

Year 9 Higher Mathematics

Curriculum Overview

Autumn 1

Fractions and Decimals

Statistical Measures

Area of 2D Shapes

Spring 1

Linear Graphs

Representing Data

Summer 1

Transformations

Solving Equations

Scatter Graphs

Autumn 2

3D Shapes

Algebraic Expressions

Ratio and Proportion

Spring 2

Angles

Collecting Data

Summer 2

Constructions

Pythagoras' Theorem



Contents

| | |
|---|----|
| Fractions and Decimals..... | 3 |
| Statistical Measures..... | 7 |
| Area of 2D Shapes..... | 10 |
| Drawing 3D Shapes | 13 |
| Algebraic Expressions | 16 |
| Ratio and Proportion..... | 20 |
| Equations of Straight-Line Graphs | 23 |
| Plotting and Interpreting Statistical Diagrams..... | 26 |
| Angle Geometry..... | 29 |
| Collecting Data | 32 |
| Performing and Describing Transformations | 35 |
| Solving Linear Equations | 38 |
| Scatter Graphs..... | 42 |
| Constructions and Loci..... | 45 |
| Pythagoras' Theorem | 48 |



Fractions and Decimals

Students learn how to solve problems with long multiplication and division using written methods. They use this knowledge to add, subtract, multiply and divide with fractions and mixed numbers.

Prerequisite Knowledge

- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- interpret fractions as operators
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- order positive and negative decimals

Key Concepts

- All rational numbers are written using exact proper or improper fractions.
- When adding or subtracting fractions the denominators need to be equal.
- Dividing fractions is equivalent to multiplying by a reciprocal.
- When calculating with decimal numbers encourage students to estimate the solution as means to check their working.
- Students may need to recap multiplying and dividing by powers of ten when calculating the product of decimal numbers.
- Use equivalent fractions when performing long division. Simplifying the fractions help to break down the calculation.

Success Criteria

- apply the four operations, including formal written methods, simple fractions (proper and improper)
- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- apply the four operations, including formal written methods, to mixed numbers both positive and negative;
- calculate exactly with fractions

Common Misconceptions

- A fraction with a smaller denominator has a lesser value.
- Fractions such as $\frac{3}{5}$ can incorrectly assumed to have a decimal equivalence of 3.5.
- Students incorrectly consider multiplications to always increase a number and divisions to decrease.
- Students fail to spot incorrect calculations due to not estimating solutions.



Lessons

Using a Calculator

Using Calculators
Learning Objective: Use a calculator efficiently to evaluate numerical problems.
Use the buttons on the left to calculate:

| | | |
|---------------------------|---------------------|--|
| x^2 | Brackets | a) $\sqrt{150} \times 2^3$ |
| \sqrt{x} | x Squared | b) $\sqrt{3^2 + 4^2}$ |
| x^y | x to the power of y | c) $\frac{2 \times 4}{5+3}$ |
| () | Square root of x | d) $\left(\frac{6.4}{5.2 \times 6.7}\right)^2$ |
| $\frac{\square}{\square}$ | Fraction | e) $\sqrt{\frac{17}{4}}$ |

Long Multiplication

Long Multiplication
Learning Objective: Calculate the product of two or more integers.
Find the product of:

a) 32×42 b) 48×56 c) 157×35

Long Division

Division
Learning Objective: Use equivalent fractions to perform long division.

A stadium fence has a perimeter of 17287 metres. A fence panel is 12 metres long and costs £25.

How much will it cost for all the stadium panels?

Adding and subtracting fractions with different denominators

Fractions with Different Denominators
Learning Objective: Add and subtract fractions with different denominators.
Use equivalent fractions to calculate the following:

a) $\frac{1}{2} + \frac{1}{4}$
 b) $\frac{1}{3} + \frac{1}{6}$
 c) $\frac{1}{2} + \frac{1}{6}$
 d) $\frac{1}{3} + \frac{1}{12}$

Addition and Subtraction with Fractions and Mixed Numbers

Addition and Subtraction with Fractions and Mixed Numbers
Learning Objective: Calculate exact addition and subtractions with fractions and mixed numbers.
Work out the following. Simplify your answers.

a) $\frac{1}{2} + \frac{1}{2}$ b) $1\frac{1}{4} - \frac{1}{4}$ c) $\frac{7}{4} - 1\frac{1}{5}$

d) $1\frac{1}{6} - 1\frac{1}{5}$ e) $\frac{3}{2} + \frac{5}{6} + 1\frac{1}{6}$ f) $1\frac{1}{10} + 2\frac{2}{5} - \frac{7}{3}$

g) The distance from Kintown to Inkwille is $3\frac{4}{5}$ miles. The distance from Marshall to Inkwille is $2\frac{1}{2}$ miles. Work out the distance from Kintown to Marshall.

Adding and Subtracting with Fractions and Mixed Numbers
Calculate $1\frac{3}{4} + 1\frac{2}{5}$
 Convert to top heavy fractions.
 $1\frac{3}{4} = \frac{7}{4}$ $1\frac{2}{5} = \frac{7}{5}$
 $\frac{7}{4} + \frac{7}{5}$
 Make the denominators common.
 $\frac{7}{4} + \frac{7}{5} \times \frac{5}{5} = \frac{35}{20} + \frac{28}{20} \times 4$
 Find the sum and simplify if needed.
 $\frac{21+20}{28} = \frac{41}{28}$

Multiplying Fractions and Mixed Numbers

Multiplying Fractions
Learning Objective: Calculate the product of two or more fractions.
Calculate the product of these fractions by cross simplifying:

a) $\frac{1}{2} \times \frac{1}{2}$
 b) $\frac{2}{3} \times \frac{1}{2}$
 c) $\frac{1}{3} \times \frac{1}{4}$
 d) $\frac{3}{6} \times \frac{1}{2}$

Dividing Fractions and Mixed Numbers

Dividing Fractions
Learning Objective: Divide one fraction by another using reciprocals.
Calculate the following divisions:

a) $\frac{2}{3} \div \frac{3}{4} =$ h) $\frac{4}{5} \div 1\frac{2}{3} =$
 b) $\frac{3}{10} \div \frac{2}{5} =$ i) $3\frac{1}{2} \div \frac{1}{3} =$
 c) $\frac{4}{5} \div \frac{2}{10} =$ j) $2\frac{2}{5} \div 1\frac{3}{4} =$
 d) $\frac{1}{8} \div \frac{7}{16} =$
 e) $1\frac{3}{7} \div \frac{2}{5} =$

Recurring Decimals

Recurring Decimals
Learning Objective: Convert recurring decimals to a fraction by setting up an equation using place value.
Use the place value table to convert these recurring decimals to a fraction.

a) 0.3
 b) 0.2
 c) 0.14
 d) 0.26
 e) 0.41



Problem Solving and Revision Lessons

Adding and Subtracting Fractions

Adding and Subtracting with Fractions

Calculate the following fractions.
Leave your answer as a simplified fraction or mixed number.

a) $\frac{1}{2} + \frac{1}{3}$
 b) $\frac{2}{3} + \frac{1}{4}$
 c) $\frac{1}{2} - \frac{1}{4}$
 d) $\frac{3}{4} - \frac{1}{2}$

e) $\frac{1}{2} + \frac{1}{3}$
 f) $\frac{2}{3} - \frac{1}{4}$
 g) $\frac{1}{2} + \frac{1}{4}$
 h) $\frac{3}{4} - \frac{1}{2}$

Calculate the following.
Leave your answer as a simplified fraction or mixed number.

i) $2\frac{1}{2} + \frac{1}{3}$
 j) $2\frac{1}{2} - \frac{1}{4}$
 k) $2\frac{1}{2} + \frac{1}{3}$

l) $3\frac{1}{2} - \frac{1}{4}$
 m) $3\frac{1}{2} + 2\frac{1}{3}$
 n) $4\frac{1}{2} - 2\frac{1}{10}$

nr-mathematics.com

Multiplication and Division of Fractions

Multiplying and Dividing with Fractions

Calculate the following fractions.
Leave your answer as a simplified fraction or mixed number.

a) $10 \times \frac{1}{2}$
 b) $\frac{2}{3} \times 40$
 c) $1\frac{1}{2} \div 4$
 d) $4\frac{1}{2} \div 6$

e) $\frac{1}{2} \times \frac{1}{3}$
 f) $\frac{2}{3} \times \frac{11}{10}$
 g) $\frac{1}{5} \div \frac{1}{3}$
 h) $2\frac{1}{2} \div \frac{1}{3}$

Calculate the following.
Leave your answer as a simplified fraction or mixed number.

i) $2\frac{1}{2} \times 1\frac{1}{3}$
 j) $1\frac{1}{2} \times 1\frac{1}{3}$
 k) $1\frac{1}{2} \div 1\frac{1}{3}$

l) $3\frac{1}{2} \div 1\frac{1}{3}$
 m) $2\frac{2}{3} \div 1\frac{1}{3}$
 n) $4\frac{1}{2} \div 2\frac{1}{3}$

nr-mathematics.com

Product of Decimal Numbers

Multiplying with Decimals

Evaluate these products.

