note: If $r > 0$, then the function increases exponentially
If $r < 0$, then the function decreases exponentially

$$800 = 500e^{r(19)}$$
$$\frac{800}{500} = e^{19r}$$
$$\frac{8}{5} = e^{19r}$$
$$\ln(8/5) = \ln e^{19r}$$
$$\ln(8/5) = 19r$$
$$\frac{\ln(8/5)}{19} = r$$
$$0.024737... = r$$

$$A = 500e^{27(r)}$$
$$A = 975.068$$

975 mice

Homework: Word Problem Worksheet

Word Problems

1. A radioactive substance is decaying according to the following formula $y = Ae^{-0.2x}$ where A = original amount and y = amount remaining after x years.
   a) If we started with 80grams of material, how much is left after 3 years?
   b) Find the half-life.

2. A $5000 investment earns interest at the annual rate of 8.4% compounded monthly.
   $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
   a) What is it worth after 1 year?
   b) What is it worth after 10 years?
   c) How much interest is earned after 10 years?

3. At the present time there are 5000 type A bacteria. The rate of increase per hour is 0.025. How many bacteria can you expect in 24 hours? $A = Pe^{rt}$

4. A radioactive substance decays at a daily rate of 0.13. How long does it take for this substance to decompose to half its size? $A = Pe^{rt}$

5. If you invest any amount of money at 11.25% compounded quarterly, how long will it take for the investment to double? $A = P\left(1 + \frac{r}{n}\right)^{nt}$

6. Craig invests $1000 in a mutual fund which is supposed to grow at 10% compounded annually. Laura has concerns about the stock market so she buys $2000 worth of bonds paying 5% compounded annually. After how many years will Craig's investment be equal in value to Laura's?
   $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

7. Determine how many monthly investments of \$200 would have to be made into an account that pays 6% annual interest, compounded monthly, for the future value to be \$100,000.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

*#8 is missing ☹

9. The population of a certain country is 28 million and grows continuously at a rate of 3% annually. How many years will it take for the population to reach 40 million? $A = Pe^{rt}$

10. A radioactive substance decays so that the amount present is "P" grams after "t" years according to the following: $P = 50e^{-0.135t}$. What is the half-life of the substance?

11. The most intense earthquake ever recorded was in Chile in May 1960, with magnitude 9.5. In January 2010, Haiti experienced an earthquake with magnitude 7.0.
   a) Calculate the intensity of the Haiti earthquake in terms of a standard earthquake.
   b) Calculate the intensity of the Chile earthquake in terms of a standard earthquake.
   c) How many times as intense as the Haiti earthquake was the Chile earthquake? Give answer to the nearest whole number.

$$M = \log\left(\frac{A}{A_0}\right)$$

Word Problems

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7. Determine how many monthly investments of \$200 would have to be made into an account that pays 6% annual interest, compounded monthly, for the future value to be \$100,000.

8. A person borrows \$15000 to buy a car. The person can afford to pay \$300 a month. The loan will be repaid with equal monthly payments at 6% annual interest, compounded monthly. How many monthly payments will the person make?

9. The population of a certain country is 28 million and grows continuously at a rate of 3% annually. How many years will it take for the population to reach 40 million?

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1

$y = Ae^{-0.2x}$

a) $y = 80e^{-0.2x}$

$y = 80e^{-0.2(3)}$

$y = 80e^{-.6}$

$y = 43.905$ gms $\therefore$ 43.905gms will remain in 3 yrs

b) $40 = 80e^{-0.2x}$

$\frac{40}{80} = e^{-0.2x}$

$.5 = e^{-0.2x}$

$\ln(.5) = \ln e^{-0.2x}$

$\ln(.5) = -0.2x \cdot \ln e$

$\frac{\ln .5}{-0.2} = x$

$x = 3.466$

$\therefore$ ~~3.466~~ 3.4657 yrs is the half life

② a) $A = P\left(1+\frac{i}{n}\right)^{n \cdot t}$

$= 5000\left(1+\frac{.084}{12}\right)^{12(1)}$

$= \$5436.55$

b) $A = 5000\left(1+\frac{.084}{12}\right)^{12(10)}$

$= \$11547.99$

c) $11547.99 - 5000.00$

$= \$6547.99$ interest

③ $A = P \cdot e^{rt}$

$= 5000\, e^{0.025(24)}$

$= 9110$ bacteria

④ half life $\frac{1}{2} = e^{-0.13t}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-0.13t}$

$\ln\left(\frac{1}{2}\right) = -0.13t \cdot \ln e$

$\frac{\ln\left(\frac{1}{2}\right)}{-0.13} = t$ $t = 5.33$ yrs

5)

Double

$$2 = \left(1+\frac{.1125}{4}\right)^{4t}$$

$$\log 2 = \log\left(1+\frac{.1125}{4}\right)^{4t}$$

$$\log 2 = 4t\left(\log\left(1+\frac{.1125}{4}\right)\right)$$

$$\frac{\log 2}{4\left(\log\left(1+\frac{.1125}{4}\right)\right)} = t$$

$$t = 6.24755$$

$$\therefore\ t = 6.25 \text{ yrs}$$

6)

$$1000(1+0.1)^t = 2000(1+.05)^t$$

$$(1.1)^t = 2(1.05)^t$$

$$\log(1.1)^t = \log\left(2(1.05)^t\right)$$

$$t\log(1.1) = \log 2 + \log(1.05)^t$$

$$t\log(1.1) = \log 2 + t\log(1.05)$$

$$t\log(1.1) - t\log(1.05) = \log 2$$

$$t\left(\log(1.1) - \log(1.05)\right) = \log 2$$

$$t = \frac{\log 2}{(\log 1.1 + \log 1.05)}$$

$t = 14.899$

$t = 14.9$ yrs

(7) $100000 = 200\left(1+\frac{.06}{12}\right)^{12t}$

$500 = \left(1+\frac{.06}{12}\right)^{12t}$

$\log 500 = \log\left(1+\frac{.06}{12}\right)^{12t}$

$\log 500 = 12t \log\left(1+\frac{.06}{12}\right)$

$\frac{\log 500}{12 \log\left(1+\frac{.06}{12}\right)} = t$

$t = 103.8355$ yrs

$\therefore t = 1246$ or 1247 ~~yr~~ months

(8) Omit

(9) $40 = 28 e^{.03t}$

$\frac{40}{28} = e^{.03t}$

$\ln \frac{40}{28} = \ln e^{.03t}$

$\ln \frac{40}{28} = .03t \ln e$

$t = \frac{\ln \frac{40}{28}}{.03}$ $\quad t = 11.89$ yrs

(10) $25 = 50\,e^{-0.13t}$

$.5 = e^{-0.13t}$

$\ln .5 = \ln e^{-0.13t}$

$\ln .5 = -0.13t \ln e$

$\dfrac{\ln .5}{-0.13} = t$

$t = 5.33$ yrs

(11) b) $9.5 = \log \dfrac{A}{A_0}$

$10^{9.5} = \dfrac{A}{A_0}$

$A = 10^{9.5} \cdot A_0$

a) $7 = \log \dfrac{A}{A_0}$

$10^{7} = \dfrac{A}{A_0}$

$A = 10^{7} \cdot A_0$

c, $\dfrac{10^{9.5} \cdot A_0}{10^{7.0} \cdot A_0}$

$= 316$ times Greater