STUDENT NAME:			
SUBJECT TEACHER:		TUTOR GROUP:	

Year 11

MATHEMATICAL METHODS CAS UNIT 1

(b1mMC1)
Written examination 2

TUESDAY 11th of June 2013

Reading time: 8:55 am to 9:10 am (15 minutes)
Writing time: 9:10 am to 10.40 am (90 Minutes)

QUESTION AND ANSWER BOOK

Structure of book

Section		*Number of questions*	*Number of questions to be answered*	*Number of marks*	*Suggested time per section*
Section A	**Vocabulary questions**	5	5	5	*5 mins*
Section B	**Multiple Choices questions**	20	20	20	*20 mins*
Section C	**Short answer questions**	5	5	35	*40 mins*
Section D	**Analysis Questions**	2	2	20	*25 mins*
				Total = 80 marks	*Total = 90 mins*

INSTRUCTIONS TO STUDENTS

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- CAS Calculator **is allowed** in this examination.
- One bound document of prepared theory notes **is allowed** to the examination.

Materials supplied

- Additional space is available at the end of the book if you need extra paper to complete an answer.

Instructions

- Write your **student name, tutor group & subject teacher's name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones, mp3 players and/or any other unauthorised electronic devices into the examination room.

Section A: Vocabulary   (5 marks)

Place the correct answer from the list below in the spaces provided.

Words List

The domain of a relation	natural numbers3	cubic
Image4	quadratic 1	up or down2
Left or right	integers	pre-image5

1. All ________________ functions could be written in 'perfect square'.

2. By adding or subtracting a constant term to $y = x^4$, the graph moves either _______.

3. The elements of {1, 2, 3, 4, . . .} are called the _______________.

4. The element y is called the _________________of x under f and x is called the _____________ of y.

JANE SOLVED THE MULTIPLE CHOICE QUESTIONS:

Multiple choice answers should be;

E, E, B, B, E, B, D, E, A, E, C, C, C, A, C, E, D, D, D, C.

<u>SECTION B: Multiple choice questions:</u> (20 × 1 = 20 marks)

Instructions for Section B
Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question.
A correct answer scores 1, an incorrect answer scores 0. Marks will not be deducted for incorrect answer.
No marks will be given if more than one answer is completed for any question.

1. The solution of the equation $\frac{x}{a} + \frac{x}{b} = c$ is

A $\frac{c}{a + b}$

B cab

C $c - a - b$

D $c(a + b)$

E $\frac{cab}{a + b}$

2. (6, 3) is the midpoint of the line joining the points with coordinates (–4, y) and (x, –6).

The value of $x + y$ is

A 0

B 16

C 20

D –10

E 28

3. The tangent of the angle between the line with equation $3y = 5 - 4x$ and the positive direction of the x-axis is

A -4

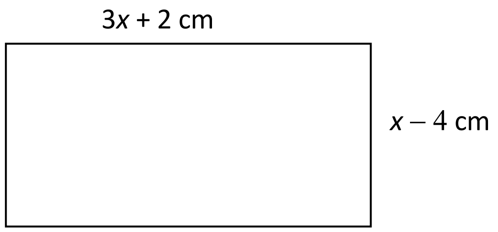
B $-\frac{4}{3}$

C $\frac{3}{4}$

D -2

E 5

4. The perimeter of the rectangle shown is 60 cm.



The area is

A 8

B 104

C 26

D 31

E 4

5. If $\mathbf{A} = \begin{bmatrix} 2 & 7 \\ -5 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 9 & -2 \\ -3 & 0 \end{bmatrix}$, $2\mathbf{A} + 3\mathbf{B}$ is equal to