a) 5.27×6
 b) 3.25×7
 c) 8.58×6

d) 1.07×9
 e) 6.5×3.1
 f) 3.8×4.9

g) 0.4×8.9
 h) 0.17×3.8
 i) 2.15×3.8

j) 41.9×5.4
 k) 2.84×0.15
 l) 21.45×0.7

nr-mathematics.com

Division with Decimal Numbers

Dividing with Decimals

Calculate

a) $6.2 \div 0.2$
 b) $1.6 \div 0.4$
 c) $8.15 \div 0.3$

d) $6.3 \div 0.99$
 e) $0.9 \div 0.83$
 f) $1.96 \div 0.06$

g) $3.54 \div 0.4$
 h) $15.2 \div 0.4$
 i) $10.8 \div 0.12$

Calculate

j) $6.95 \div 30$
 k) $5.78 \div 480$
 l) $7.15 \div 90$

nr-mathematics.com

Problem Solving – Fractions

Problem Solving - Fractions

Which of the following has a value that is closest to $\frac{5}{7}$?

a) $\frac{8}{9} - \frac{1}{3} = \frac{5}{9}$
 b) $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

c) $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$
 d) $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

nr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Statistical Measures

Students learn how to calculate the area and circumference of circles. Later, as learning progresses, they find the composite area and perimeter of circular shapes, including sectors.

Prerequisite Knowledge

- Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.
- Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

Key Concepts

- A frequency table is used when the sample size increases beyond simple calculations being possible from a list.
- The median average of a class width is used as the mid-point when calculating the mean from grouped data.

Success Criteria

- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
- Appropriate graphical representation involving discrete, continuous and grouped data
- Appropriate measures of central tendency (median, mean, mode and modal class) and spread
- Construct and interpret stem and leaf diagrams
- Apply statistics to describe a population

Common Misconceptions

- Students often find it difficult to calculate the median average from data presented in a frequency table.
- When sorting continuous data into a grouped data table students often struggle to fully understand the inequality notation.



Lessons

Using the Range

Using the Range

Learning Objective: Use the range to determine the consistency of data.
Manchester United and Arsenal each played four games. Here is the number of goals they scored in each match:

Manchester United: 4, 0, 6, 2
Arsenal: 4, 3, 3, 2

Calculate the mean average and range to determine who is most likely to score in their next game.

Two competitors in a competition list their marks as follows:

| | | | | | | | | | | |
|-------|----|----|----|----|----|----|----|----|----|----|
| Becki | 21 | 14 | 15 | 15 | 17 | 18 | 20 | 15 | 20 | 20 |
| Sally | 18 | 13 | 13 | 23 | 20 | 21 | 22 | 16 | 21 | 14 |

Who is the better competitor? Why?

Stem and Leaf Diagrams

Stem and Leaf Diagrams

Learning Objective: Draw a Stem and Leaf diagram and interpret the data using averages and the range.
a) Simon records the heights of the plants in his garden to the nearest cm.

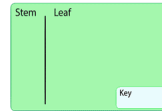
| | | | | | |
|----|----|----|----|----|----|
| 36 | 33 | 26 | 13 | 38 | 40 |
| 33 | 47 | 12 | 10 | 26 | 28 |

Use the data to produce an ordered stem and leaf diagram.

b) Use the following sets of data to make ordered stem and leaf diagrams.

i) 158 | 149 | 109 | 127 | 101 | 106 | 136 | 126 | 151 | 102 | 142 | 133 | 132 | 102 | 126

ii) 3.7 | 5.3 | 2.7 | 3.2 | 5.7 | 3 | 4.8 | 3 | 4.7 | 4.5 | 6 | 5.4 | 4.7 | 3.3 | 3.3



Mean from a Frequency Table

Mean Average from a Frequency Table

Learning Objective: Calculate the mean average from data represented in a frequency table.
Thomas wanted to know the difference between the average shoe size for boys and girls. He recorded the results of 53 samples.

$$\text{Mean Average} = \frac{\text{Sum of the Data}}{\text{Sample Size}}$$

Do boys or girls have a larger shoe size? If so, by how much?

| | | | | | |
|----------------|---|---|---|---|---|
| Boys Shoe Size | 5 | 6 | 7 | 8 | 9 |
| Frequency | 1 | 2 | 4 | 3 | 2 |

| | | | | | |
|-----------------|---|---|----|----|---|
| Girls Shoe Size | 4 | 5 | 6 | 7 | 8 |
| Frequency | 8 | 9 | 12 | 10 | 1 |

Mean from a Grouped Data

Averages from Grouped Data

Learning Objective: Estimate the Mean Average from Grouped Data
Calculate an estimate for the Mean Average from the grouped data table.

$$\text{Mean Average} = \frac{\text{Sum of the Data}}{\text{Sample Size}}$$

| Weight, w, Kg | Frequency | | |
|---------------|-----------|--|--|
| 40 < w ≤ 50 | 2 | | |
| 50 < w ≤ 60 | 15 | | |
| 60 < w ≤ 70 | 18 | | |
| 70 < w ≤ 80 | 10 | | |
| 80 < w ≤ 90 | 2 | | |

Problem Solving and Revision Lessons

Problem Solving with Averages

Problem Solving with Averages

The table shows information about how far Jonathan ran each day over the course of a week.

| Day | Distance (km) |
|-----------|---------------|
| Monday | 5.9 |
| Tuesday | 5.2 |
| Wednesday | |
| Thursday | 6.5 |
| Friday | 7.3 |
| Saturday | 7.6 |
| Sunday | 6.4 |

The value for Wednesday is missing.

The mean distance for the seven days is 6.4 km.

Work out the distance for Wednesday.

ANS

mr-mathematics.com

Estimating the Mean from Grouped Data

Data

Averages from Grouped Data

From each frequency table find
i) Mean Estimate, ii) Modal Class, iii) Modal Class

Train Travel Times

| Time, T (hours) | Frequency |
|-----------------|-----------|
| 0 < T ≤ 1 | 2 |
| 1 < T ≤ 2 | 2 |
| 2 < T ≤ 3 | 4 |
| 3 < T ≤ 4 | 5 |
| 4 < T ≤ 5 | 1 |

Temperatures in UK Cities

| Temperature, T (°C) | Frequency |
|---------------------|-----------|
| 0 < T ≤ 5 | 2 |
| 5 < T ≤ 10 | 6 |
| 10 < T ≤ 15 | 8 |
| 15 < T ≤ 20 | 5 |
| 20 < T ≤ 25 | 2 |

ANS

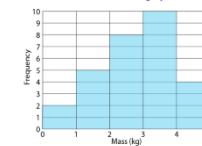
ANS

mr-mathematics.com

Problem Solving – Grouped Data

Problem Solving - Grouped Data

The frequency diagram shows the weights of cats who visited a vet's surgery over the course of a week.



a) Estimate the mean average weight of cat.

b) Calculate the percentage of cats that weighed more than 3 kg.

ANS

mr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Area of 2D Shapes

Students learn how to find the area of various 2D shapes including parallelograms, trapezia, compound shapes and circles. Throughout the topic links are made to algebraic reasoning and estimation.

Prerequisite Knowledge

- Know and apply formulae to calculate the area of rectangles
- Calculate the perimeters of 2D shapes, including composite shapes;
- Compare and order lengths, mass, volume / capacity and record the results using $>$, $<$ and $=$
- Measure, compare, add and subtract lengths (m/cm/mm); mass(kg/g); volume/capacity (l/ml)
- Identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

Key Concepts

- Demonstrate a triangle as being half a rectangle so students know to use the perpendicular height in their calculation. Demonstrate a parallelogram as having an equal area to a rectangle.
- To calculate the area of composite rectilinear shapes have students break them up in different ways.
- A sector is a fraction of 360° of the entire circle.

Success Criteria

- Know and apply formulae to calculate: area of triangles, parallelograms and trapezia;
- Know the formulae: circumference of a circle $= 2\pi r = \pi d$, area of a circle $= \pi r^2$; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;
- Calculate arc lengths, angles and areas of sectors of circles

Common Misconceptions

- Students often confuse area and perimeter.
- When calculating the area of a triangle or parallelogram students tend to use the slanted height rather than the correct perpendicular height.
- Arc length and area of a sector are often rounded incorrectly. Encourage students to evaluate as a multiple of pi and calculate the decimal at the end.



Lessons

Area of Compound Shapes

Area of Compound Shapes
Learning Objective: Calculate the area and perimeter of compound shapes made with rectangles.

Determine the shaded areas:

a) b) c)

d) e)

Parallelograms and Trapezia

Parallelograms & Trapezia
Learning Objective: Calculate the area of parallelograms and trapezia.

Calculate the area of these shapes:

a) b)

Area of Parallelograms & Trapezia
 Parallelogram: Area = Base \times Perpendicular Height, Area = $b \times h$
 Trapezium: Area = $1/2(a + b)h$

Circumference of Circles

Circumference of a Circle
Learning Objective: Investigate the circumference of a circle and discover its formula. **$C = \pi D$**

a) Circumference =
 b) Circumference =
 c) Circumference =

d) Diameter =
 e) Diameter =
 f) Radius =

Mr Mathematics
www.mr-mathematics.com

Circumference Problems

Circumference of a Circle
Learning Objective: Solve problems involving the circumference of circular shapes. **$C = \pi D$**

Calculate the circumference or perimeter of each shape.

a) b) c)

Area of a Circle

Area of a Circle
Learning Objective: To investigate the area of a circle and discover its formula. Cut a circle into quarters and arrange the pieces to resemble a quadrilateral.

Smooth out the curves ...
 Continue to smooth out the curves to create a ...

Mr Mathematics
www.mr-mathematics.com

Area of Circle Problems

Problems involving the area of a circle
Learning Objective: Calculate the area of circular shapes. **$Area = \pi r^2$**

Calculate the area of the following shapes to 3 significant figures.

a) Area =
 b) Area =
 c) Area =

d) Area =
 e) Radius =
 f) Diameter =

Problem Solving and Revision Lessons

Area of Triangular Shapes

Area of Triangular Shapes

Find the area of the following triangles.

a) b) c)

Calculate the base of the following triangles.

d) Base =
 e) Base =

mr-mathematics.com

Area of 2D Shapes

Area of 2D Shapes

Calculate the area of the following shapes.

A rectangle measures 5 m by 4 m. Part of the rectangle is covered by a square of length 2 m.

Show that 20% of the square is covered by the square.

mr-mathematics.com

Problems with Circles

Problems with Circles

The diagram shows an equilateral triangle and a circle. The circle has an area of $16\pi \text{ cm}^2$. Calculate the shaded area.

The diagram shows an equilateral triangle and a circle. What percentage of the diagram is shaded?

mr-mathematics.com

Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Drawing 3D Shapes

This topic teaches students how to draw them using isometric and elevation drawings. Learning progresses onto constructing nets of a range of solids.

Prerequisite Knowledge

- draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them
- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes

Key Concepts

- Students need to understand the geometrical difference between a prism and pyramid.
- Horizontal lines are not drawn on isometric paper.
- Students need to sketch a solid from elevation drawings and vice-a-versa

Success Criteria

- identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- construct and interpret plans and elevations of 3D shapes.

Common Misconceptions

- Students often get confused which elevation to draw and how to include hidden detail.
- Some students find it difficult to draw 3D shapes on isometric paper. Encourage the use of multilink cubes to aid drawings.



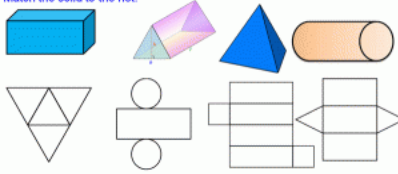
Lessons

Net Drawings

Nets of Solids

Learning Objective:

Create a net drawing of the a Cube, Cuboid, Triangular Prism and Tetrahedron.
Match the solid to the net.

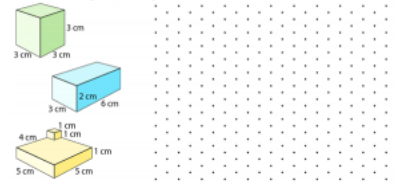


Isometric Drawings

Isometric Drawings

Learning Objective: Draw 3D shapes to scale using isometric paper.

Draw the following shapes using the isometric dots.

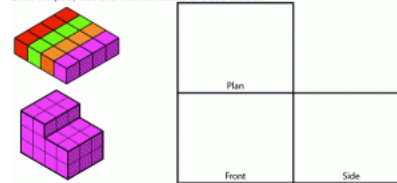


Elevation Drawings

Elevation Drawings of 3D Shapes

Learning Objective: Create 2D representations of 3D shapes using elevation drawings.

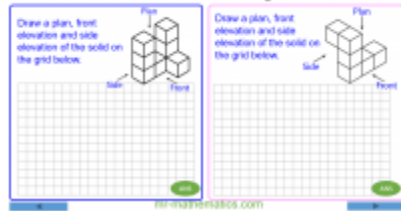
Draw the plan, side and front elevations of these cuboids.



Problem Solving and Revision Lessons

Isometric and Elevation Drawings of 3D Shapes

Isometric & Elevation Drawings



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Algebraic Expressions

Students learn how to expand and factorise algebraic linear and quadratic expressions. Learning progresses to expanding cubic and factorising quadratics in the form $ax^2 + bx + c$.

Prerequisite Knowledge

- ab in place of $a \times b$
- $3y$ in place of $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$ and a^2b in place of $a \times a \times b$
- a/b in place of $a \div b$
- Coefficients written as fractions rather than decimals

Key Concepts

- Students need to appreciate that writing with algebra applies the rules of arithmetic to unknown numbers which are represented as letters.
- It is important to define the difference between an expression, equation and formula.
- Linear (x), quadratic (x^2) and cube terms (x^3) cannot be collected together.
- Understanding quadratics in the general form $x^2 + bx + c$ helps to factorise and expand expressions.
- Developing mental methods to factorise quadratics is key to gaining confidence with quadratics equations later on.

Success Criteria

simplify and manipulate algebraic expressions by:

- Multiplying a single term over a bracket
- Taking out common factors
- Expanding products of two or more binomials
- Factorising quadratic expressions of the form $(a)x^2 + bx + c$, including the difference of two squares
- Simplifying expressions involving sums, products and powers including the laws of indices

Common Misconceptions

- When multiplying out brackets students incorrectly forget to multiply the second term especially with negative products. E.g., $2(x + 5) = 2x + 10$ and $-2(x + 5) = -2x - 10$
- When factorising expressions, a common misconception is to not fully factorise. E.g., $18x + 24y$ can be written as $9x + 12y$
- When expanding the product of two or more brackets students often incorrectly collect the like terms associated to the linear unknown



Lessons

Expanding Brackets

Expanding Brackets

Learning Objective: Multiply out a bracket and simplify the result. Multiply out these brackets.

- a) $3(x + 5)$
- b) $2(y - 6)$
- c) $5(4 - a)$
- d) $3(2x + 4)$
- e) $6(10 + 5g)$
- f) $5(x + 7) + 3(8 - x)$

Factorising Terms

Factorising Expressions

Learning Objective: Fully factorise an algebraic expression using the Highest Common Factor.

- a) Factorise $5x + 10$
- b) Factorise $4y + 12$
- c) Factorise $7y + 14t + 21h$
- d) Factorise $w^2 + w$
- e) Factorise $6h^2 + 4h$

Expanding Quadratics

Product of Two Brackets

Learning Objective: Multiply out a pair of brackets into the form $ax^2 + bx + c$. Multiply out the following expressions and simplify:

- a) $(x + 1)(x + 3)$
- b) $(y + 2)(y + 5)$
- c) $(l + 7)(l - 1)$
- d) $(z - 3)(z - 2)$
- e) $(c + 4)^2$
- e) $(2x + 4)(x - 3)$

Factorising Expressions with Powers

Factorising Complex Expressions

Learning Objective: Use index notation to factorise complex expressions and maintain equivalence.

Factorise the following expressions.

- a) $4^m + 4^m$
- b) $6w^2y - 8wy^2$
- c) $8u^2c^2 - 20u^2c$
- d) $21y^4h + 14yh - 49yh^2$
- e) $15v^4x^2 - 5v^2x^3 + 10v^2x^2$
- f) $3a^3b^3 + 7a^4b^4$

Match together the equivalent expressions

- $12xy + 8xy$
- $6xy + 4xy^2$
- $6xy^2 + 12xy^2$
- $8xy + 6xy^2$
- $4xy(3x + 2)$
- $2xy(4x + 3y)$
- $6xy^2(1 + 2x)$
- $2x^2y(3x + 2y)$

Factorising Complex Expressions

Factorise $w^2y + wy^2$

Take out the H.C.F. = wy

$wy \times w + wy \times y$

$wy(w + y)$

Factorise $18a^2b^3 - 27a^2bc$

Take out the H.C.F. = $9ab$

$9ab \times 2ab - 9ab \times 3c$

$9ab(2ab - 3c)$

Factorise $(a + b)^2 + 4a + 4b$

Take out the H.C.F. = $a + b$

$(a + b)(a + b) + (a + b) \times 4$

$(a + b)(a + b + 4)$

Factorising Quadratics

Factorising Quadratics

Learning Objective: Factorise quadratics in the form $x^2 + bx + c$ using mental methods. Factorise the following quadratics.