A $\begin{bmatrix} -23 & 20 \\ -1 & 8 \end{bmatrix}$

B $\begin{bmatrix} -19 & 23 \\ -7 & 16 \end{bmatrix}$

C $\begin{bmatrix} -11 & 7 \\ -8 & 24 \end{bmatrix}$

D $\begin{bmatrix} 23 & -20 \\ 1 & -8 \end{bmatrix}$

E $\begin{bmatrix} 31 & 8 \\ -19 & 8 \end{bmatrix}$

6. If $x + \dfrac{1}{x} = 9$ then the value of $x^2 + \dfrac{1}{x^2}$ is

A 81

B 79

C 36

D 54

E 18

7. The equation of the parabola that passes through the point (0, 11) and has its vertex at (3, −7) is:

A $y = 2(x + 3)^2 + 7$

B $y = (x + 3)^2 + 7$

C $y = (x + 3)^2 - 7$

D $y = 2(x - 3)^2 - 7$

E $y = 2(x - 3)^2 + 7$

8. The solution of the inequality $3x^2 \leq 5x$ is

A $0 \leq - x \leq \frac{3}{5}$

B $x \geq - \frac{5}{3}$

C $x \geq 0$

D $x \leq \frac{5}{3}$

E $0 \leq x \leq \frac{5}{3}$

9. $4y + 3x = 25$ is the tangent to the circle $x^2 + y^2 = 25$ at the point $P(3, 4)$. The equation of the radius of the circle that passes through P is

A $3y - 4x = 0$

B $4y + 3x = 25$

C $3y + 4x = 25$

D $3y + 4x = 0$

E $4y + 3x = 0$

10. The maximum value of $-4(\sqrt{x-3} + 26)$ is

A 104

B –12

C 12

D 0

E –104

11. If $x - 3$ is a factor of $x^2 + ax + b$, then $3a + b + 10$ equals

A –7

B –1

C 1

D 7

E 9

12. The curve with equation $y = x^3$ is transformed under a dilation of factor 4 from the y-axis and then by a translation of 6 units in the positive direction of the x-axis. The equation of the image is

A $y = 4(x-3)^3$

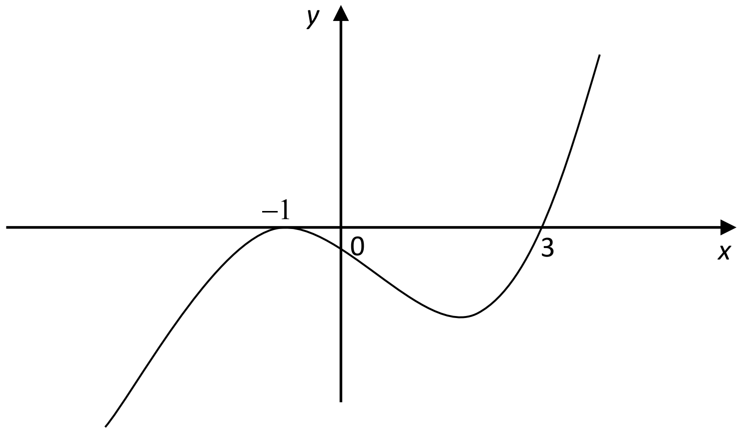
B $y = 4x^3 + 3$

C $y = \dfrac{(x-6)^3}{64}$

D $y = \dfrac{(x-6)^3}{4}$

E $y = \dfrac{(x+6)^3}{64}$

13. The equation of the graph shown is



A $y = x(x - 1)(x + 3)$

B $y = x(x + 1)(x - 3)$

C $y = (x + 1)^2(x - 3)$

D $y = (x + 1)(x - 3)^2$

E $y = (x + 1)^2(x - 3)^2$

14. The simultaneous equations $mx + 2y = 8$ and $3x + 4y = 10$ have no solution for m equal to

A $\frac{3}{2}$

B $\frac{-2}{3}$

C $\frac{3}{4}$

D 2

E $\frac{1}{2}$

15. The simultaneous equations $(m - 3)x + 8y = 10$ and $2x + (m + 3)y = 11$ have a unique solution for

A $m \in R \setminus \{0\}$

B $m \in R \setminus \{-3,3\}$

C $m \in R \setminus \{-5, 5\}$

D $m \in R \setminus [-5, 5]$

E $m \in R$

16. The matrix which describes the composition of mappings

- dilation of factor 3 from the x-axis
- reflection in the line $y = x$
- reflection in the x-axis