- a) $x^2 + 4x + 3$
- b) $x^2 + 7x + 12$
- c) $x^2 + 5x + 6$
- d) $x^2 - 7x + 10$
- e) $x^2 - 2x - 12$
- f) $x^2 - 9$

Factorising Quadratics $ax^2 + bx + c$

Factorising Quadratics

Learning Objective: Factorise a quadratic expression in the form $ax^2 + bx + c$ using mental methods. Factorise the following quadratics:

- a) $2x^2 + 13x + 20$
- b) $2x^2 + 18x + 28$
- c) $3x^2 + 14x + 15$
- d) $3x^2 + 16x + 16$
- e) $2x^2 - 18$
- f) $3x^2 + 17x - 6$

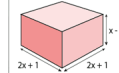
Expanding Cubic Expressions

Expanding Cubic Expressions

Learning Objective: Expand the product of two or more binomials. Expand and simplify:

- a) $(x + 1)(x + 2)(x + 3)$
- b) $(x + 4)(x - 1)(x + 2)$
- c) $(x - 4)^2(x + 3)$
- d) $3(x - 1)(x - 2)(x - 5)$

Write an expression for the volume of the cuboid.



Expanding cubic expressions

Expand $(y + 1)(y - 2)(y + 4)$

$(y + 1)(y - 2)(y + 4)$

$(y^2 + y - 2)(y + 4)$

$(y^2 + y - 2)(y + 4)$

$(y^3 - y^2 - 2y + 4y - 8)$

$y^3 - y^2 - 2y + 4y - 8$

$y^3 + 3y^2 - 6y - 8$

$y^3 + 3y^2 - 6y - 8$



Problem Solving and Revision Lessons

Expanding Brackets

Expanding Brackets

Expand and simplify

a) $4(x + 3) + 2(3x + 1)$ b) $5(x - 6) + 2(x + 3)$ c) $4(x + 1) + 6(x - 4)$

d) $5(w + 3) - 2(1 - 2w)$ e) $3(1 + x) - 2(3 - 4x)$ f) $x(a + b) - x(2a - 3b)$

Expand and simplify

g) $\frac{6x+18}{2} + 7(x + 4)$ h) $\frac{6x+3}{4} + 5x + 1$ i) $2(3x - 5) - \frac{6x+8}{2}$

j) $\frac{6x-12}{4} - 3(1 - 4x)$ k) $4(3 - 7a) - \frac{8a+15}{5}$ l) $\frac{21x-27}{3} - \frac{2(6-8x)}{4}$

mi-mathematics.com

Factorising Expressions

Factorising Expressions

Factorise these expressions

a) $12x + 20$ b) $2x + 10$ c) $3y + 15$

d) $8a - 4$ e) $14 + 6k$ f) $8u - 3$

g) $28 - 6x$ h) $18 - 30x$ i) $40w - 24$

Factorise these expressions

j) $x^2 - x$ k) $3r - r^2$ l) $8u^2 + u^3$

m) $4ab - 12b^2$ n) $3xy^2 + 6xy$ o) $20ab^2 + 12ba^2b$

mi-mathematics.com

Expanding Quadratics

Expanding Quadratics

$(2x + 1)(x - 3) + ax + 3 = 2x^2 + 4x - 5$

Work out the values of a and b

$(x + a)(x + b) = x^2 + px + q$

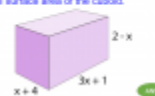
Work out the values of p and q in terms of a and b.

Show that $x(x^2 - y^2) = (x + y)(x - y)$

$x = 5.2$ and $y = 3.6$

Use your answer to part a) to work out $x^3 - y^3$

Write a simplified expression for the total surface area of the cuboid.



mi-mathematics.com

Factorising Quadratics

Factorising Quadratic Expressions

Factorise the following quadratics

a) $x^2 + 6x + 5$ b) $x^2 + 10x + 16$

c) $x^2 + 9x + 20$ d) $x^2 - 2x - 8$

e) $x^2 + 3x - 18$ f) $x^2 - 12x + 27$

g) $x^2 - 8x + 16$ h) $x^2 + 4x - 96$

Factorise the following quadratics

i) $x^2 - 16$ j) $x^2 - 81$ k) $x^2 - 100$

mi-mathematics.com

Expanding Cubics

Expanding Cubic Expressions

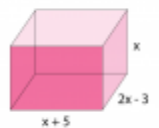
Expand and simplify

a) $x(x - 2)(x + 5)$ b) $(x^2 + 3x - 5)(x + 4)$

Expand and simplify the following

a) $(x + 2)^3$ b) $(2x + 3)^3$ c) $(4 - 3x)^3$

Show below is a cuboid



Form and simplify an expression for the volume of the cuboid.

mi-mathematics.com



Ratio and Proportion

Students learn how to simplify and use equivalent ratios to calculate proportionate amounts. They use this knowledge to share an amount using a ratio.

Prerequisite Knowledge

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Key Concepts

- It is important for students to visualise equivalent ratios by categorising objects and breaking them down into smaller groups.
- It is important to apply equivalent ratios when solving problems involving proportion. Including the use of the unitary method.
- To share amount given a ratio it is necessary to find the value of a single share.
- When two or more measurements increase at a linear rate they are in direct proportion. Inverse proportion is when one increases at the same rate the other decreases.

Success Criteria

- use ratio notation, including reduction to simplest form
- express a multiplicative relationship between two quantities as a ratio
- understand and use proportion as equality of ratios
- relate ratios to fractions
- express the division of a quantity into two parts as a ratio
- apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- understand and use proportion as equality of ratios

Common Misconceptions

- Ratios amounts are often confused with fractions involving the same digits. For instance $2 : 3$ is confused with $\frac{2}{3}$ or $1 : 2 = \frac{1}{2}$.
- When solving problems involving proportion students tend to struggle with forming a ratio. For instance, 3 apples cost 45p would form the ratio apples : cost.
- When writing ratios into the form $1 : n$ students incorrectly assume that n has to be an integer or greater than 1.



Lessons

Simplifying Ratios

Simplifying Ratios

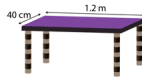
Learning Objective: Simplify a ratio using the highest common factor.

Simplify the following ratios:

- a) 20 : 50 b) 8 : 24 : 40 c) £1.50 : 50

- d) 2 m : 25 cm : 400 m e) Write 25p to £1 as a simplified ratio.

f) A table has a length of 1.2 m and a width of 40 cm.



- i) Simplify the ratio of the length of the table to its width.
ii) Simplify the ratio of the width of the table to its length.

3-Part Ratios

Three Part Ratios

Learning Objective: Use ratio notation, including reduction to simplest form.

Find the values of x and y for each 3 part ratio.

Ans a) $\frac{a}{6} : \frac{b}{3} : \frac{c}{2}$ **Ans b)** $\frac{p}{3} : \frac{q}{4} : \frac{r}{2}$ **Ans c)** $\frac{m}{4} : \frac{n}{5} : \frac{o}{3}$

$\frac{6}{x} : \frac{3}{y} : \frac{2}{1}$ $\frac{3}{x} : \frac{4}{y} : \frac{2}{1}$ $\frac{4}{x} : \frac{5}{y} : \frac{3}{1}$

$\frac{3}{x} : \frac{2}{y} : \frac{1}{1}$ $\frac{1}{x} : \frac{2}{y} : \frac{1}{1}$ $\frac{3}{x} : \frac{8}{y} : \frac{1}{1}$

$\frac{x}{3} : \frac{y}{2} : \frac{1}{1}$ $\frac{x}{4} : \frac{y}{2} : \frac{1}{1}$ $\frac{x}{15} : \frac{y}{4} : \frac{1}{1}$

Three Part Ratios

The ratio of red to blue counters in bag is 3 : 2.
The ratio of blue to orange is 5 : 3.
a) What is the ratio of red to orange counters?

Red : Blue : Orange
3 : 2 : 5

$\frac{3}{15} : \frac{2}{10} : \frac{5}{5} \times \frac{1}{2} = \frac{1}{2}$

b) There are 10 orange counters. How many counters are red?

Red : Orange
5 : 2
25 : 10

- Ans d)**
The ratio of blue to red counters in a bag is 3 : 1.
The ratio of red to green counters in the same bag is 2 : 5.

- i) What is the ratio of blue to green counters?
ii) There are 15 green counters. How many are blue?

mr-mathematics.com

Proportional Reasoning

Proportional Reasoning

Learning Objective: Use equivalent ratios to solve problems involving proportions.

1. James makes a pastry by mixing flour and butter in the ratio 3 : 2. He has 450 grams of flour and plenty of butter.

How much of the pastry can he make?

2. Stephen is mixing light pink paint. He mixes red paint to white paint in the ratio 1 part red to 3 parts white.

- a) How much white paint should he mix with 3 litres of red paint?
b) How much red paint should he mix with 12 litres of white paint?

Sharing to a Ratio

Sharing to a Ratio

Learning Objective: Share an amount to a ratio.

Q1. Divide £30 in the ratio 2 : 3.

Q2. Divide £40 in the ratio 1 : 4 : 3

Q3. Paul works 5 hours and Emily works 8. Their employer shares £80 between them according to the number of hours worked. How much does each person get?