is

A $\begin{bmatrix} 3 & 0 \\ -1 & 3 \end{bmatrix}$

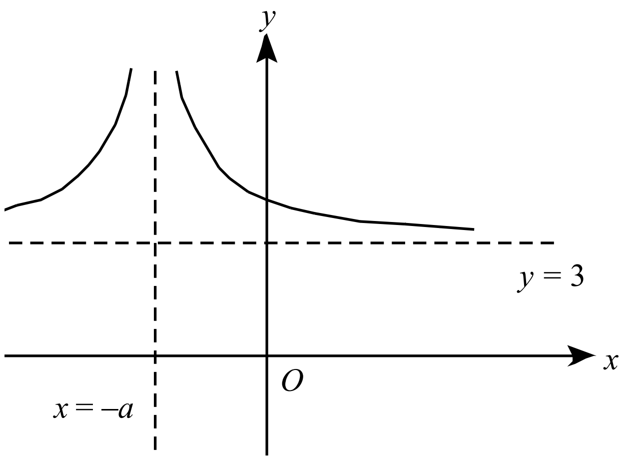
B $\begin{bmatrix} 0 & 0 \\ -1 & 3 \end{bmatrix}$

C $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 0 & 0 \\ -3 & 1 \end{bmatrix}$

E $\begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$

17. The equation of the curve shown below is



A $y = \left(\dfrac{1}{x+3}\right)^2 + a$, where $a > 0$

B $y = \left(\dfrac{1}{x-a}\right)^2 + 3$, where $a > 0$

C $y = \left(\dfrac{1}{x-3}\right)^2 + a$, where $a > 0$

D $y = \left(\dfrac{1}{x+a}\right)^2 + 3$, where $a > 0$

E $y = \left(\dfrac{a}{x-3}\right)^2 + a$, where $a > 0$

18. If $f(x) = 2x^2 - 2$ then $f(x - 1)$ is equal to

A 0

B 2

C $2x^2 - 4$

D $2x^2 - 4x$

E $2x^2 - 4x + 4$

19. The maximal domain of the function f with rule $f(x) = \dfrac{2}{x-1} + 3$ is

A $R \setminus \{3\}$

B $R \setminus \{-1\}$

C $(-\infty, 1) \cap (1, \infty)$

D $R \setminus \{1\}$

E $(-\infty, 1] \cup (2, \infty)$

20. The range of the function with rule $f: [-1, 5] \to R,\ f(x) = (x - 3)^2$ is given by the interval

A $[4, 16]$

B $[0, 4]$

C $[0, 16]$

D $[4, \infty)$

E $[0, \infty)$

End of Section B

Section C   Short – Answer Questions   (5x7 = 35 marks)