Q4. £420 is divided by Simon, Clare and Freya in the ratio 4 : 3 : 5. How much does Freya receive?

Proportion and Recipes

Proportion and Recipes

Learning Objective: Apply ratio to real contexts and problems such as those involving scaling.

Q1. Here is a list of ingredients for making a strawberry trifle for 6 people.

- Strawberry Trifle**
Serves 6
210g jelly
12 sponge fingers
600 ml custard
240 g strawberries

Work out the amount of ingredients needed to serve 15 people.

Q2. Here are the ingredients needed to make soup for 4 people.

- Carrot and Leek Soup**
Serves 4
4 leeks
325 g carrots
800 ml vegetable stock
320 ml milk

Work out the amount of each ingredient needed to serve 6 people.

Proportion and Recipes

Here is an ingredient list to make 6 shortbread biscuits.

- Shortbread Biscuits**
Ingredients for 6 People
160 g butter
40 g sugar
1 egg
240 g flour

How much of each ingredient is needed for 18 people?

- Shortbread Biscuits**
Ingredients for 18 People
3 x 160 g butter = 480 g
3 x 40 g sugar = 120 g
3 x 1 egg = 3 eggs
3 x 240 g flour = 720 g

mr-mathematics.com

Problem Solving and Revision Lessons

Equivalent Ratios

Applying Equivalent Ratios

The ratio of teachers to students in a school is 3 : 40.
There are 27 teachers in the school.
How many students are there?

A paint mix uses yellow and blue in a ratio 1 : 12.
How much yellow paint will be needed to mix with 2.4 litres of blue?

Bethany, Ryan and Ben travel in a car from Liverpool to London.
They share the driving in the ratio 2 : 3 : 5.
Ryan drives 72 miles.
How much further does Ben drive than Bethany?

Daniel, Michelle and Vicki save some money in the ratio 5 : 3 : 7.
Daniel saved £84 more than Michelle.
Calculate the total money saved.

mr-mathematics.com

Sharing to a Ratio

Sharing to a Ratio

Purple paint is made by mixing red paint and blue paint in the ratio 5 : 2.
Red paint costs £5.50 per litre.
Blue paint costs £4.80 per litre.
Work out the cost of 42 litres of green paint.

Kira is paid £320 a week.
The ratio for money spent paying her bills to money left over is 5 : 3.
Of the money left over she saves 12%.
How many weeks will it take Kira to save £250?

Concrete is made by mixing cement, sand and gravel in the ratio 1 : 2 : 5.
A builder mixes 416 kg of concrete.
How much gravel does he need?

A sample of 400 people are chosen.
• 340 people are right handed.
• The ratio of women to men is 11 : 9.
• $\frac{1}{5}$ of them are right handed men.
How many left handed women are there?

mr-mathematics.com

Calculating Exchange Rates

Exchange Rates

Given that £1 = \$1.65
Convert the following into dollars.
a) £50 b) £190 c) £450

Convert the following into pounds.
d) \$68 e) \$140.25 f) \$412.50

Howard goes on holiday.
He changes £500 to euros.
The exchange rate is £1 = 1.2 euros.
Howard spends 500 euros.
How much, in pounds, does he get back?

In Spain, a 4K TV costs 604E.
In England, the same TV costs £580.

Work out the difference between the cost of the TV in Spain and the cost of the TV in England.

mr-mathematics.com

Problem Solving with Ratio

Problem Solving with Ratio

Emma, Jake and Sarah live together and decide to decorate their house.
They buy:

- 4 tins of white paint costing £10.15
- 3 tins of blue paint costing £21.45
- 2 sets of brushes £45.

Emma, Jake and Sarah split the total cost in the ratio 6 : 2 : 3.

a) Work out an estimate for the amount Sarah pays.
b) Is your answer to part a) an underestimate or overestimate?

mr-mathematics.com



Equations of Straight-Line Graphs

Students learn how to plot and derive the equation of straight-line graphs in the form $y = mx + c$. Learning progresses on from this to find the equation of parallel and perpendicular lines in the form $ax + by + c = 0$.

Prerequisite Knowledge

- Recognise and describe linear number sequences, including those involving fractions and decimals, and find the term-to-term rule.
- Generate and describe linear number sequences

Key Concepts

- Gradient is a measure of rate of vertical change divided by horizontal change.
-
- Parallel lines have the same gradient
- The intercept always has the x value equal zero.

Success Criteria

- Plot graphs of equations that correspond to straight-line graphs in the coordinate plane;
- Use the form $y = mx + c$ to identify parallel lines
- Find the equation of the line through two given points, or through one point with a given gradient
- Identify and interpret gradients and intercepts of linear functions graphically and algebraically

Common Misconceptions

- Students often confuse linear graphs to have the same notation as statistical graphs.
- The gradient can be calculated from any two points along the graph. Not necessarily from the origin.
- A linear function does not have to pass through the origin.
- It is beneficial to create a table of results when plotting a linear function. The coordinate pairs arise from the x and y values.



Lessons

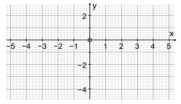
Plotting Straight Line Graphs $y = mx + c$

C

Plotting Linear Graphs

Learning Objective: Plot a linear function in the form $y = mx + c$ on Cartesian axes.
Plot the graph $y = \frac{x}{2} - 1$

| | | | | | | | |
|---|----|----|----|---|---|---|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| y | | | | | | | |



Draw the graphs of these straight lines on a coordinate grid with axes drawn from -3 to +3 on the x axis.

a) $y = 2x - 3$ b) $y = 3(x - 1)$ c) $y = 2 - x$
 d) $y = \frac{x}{2} + 3$ e) $y = 4 - 5x$ f) $y = \frac{x}{3} - 6$

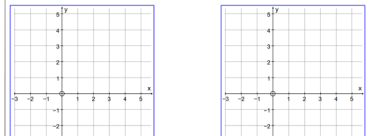
Straight Line Graphs in the Form $ax + by = c$

Plotting Straight Line Graphs

Learning Objective: Plot a straight line graph in the form $ax + by = c$ using two points.

Plot the graph of $2x + 3y = 6$
 $x = 0$ $y = 2$

Plot the graph of $3x + 4y = 12$
 $x = 0$ $y = 3$

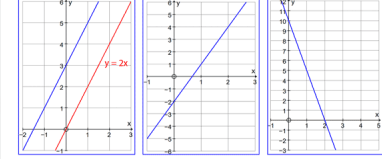


Finding the Gradient of Straight Lines

The Gradient

Learning Objective: Calculate the gradient of a straight line graph.
 Calculate the gradient of each straight line graph.

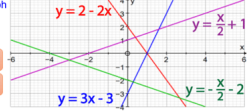
Gradient (M) = $\frac{\Delta Y}{\Delta X}$



Gradient of Parallel Lines

Parallel Gradients

Learning Objective: Recognise and calculate the gradient of parallel lines.
 Match the equation with the graph it runs parallel to.



QNS: Match the pairs of parallel lines.

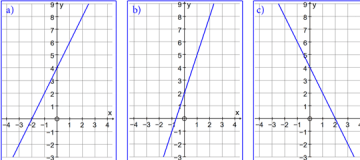
| | | | |
|---------------------|---------------|-----------------------|--------------------|
| $y = \frac{x+8}{4}$ | $y = 5 + x$ | $y = \frac{x}{4} + 1$ | $y = \frac{2x}{3}$ |
| $y = 2(x+1)$ | $2x + 3y = 6$ | $y = 2x - 3$ | $y - x = 3$ |

mr-mathematics.com

Deriving the Equation of a Straight Line

$y = mx + c$

Learning Objective: Derive the gradient of a straight line in the form $y = mx + c$.
 Find the gradient and intercept of these graphs.



Gradient of Perpendicular Lines

Perpendicular Gradients

Learning Objective: Derive the equation of perpendicular lines in the form $y = mx + c$.

QNS: The line $y = 2x$ is drawn.
 On the same grid plot the line $y = 5 - 0.5x$.

What do you notice about the angle where the two lines intersect?

What do you notice about the gradients of the two lines?

QNS: Match the pairs of perpendicular lines.

| | | | |
|--------------|-----------------------|---------------|----------------------|
| $y = 3x + 5$ | $y = 4 - \frac{x}{3}$ | $2y = 6 - 3x$ | $y = 5x$ |
| $y = 2(1-x)$ | $y = \frac{x+3}{2}$ | $5y = 1 - x$ | $y = \frac{2x+1}{3}$ |

mr-mathematics.com

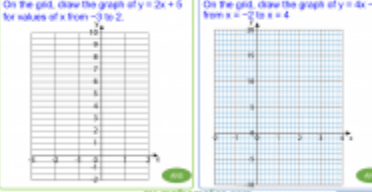
Problem Solving and Revision Lessons

Plotting Linear Graphs

Plotting Linear Graphs

On the grid, draw the graph of $y = 2x + 5$ for values of x from -3 to 2.

On the grid, draw the graph of $y = 4x - 1$ from $x = -2$ to $x = 4$.

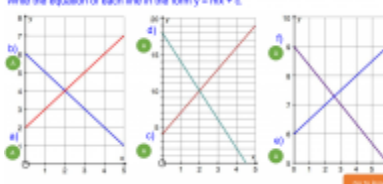


mr-mathematics.com

Equation of Straight Line Graphs

Equation of Straight Line Graphs

Write the equation of each line in the form $y = mx + c$.



mr-mathematics.com

Perpendicular and Parallel Gradients

Parallel and Perpendicular Gradients

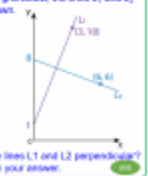
The straight line L_1 has equation $y = 3x + 2$. The straight line L_2 is parallel to line L_1 , and passes through the point $(5, -1)$. Find the equation of line L_2 .

Line L_1 has the equation $12x - 6y = 5$.

a) Circle the equation of the line that runs parallel to line L_1 .
 $4y = 5 - 8x$ $4y = 8x - 5$

b) Circle the equation of the line that runs perpendicular to line L_1 .
 $8y = 4x + 5$ $8y = 5 - 4x$

On the grid below, the lines L_1 and L_2 are shown.



Are the lines L_1 and L_2 perpendicular? Explain your answer.


mr-mathematics.com

Problem Solving – Straight Line Graphs

Problem Solving with Straight Line Graphs

The diagram shows the graph of $y = 5x + c$, where c is a constant.

Work out the value of n .



mr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Plotting and Interpreting Statistical Diagrams

Students learn how to plot and interpret various statistical diagrams ranging from pie charts to histograms with unequal class widths. Throughout the unit emphasis is placed on interpreting the diagrams as much as it is plotting them.

Prerequisite Knowledge

- Interpret and construct:
 - frequency tables
 - bar charts
 - pictograms
- for categorical data.
- Construct and interpret stem and leaf diagrams
- Apply statistics to describe a population

Key Concepts

- Students need to spend time interpreting the diagrams as well as creating them.
- When using pie charts to compare distributions the frequency of corresponding sectors is dependent on the total sample size.
- Frequency diagrams are used to represent discrete data whereas histograms are used for continuous data.
- Histograms with unequal class widths represent data with an unequal spread. Frequency is found using the area of a bar rather than its height.
- Cumulative frequency is the running total of the frequency.
- The interquartile range (IQR) shows the boundaries of where the most representative data is located.

Success Criteria

- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- Interpret and construct tables and line graphs for time series data and know their appropriate use
- construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use.

Common Misconceptions

- Histograms are often confused with frequency diagrams.
- Students tend to be more competent with constructing the various representations than using them to analyse and make summative comments about distributions.



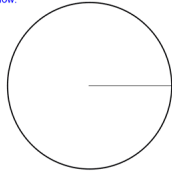
Lessons

Drawing Pie Charts

Pie Charts

Learning Objective: Construct and interpret a pie chart.
36 people were asked about the make of their mobile phone. The results are shown in the frequency table below.

| Mobile Phone | Frequency |
|--------------|-----------|
| Apple | 12 |
| Samsung | 9 |
| HTC | 8 |
| Sony | 6 |
| Nokia | 1 |



Draw a pie chart to represent this data.

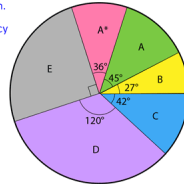
Interpreting Pie Charts

Interpreting Pie Charts

Learning Objective: Calculate frequencies from a pie chart.
The pie charts show the mathematics results for a group of students. 120 students took the mathematics exam.

Complete the table to show the frequency of the results.

| Grade | Frequency |
|-------|-----------|
| A* | |
| A | |
| B | |
| C | |
| D | |
| E | 30 |



Plotting Time Series

Time Series

Learning Objective: Plot a time series and draw a trend line to make predictions.
This table shows the quarterly sales of cars at AW Cars.

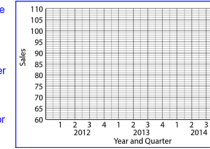
| Year | 2012 | | | | 2013 | | | | 2014 | | | |
|---------|------|----|----|----|------|----|----|----|------|----|----|---|
| Quarter | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Sales | 85 | 79 | 74 | 68 | 91 | 82 | 76 | 71 | 103 | 86 | 81 | |

i) Draw a time series graph to illustrate the trend of this data.

ii) Draw a trend line.

iii) Comment on the trend of sales over time.

iv) Make a prediction of sales for the last quarter of 2014. Give a reason for your answer.



Frequency Diagrams and Polygons

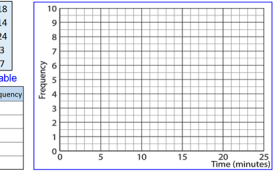
Frequency Polygons

Learning Objective: Plot and interpret a frequency polygon for grouped data.
Here are the time the pupils in 9R took to get to school this morning. The times are in minutes, correct to the nearest minute.

| | | | | | |
|----|----|----|----|----|----|
| 15 | 15 | 19 | 22 | 1 | 18 |
| 3 | 8 | 24 | 7 | 22 | 14 |
| 3 | 14 | 18 | 18 | 24 | |
| 20 | 18 | 18 | 8 | 6 | 3 |
| 12 | 2 | 20 | 6 | 11 | 7 |

Organise the data into this table

| Time (min) | Tally | Frequency |
|------------|-------|-----------|
| 1-5 | | |
| 6-10 | | |
| 11-15 | | |
| 16-20 | | |
| 21-25 | | |



Cumulative Frequency Diagrams

Cumulative Frequency Graphs

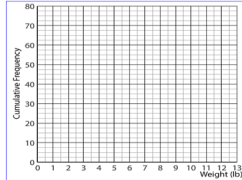
Learning Objective: Plot and interpret cumulative frequency graphs from a grouped data table.

| Weight, w, Lbs | Frequency | Cumulative Frequency |
|----------------|-----------|----------------------|
| 0w<=2 | 2 | |
| 2w<=4 | 17 | |
| 4w<=6 | 24 | |
| 6w<=8 | 23 | |
| 8w<=10 | 10 | |
| 10w<=12 | 4 | |

Calculate the Median and IQR.

How many babies were born weighing less than 5 Lbs?

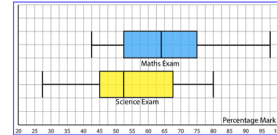
How many babies were born in the range 7 to 3 Lbs?



Box and Whisker Diagrams

Box and Whisker Diagrams

Learning Objective: Plot and interpret box and whisker diagrams.
The Box and Whisker Diagrams represent the marks of a Science and Maths exam taken by the same students.



Compare the two distributions mentioning the Median Average and Interquartile Range.

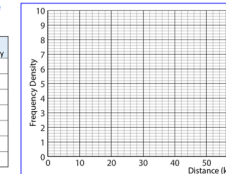
Drawing Histograms

Histograms

Learning Objective: Use frequency density to plot histograms with unequal class widths.
180 commuters were surveyed to find out the distances they travel to work.

Draw a Histogram to represent the results.

| Distance (km) | Frequency | Class Width | Freq. Density |
|---------------|-----------|-------------|---------------|
| 0 < d <= 5 | 3 | | |
| 5 < d <= 10 | 9 | | |
| 10 < d <= 15 | 34 | | |
| 15 < d <= 20 | 49 | | |
| 20 < d <= 30 | 40 | | |
| 30 < d <= 40 | 31 | | |
| 40 < d <= 60 | 16 | | |



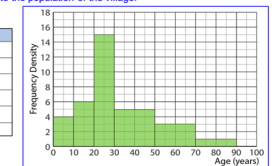
Interpreting Histograms

Interpreting Histograms

Learning Objective: Calculate the frequencies from a histogram with unequal class widths to analyse distributions.

The histogram shows the ages of people who live in a small town. Use the histogram to calculate the population of the village.

| Age, A (years) | Frequency |
|----------------|-----------|
| 0 < A <= 10 | |
| 10 < A <= 20 | |
| 20 < A <= 30 | |
| 30 < A <= 50 | |
| 50 < A <= 70 | |
| 70 < A <= 90 | |



Problem Solving and Revision Lessons

Pie Charts

Pie Charts

There were four candidates standing for election: Simon, Jo, Diane and Jenia.

A sports team can either win, lose or draw a game. Here are some results over the course of a season.

Wins = $\frac{1}{4}$ Loss = $\frac{1}{2}$

Jo received half as many votes as Simon.
a) Complete the pie chart.
b) How many people voted for Simon?

Complete the pie chart.

Histograms

Drawing and Interpreting Histograms

The histogram shows information about the weight, w, kg, of plants in a garden centre.

a) Estimate the number of plants that weigh between 15 kg and 25 kg.

b) Estimate the mean average weight of all the plants.

Problem Solving with Cumulative Frequency Graphs

Problem Solving with Cumulative Frequency

A school records the age, in months, of each of its 60 projectors.