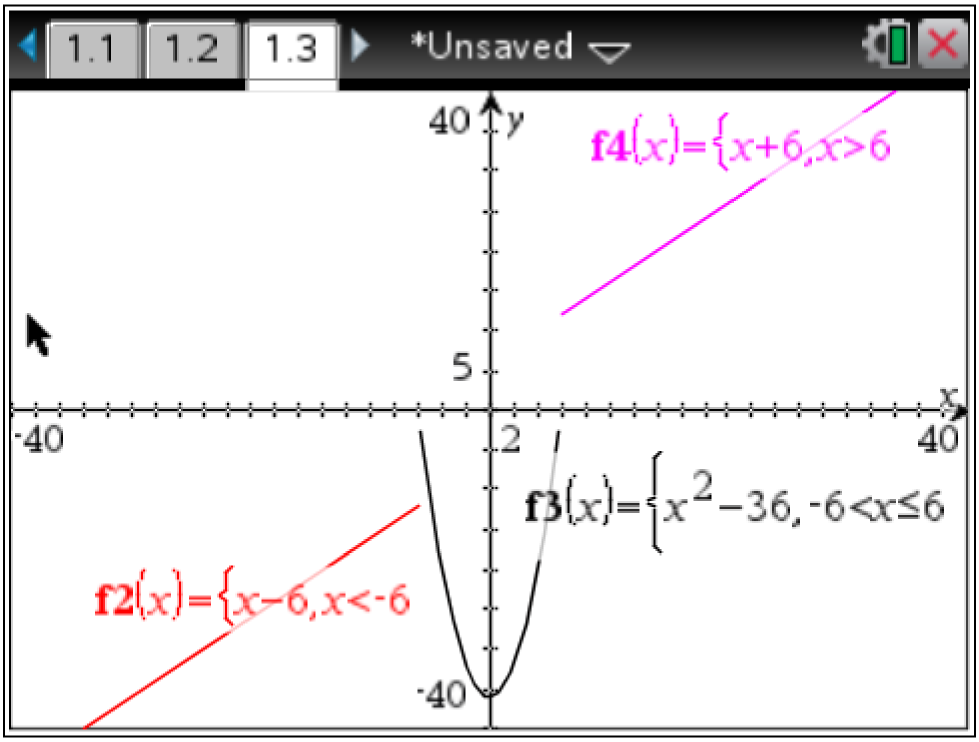
Instructions

- Answer each question in the space provided.
- Please provide appropriate workings and use **exact answers** when required.
- Unless otherwise stated, all answers should be given correct to 2 decimal places.

Question 1

a Sketch the graph of the following function:

$$f(x) = \begin{cases} x-6 & x<-6 \\ x^2-36 & -6<x\leq 6 \\ x+6 & x>6 \end{cases}$$



5 marks

b State the range of $f(x)$. **2 mark**

$y \in (-\infty,-12)\cup(-12,0]\cup(12,\infty]$

Total Question 1 = 7 marks

Question 2 **7 marks**

A transformation $T: R^2 \rightarrow R^2$ is defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

a. Find $T = \begin{bmatrix} x' \\ y' \end{bmatrix}$ *where* $T = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

3 marks

$$T = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2x \\ 4y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2x+1 \\ 4y-3 \end{bmatrix}$$

b. Show that the curve with equation $y = \dfrac{1}{x}$ is transformed according to T to produce the image equation $y = \dfrac{a}{x+b} + c$.

3 marks

$$x' = 2x+1 \Rightarrow x = \frac{x'-1}{2} \quad \text{and} \quad y' = 4y-3 \Rightarrow y = \frac{y'+3}{4}$$

$$y = \frac{1}{x} \qquad \Rightarrow y = \frac{y'+3}{4} = \frac{1}{\frac{x'-1}{2}} = \frac{2}{x'-1}$$

$$\Rightarrow y' = \frac{8}{x'-1} - 3$$

c. Find the values of a, b and c. **1 mark**

By comparison

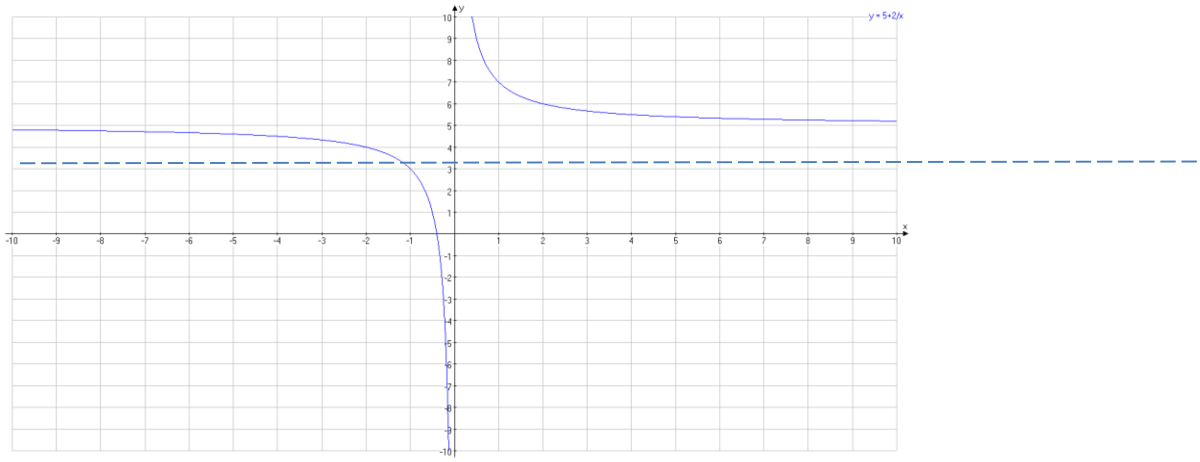
Therefore; $a = 8$, $b = -1$ *and* $c = -3$

Total Question 2 = 7 marks

Question 3 **7 marks**

a. Sketch the graph of $f(x)= 5 + \dfrac{2}{x}$ showing all intercepts and asymptotes.

3 marks



Asymptotes are: *Vertical* $x=0$ *and* *Horizontal* $y=5$

x-intercept: $0=5+\dfrac{2}{x} \Rightarrow \dfrac{2}{x}=-5$ or $x=-\dfrac{2}{5}=-0.4$

b. Write down the rule for the inverse function of *f(x)*.

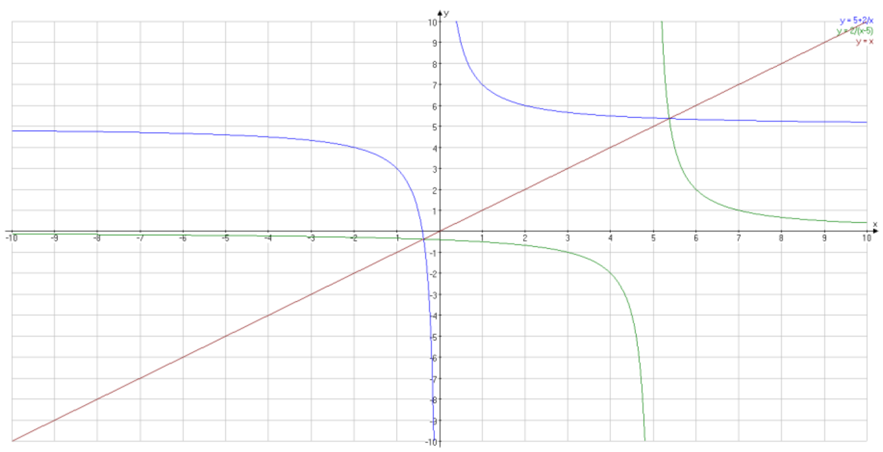
2 marks

$x=5+\dfrac{2}{y} \Rightarrow \dfrac{2}{y}=x-5$ therefore, $y=\dfrac{2}{x-5}$

$f^{-1}(x)=\dfrac{2}{x-5} \quad ,x\in R\backslash\{5\}$

c. On the same set of axes above, sketch the inverse function of *f(x)*.

2 marks



Total Question 3 = 7 marks

Question 4 **7 marks**

A quartic polynomial $y = f(x)$ function touches the x-axis at the point (2, 0). It has two other x-intercepts at (-5, 0) and (4, 0). It has a y-intercept at (0, 8). Find the equation of this polynomial.

Let $P(x) = a(x+5)(x-4)(x-2)^2$ 3marks

At (0,8)

Thus, $8 = a(0+5)(0-4)(0-2)^2 = -80a \quad \therefore a = \frac{8}{-80} = -.1$ 2 marks

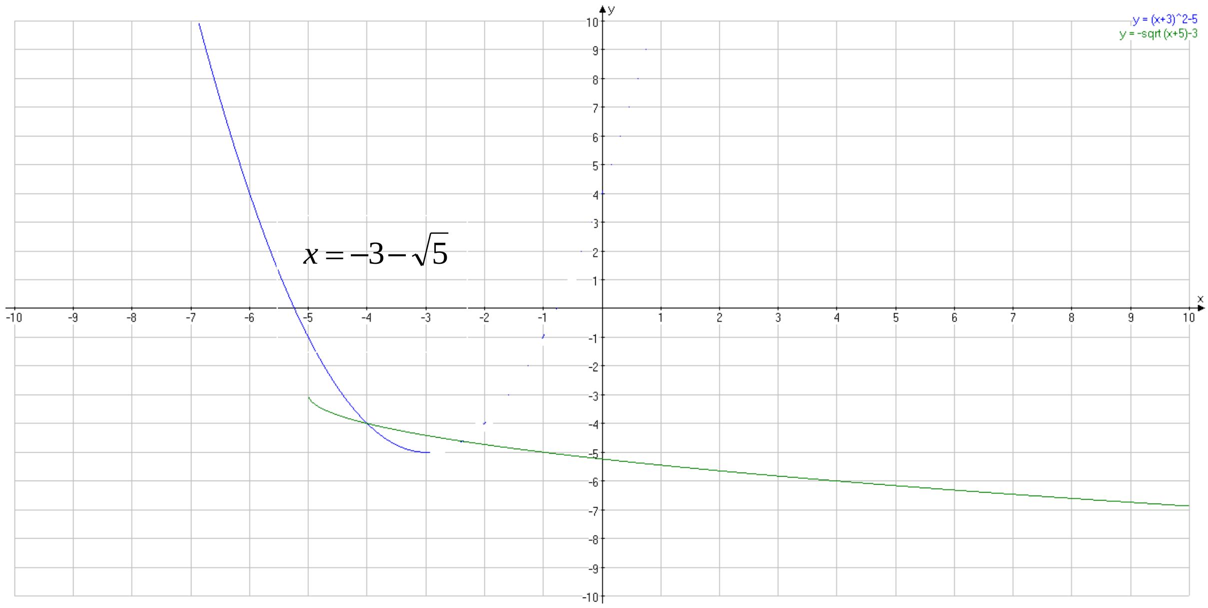
$\therefore P(x) == 0.1(x+5)(x-4)(x-2)^2$ 2 marks