The cumulative frequency graph summarises this information.

The school decides to replace all the projectors over 3 years old.

The cost of replacing each projector is £850.

Work out the cost of replacing all the projectors over 3 years old.



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Angle Geometry

Students learn how to discover various angle properties such as angles on a straight line, about a point, in a triangle and on parallel lines. As learning progresses, they are challenged to construct polygons and solve problems involving their interior and exterior angles.

Prerequisite Knowledge

- Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- Draw given angles, and measure them in degrees ($^{\circ}$)
- Identify:
 - angles at a point and one whole turn (total 360°)
 - angles at a point on a straight line and $1/2$ a turn (total 180°)
 - other multiples of 90°
- Apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles;

Key Concepts

- Rather than being told (or given) angle properties students should have the opportunity to discover and make sense of them practically.
- Geometric problems can often be solved using various angle properties. Encourage students to look for and apply alternative properties.
- Demonstrate how a polygon is made up from interior triangles when calculating their angles.
- Bearings always go clockwise from North and have three digits. North lines are parallel.

Success Criteria

- Understand and use alternate and corresponding angles on parallel lines;
- Derive and use the sum of angles in a triangle (e.g. to deduce use the angle sum in any polygon, and to derive properties of regular polygons)
- Measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

Common Misconceptions

- Students often forget the definition of properties associated to angles in parallel lines.
- Exterior angles in a polygon have to travel in the same direction for the sum to be 360° .



Lessons

Angles in a Triangle

Angles in a Triangle
Learning Objective: Calculate missing angles in a triangle and on a straight line
 Calculate the marked angle in each triangle.

Triangle 1: Angles 40° and 60° , missing angle a .
 Triangle 2: Angles 60° and 70° , missing angle b .
 Triangle 3: Angles 55° and 35° , missing angle c .

Angles in a Quadrilateral

Angles in a Quadrilateral
Learning Objective: Calculate missing angles in a quadrilateral
 Calculate the value of the unknown angles.

Quadrilateral 1: Angles 45° , 120° , and 135° , missing angle a .
 Quadrilateral 2: Angles 70° and 120° , missing angle b .
 Quadrilateral 3: Angles 284° and 90° , missing angle c .
 Quadrilateral 4: Angles 127° and 97° , missing angle d .

Angles in Parallel Lines

Angles in Parallel Lines
Learning Objective: Recognise Alternate, Corresponding and Interior angles in parallel lines.
 Using the three properties of angles in parallel lines calculate the missing angles.

Diagram a: Parallel lines with angles 40° and 60° .
 Diagram b: Parallel lines with angles 50° and 70° .
 Diagram c: Parallel lines with angles 148° and 70° .
 Diagram d: Parallel lines with angles 64° and 125° .
 Diagram e: Parallel lines with angles 70° and 125° .
 Diagram f: Parallel lines with angles 70° and 125° .

Problems with Parallel Lines

Angle Properties
Learning Objective: To solve complex angle problems involving parallel lines, triangles, about a point and a line.
 Calculate the values of the angles represented with letters.

Diagram a: Parallel lines with angles 110° and 80° , missing angles a and b .
 Diagram b: Parallel lines with angles 65° and 32° , missing angles c and e .
 Diagram c: Parallel lines with angles 20° and 75° , missing angles g and h .
 Diagram d: Parallel lines with angles 84° and 32° , missing angles k and p .

Bearings and Scale Drawings

Bearings
Learning Objective: Construct scale drawings using correct bearing notation.
 Use the compass to measure the bearings of London to:
 a) Stratford
 b) Bromley
 c) Mitcham
 d) Croydon
 e) Shepherd's Bush
 Use sketch maps to calculate the back bearings of the above locations to London.

Constructing Polygons

Constructing Polygons
Learning Objective: Construct Regular Polygons and Measure their Interior & Exterior Angles.

Triangle: Interior Angle and Exterior Angle.
 Quadrilateral: Interior Angle and Exterior Angle.

Interior and Exterior Angles

Interior and Exterior Angles in Polygons
Learning Objective: Calculate interior and exterior angles in polygons.
 Calculate the value of the angles marked.

Diagram a: Pentagon with interior angle a .
 Diagram b: Hexagon with exterior angle b .
 Diagram c: Hexagon with interior angle c .
 Diagram d: Octagon with exterior angle d .

Problem Solving and Revision Lessons

Angles in Parallel Lines

Angles in Parallel Lines
Learning Objective: Calculate the value of the marked angles.

Diagram a: Parallel lines with angles 50° and 130° .
 Diagram b: Parallel lines with angles 100° and 130° .
 Diagram c: Parallel lines with angles 130° and 130° .
 Diagram d: Parallel lines with angles 74° and 130° .
 Diagram e: Parallel lines with angles 112° and 130° .
 Diagram f: Parallel lines with angles 130° and 130° .

Angles in Regular Polygons

Angles in Regular Polygons
Exam Style Questions:
 Q1) ABCDEFGH is a regular octagon.
 AD and DD are lines drawn from A and D respectively to the centre of the octagon.
 Work out the size of $\angle ADD$.
 Q2) ABCDE is a regular pentagon.
 AGF and FDC are straight lines.
 Work out the size of angle $\angle EFD$.

Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Collecting Data

Students learn how to design a questionnaire without bias to collect primary qualitative and quantitative data sets. As learning progresses, they use stratified sampling to determine sample size and how to design two-way tables and frequency trees to organise these data.

Prerequisite Knowledge

- Interpret and construct statistical diagrams for discrete and continuous data and know their appropriate use.
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
 - appropriate graphical representation involving discrete, continuous and grouped data
 - appropriate measures of central tendency (median, mean, mode and modal class) and spread

Key Concepts

- Students need to understand the benefits of using two-way tables to exhaustively cover each outcome for multiple events and use them to calculate probabilities.
- When designing questionnaires students need to consider time periods, multiple check boxes which do not overlap and the need to collect a wide-ranging sample to reduce bias.
- It is important to recognise the different statistical techniques that are used to analyse and represent qualitative, quantitative, discrete and continuous data.

Success Criteria

- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling.
- apply statistics to describe a population
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data.

Common Misconceptions

- Students often have difficulty designing two-way tables.
- When designing questionnaires common errors include:
 - No time period
 - Overlapping responses
 - Lack of 'none' or 'other' option.
 - Check boxes with unequal widths.
 - Double negative questions.
- Students often try to represent continuous data using methods that are only applicable for discrete sets.



Lessons

Types of Data

Types of Data
Learning Objective: Understand the difference between quantitative and qualitative data and how they are processed.
Arrange the data sets in the correct categories.

| | | |
|------------------------------|-----------------|----------------------------|
| Qualitative Data | | Eye colour |
| Discrete Quantitative Data | | Capacity of bottles |
| Continuous Quantitative Data | | Favourite colour |
| | | Results of a questionnaire |
| | | Engine size |
| | | Number of windows |
| | Weight of a pet | |
| | Shoe size | |

Frequency Trees

Frequency Trees
Learning Objective: Record, describe and analyse the frequency of outcomes using frequency trees

160 students in Year 10 had some homework.
 73 of these students are boys.
 64 of the 160 students did not do their homework.
 39 of the girls did do their homework.

a) Use this information to complete the frequency tree.

One of the boys is chosen at random.
 b) Work out the probability that this boys did not do his homework.

| Category | Maths | English |
|----------|-------|---------|
| Girls | 7 | 4 |
| Boys | 10 | 6 |

Designing Questionnaires

Questionnaires
Learning Objective: Design a fair data collection sheet.

Niall wants to find out how much people spend on Christmas. He uses this question on a questionnaire.

How much do you spend on Christmas?

A little A lot

Why is this question biased?
 Are the response boxes detailed enough?
 Design a suitable questionnaire as a way to record this information.
 Include response boxes.

Stratified Sampling

Stratified Sampling
Learning Objective: Use stratified sampling to determine a suitable sample size for a set of data.

Stratified Sampling - Data is split into groups called strata. An equal proportion of each strata is included rather than an equal amount. This is useful when the data is not equally distributed.

The table shows the number of girls in four different groups.
 Janine intends use collect a stratified sample of 40 girls to interview.

| Group | A | B | C | D | Total |
|-----------------|----|----|----|----|-------|
| Number of girls | 38 | 30 | 18 | 40 | 126 |

a) Explain why taking a stratified sample is more appropriate than quota sampling.
 b) Calculate the number of girls from each group that should be in the sample

Problem Solving and Revision Lessons

Two-Way Tables and Frequency Trees

Problem Solving with Two-Way Tables and Frequency Trees

$5x + 12$ people were included in a survey.

- $2x + 3$ were boys.
- $2(x + 1)$ girls were right-handed.
- $2x + 1$ people were left-handed in total
- 12 boys were right-handed.

Work out the value x .