Total Question 4 = 7 marks

Question 5 **7 marks**

The equation of the path of $f(x) = (x+3)^2 - 5,\ x \leq -3$

a. Sketch the graph of $f(x)$ on the set of axes below. Give the **exact** co-ordinates of the x–intercept.



For x–intercept $y = 0$ $(x+3)^2 - 5 = 0$

$(x+3)^2 = 5 \Rightarrow x = -3 \pm \sqrt{5}$ consider only the negative value so, $x = -3 - \sqrt{5}$

2 marks

b. Find the **rule** and state the **domain** and **range** of the path of $f^{-1}(x)$

$x = (y+3)^2 - 5$

$x + 5 = (y+3)^2$ Therefore, $y = -\sqrt{x+5} - 3$

The Domain is $x \geq -5$

The Range is $y \leq -3$

3 marks

c. On the same set of axes above, sketch the graph of $f^{-1}(x)$.

2 marks

Total Question 5 = 7 marks

End of Section C

Section D: Analysis Questions (2x10 = 20 marks)

Question 1 **10 marks**

A piece of wire 12 cm long is to be cut into two pieces. One piece will make a circular ring and the other a square pendant.

Let x be the length of the piece to be made into a ring.

that is (Ring length = x).

a. Derive an expression in terms of x to denote the total area of the two shapes.

5 marks

Circumference of the circle = x= $2\pi r$ so $r = \frac{x}{2\pi}$ so, $A = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$

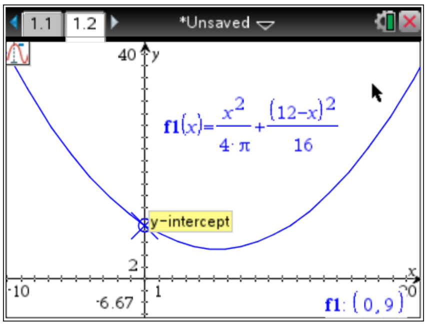
Perimeter of the square= *12-x* *so, side of the square b is* $b = \frac{12-x}{4}$, Area = $\left(\frac{12-x}{4}\right)^2$

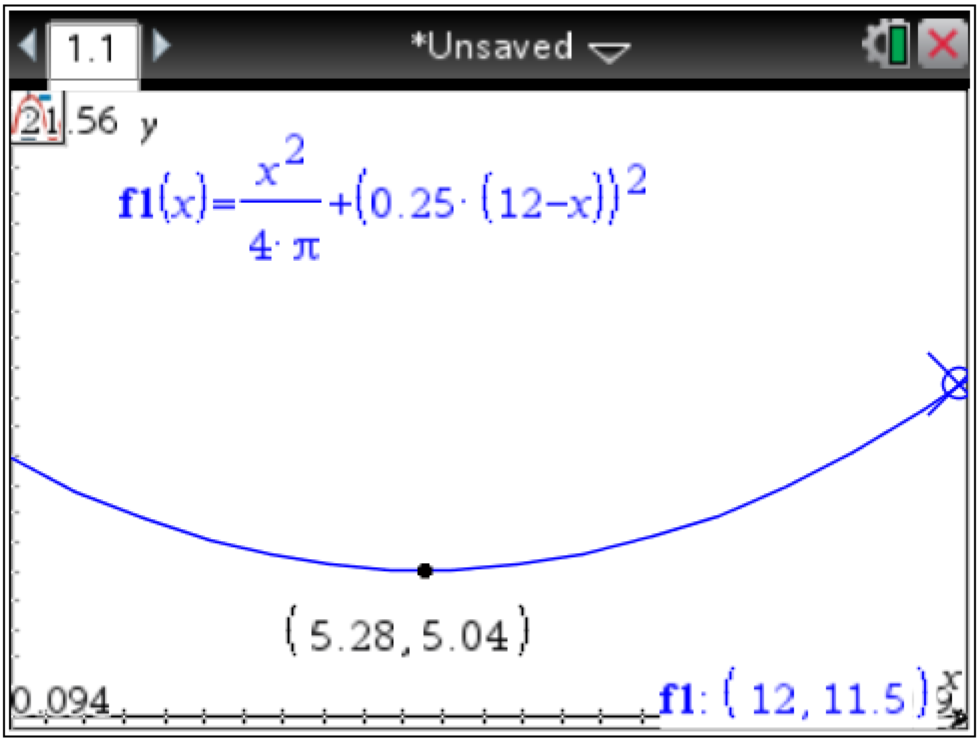
The total area of the two shapes = $A_{Total} = \frac{x^2}{4\pi} + \left(\frac{12-x}{4}\right)^2$

b. What is the domain of x? 1 mark

$0 < x < 12$

c. By sketching a graph for relationship between x and the area and hence find the range of the total area. 4 marks





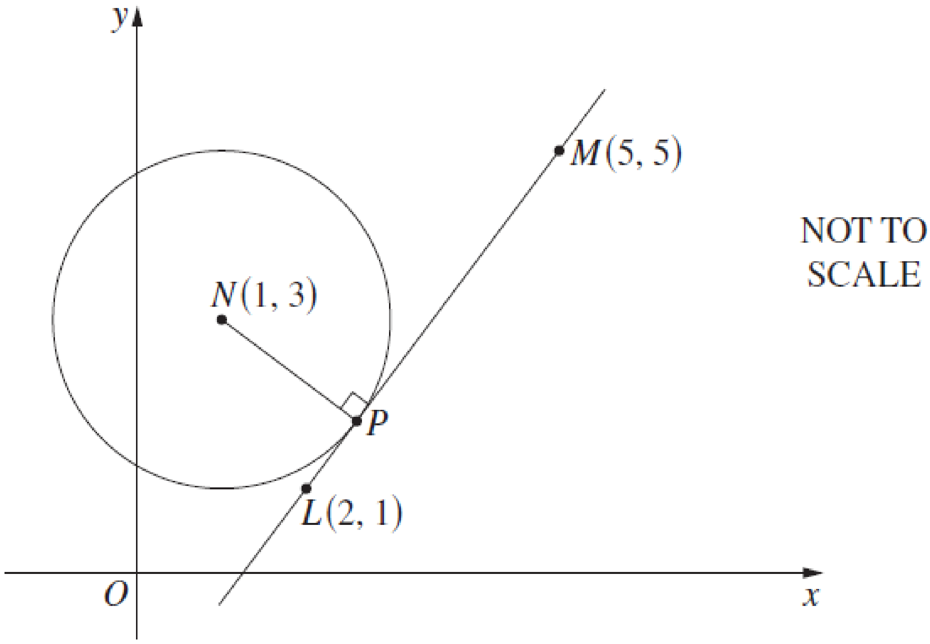
The Graph shouldn't show a negative x

Range is : $[5.28, 11.46)$

As *0<x<12* and $A_{Total} = \frac{12^2}{4\pi} + \left(\frac{12-12}{4}\right)^2 = 11.46$

Total Question 1= 10 marks

Question 2 **10 marks**



The circle in the diagram above has centre *N*. The line *LM* is tangent to the circle at *P*.

(i) Find the equation of *LM* in the form $ay + bx + c = 0$. **2 Marks**

$m = \frac{5-1}{5-2} = \frac{4}{3}$

$5 = \frac{4}{3} \times 5 + c \Rightarrow c = 5 - \frac{20}{3} = -\frac{5}{3}$

$\Rightarrow y = \frac{4}{3}x - \frac{5}{3}$ $\therefore 3y - 4x + 5 = 0$

(ii) Find the equation of *NP* in the form $ay + bx + c = 0$. **2 Marks**

$m_{NP} = \dfrac{-1}{\frac{4}{3}} = -\dfrac{3}{4}$

$3 = -\dfrac{3}{4} \times 1 + c \Rightarrow c = 3 + \dfrac{3}{4} = \dfrac{15}{4}$

$y = -\dfrac{3}{4}x + \dfrac{15}{4}$

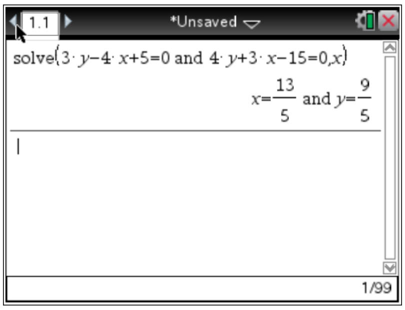
$\therefore 4y + 3x - 15 = 0$

(iii) Find the coordinate of point P **2 Marks**

$\therefore 3y - 4x + 5 = 0$ ....(1) or $\Rightarrow y = \dfrac{4}{3}x - \dfrac{5}{3}$

$\therefore 4y + 3x - 15 = 0$ ....(2) or $y = -\dfrac{3}{4}x + \dfrac{15}{4}$

Solve the above equations simultaneously gives the point of intersection P



$\dfrac{4}{3}x - \dfrac{5}{3} = -\dfrac{3}{4}x + \dfrac{15}{4}$

$\dfrac{4}{3}x + \dfrac{3}{4}x = \dfrac{5}{3} + \dfrac{15}{4}$

$\times 12 \Rightarrow 16x + 9x = 20 + 45 \quad 25x = 65 \therefore x = \dfrac{65}{25} = \dfrac{13}{5}$

And thus $y = \dfrac{4}{3}\left(\dfrac{13}{5}\right) - \dfrac{5}{3} = \dfrac{52}{15} - \dfrac{25}{15} = \dfrac{27}{15} = \dfrac{9}{5}$

So P is $\left(\dfrac{13}{5}, \dfrac{9}{5}\right)$

(iv) Find the distance *NP*. **2 Marks**

Distance NP= $\sqrt{\left(3-\frac{9}{5}\right)^2+\left(1-\frac{13}{5}\right)^2}$

= $\sqrt{\left(\frac{6}{5}\right)^2+\left(\frac{-8}{5}\right)^2}=\sqrt{\frac{36}{25}+\frac{64}{25}}=\sqrt{\frac{100}{25}}=\frac{10}{5}=2$ units

(v) Find the equation of the circle. **2 Marks**

$(x-1)^2+(y-3)^2=2^2$

That is $(x-1)^2+(y-3)^2=4$

Total Q2 =10 marks

End of Section D

End of Semester 1 Examination-2013
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MATHEMATICAL METHODS CAS UNIT 1

Multiple choice Answer Sheet   Name:

Instructions for Section B

Answer all questions in pencil on the answer sheet provided for multiple-choice questions. Choose the response that is correct for the question.

A correct answer scores 1, an incorrect answer scores 0. Marks will not be deducted for incorrect answer.

No marks will be given if more than one answer is completed for any question.

Question	A	B	C	D	E
Q1	○	○	○	○	●
Q2	○	○	○	○	●
Q3	○	●	○	○	○
Q4	○	●	○	○	○
Q5	○	○	○	○	●
Q6	○	●	○	○	○
Q7	○	○	○	●	○
Q8	○	○	○	○	●
Q9	●	○	○	○	○
Q10	○	○	○	○	●
Q11	○	○	●	○	○
Q12	○	○	●	○	○
Q13	○	○	●	○	○
Q14	●	○	○	○	○
Q15	○	○	●	○	○
Q16	○	○	○	○	●
Q17	○	○	○	●	○
Q18	○	○	○	●	○
Q19	○	○	○	●	○
Q20	○	○	●	○	○