ANS

mr-mathematics.com

Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Performing and Describing Transformations

At the start of this unit students learn how to perform and describe reflections, rotations, translations and enlargements on a grid. As learning progresses, they are challenged to describe a combination of transformations using the correct terminology.

Prerequisite Knowledge

- Use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries;
- Identify an order of rotational and reflective symmetry for two dimensional shapes
- Use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
- Recognise linear functions in the form $y = \pm a$ and $x = \pm$

Key Concepts

- An object is transformed to create an image.
- Rotation, Translation and Reflections involve congruent objects and images whereas enlargement leads to the object being similar to the image.
- Translation vectors are used to describe movements along Cartesian axes.
- When reflecting objects the image is always the same distance from the line of reflection as the object.
- Rotations and enlargements are constructed from a centre.
- A negative scale factor transforms the object through the centre.

Success Criteria

- Identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional and negative scale factors)

Common Misconceptions

- Translation vectors can incorrectly be written using the name notation as coordinate pairs.
- Translations, Rotations, Enlargement and Reflections all come under the umbrella term of transformation. Students often confuse the term translation for transformation.
- Students often have more difficulty describing single transformations rather than performing them.
- Enlargements can involve making a shape smaller as well as bigger. Fractional scale factors between 0 and 1, not negative, decrease the size.



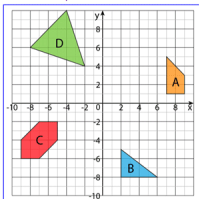
Lessons

Reflections

Reflection

Learning Objective: Use an equation as a mirror line to perform and describe reflections.

- Reflect object A in the line $x = 6$.
- Reflect object B in the line $y = -5$.
- Reflect object A in the line $y = x$.
- Reflect object B in the line $y = -x$.
- Reflect object C in the line $y = x$.
- Reflect object C in the line $y = -x$.

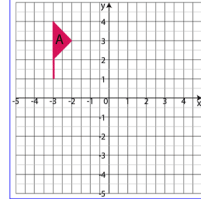


Rotations

Rotations

Learning Objective: Use a centre, direction and amount of turn to perform and describe rotations.

- Rotate object A 90° anti-clockwise about $(-1, 0)$. Label the image B.
- Rotate object A 180° about $(0, 0)$. Label the image C.
- Rotate object A 90° clockwise about $(1, 0)$. Label the image D.
- Describe the rotation that maps B on to D.

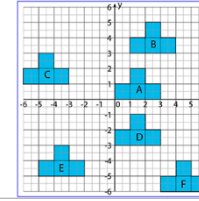


Translations

Translation

Learning Objective: Use vector notation to perform and describe translations. Use vector notation to describe the following translations:

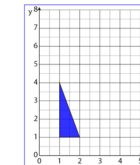
- A to B
- A to C
- A to D
- A to E
- A to F
- C to D



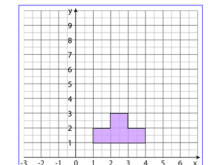
Enlargements

Enlargement

Learning Objective: Enlarge an object on a grid using a centre and scale factor.



Enlarge by a scale factor of 2 about $(0, 0)$.

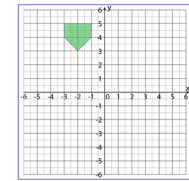


Enlarge by a scale factor of 3 about $(0, 0)$.

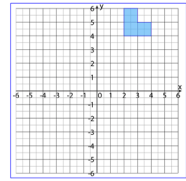
Negative Scale Factor Enlargements

Negative Scale Factors

Learning Objective: Perform and describe enlargements to a negative scale factor.



- Enlarge the object by scale factor -1 from $(2, 2)$.
- Enlarge the object by scale factor -2 from $(0, 3)$.



- Enlarge A by scale factor -1 from centre $(0, 2)$.
- Enlarge A by scale factor -2 from centre $(0, 3)$.

Describing Transformations

Column Vectors

Learning Objective: Perform addition and subtraction with column vectors.

Here are three column vectors:

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad c = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

On the grid to the right:
i) Draw and label the following vectors:
ii) Give the single column vector

- $2a - b$
- $b + a + c$
- $2c + 3a$
- $a - b + 2c$

mr-mathematics.com



Addition and Subtraction with Column Vectors

Column Vectors

Learning Objective: Perform addition and subtraction with column vectors.

Here are three column vectors:

$$a = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad c = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

On the grid to the right:
i) Draw and label the following vectors:
ii) Give the single column vector

- $2a - b$
- $b + a + c$
- $2c + 3a$
- $a - b + 2c$

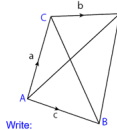
mr-mathematics.com



Vector Notation

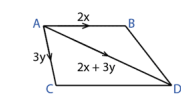
Vectors

Learning Objective: Add and subtract with vectors. ABCD is a quadrilateral.



- Write:
- \vec{AB} in terms of a and b .
 - \vec{CB} in terms of a and b .
 - Show that $\vec{BD} = a + b - c$.

ABCD is a quadrilateral.



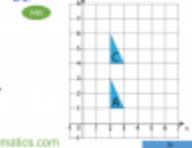
- Write:
- Express \vec{BD} , \vec{DC} in terms of x and y .
 - What can you say about the quadrilateral ABCD?

Problem Solving and Revision Lessons

Enlargements on a Grid

Positive Scale Factor Enlargements on a Grid

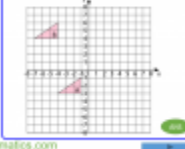
- Q1. a) Enlarge triangle P by a scale factor of 2 from centre $(1, 2)$. Label the image Q.
b) Describe the transformation that will map P on to R.



Performing and Describing Transformations

Transformations on a Grid

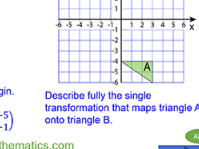
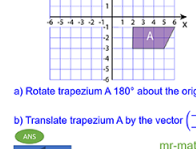
- Describe fully the single transformation that maps triangle A onto triangle B.
- Reflect triangle B in the line $y = x$.



Rotations, Translations and Reflections

Rotations, Reflections and Translations

- Q3. a) Rotate trapezium A 180° about the origin.
b) Translate trapezium A by the vector $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$.



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Solving Linear Equations

Students learn how to solve an equation using the balance method and trial and improvement. As learning progresses, they are taught how to form and solve a pair of simultaneous equations from known geometrical facts and real-life problems.

Prerequisite Knowledge

- Use simple formulae
- Generate and describe linear number sequences
- Express missing number problems algebraically
- Find pairs of numbers that satisfy an equation with two unknowns
- Use and interpret algebraic notation
- Simplify and manipulate algebraic expressions by:
 - collecting like terms
 - multiplying a single term over a bracket

Key Concepts

- To solve an equation is to find the only value (or values) of the unknown that make the mathematical sentence correct.
- For every unknown an equation is needed.
- Students need to have a secure understanding of adding and subtracting with negatives when eliminating an unknown.
- Coefficients need to be equal in magnitude to eliminate an unknown

Success Criteria

- Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation)
- Solve two simultaneous equations in two variables algebraically;
- Find approximate solutions to simultaneous equations in two variables using a graph;
- solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation)
- Translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

Common Misconceptions

- Students can forget to apply the same operation to both sides of the equation therefore leaving it unbalanced.
- Students often struggle knowing when to add or subtract the equations to eliminate the unknown. Review addition with negatives to address this.
- Equations need to be aligned so that unknowns can be easily added or subtracted. If equations are not aligned students may add or subtract with non like variables.
- Students often try to eliminate variables with their coefficients being equal.



Lessons

Equations and the Balance Method

Learning Objective: To solve an equation using the balance method.
Solving Equations

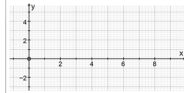
Solve the following equations:

- a) $x + 4 = 7$
- b) $2g = 26$
- c) $14 + u = 30$
- d) $y - 3 = 10$
- e) $\frac{t}{7} = 3$
- f) $2r + 5 = 17$
- g) $7x + 3 = 80$
- h) $7x - 4 = 80$
- i) $3x + 1 = 40$
- j) $3x - 2 = 40$
- k) $\frac{t}{2} + 3 = 8$
- l) $\frac{z}{6} - 3 = 4$

Solving Simultaneous Equations Graphically

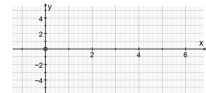
Learning Objective: Solve a pair of simultaneous equations using graphical methods.
Solving Simultaneous Equations Graphically

Plot the graphs
a) $2x + 3y = 12$ & b) $x + 3y = 9$



Solve:
 $2x + 3y = 12$ & $x + 3y = 9$

Plot the graphs
a) $3x - 5y = 15$ & b) $x + 2y = 9$



Solve:
 $3x - 5y = 15$ & $x + 2y = 9$

Unknown on Both Sides

Learning Objective: Solve an equation with the unknown on both sides using the balance method.
Equations with the unknown on both sides

Solve these equations using the balance method.

- a) $3x = 2x - 4$
- b) $4y + 3 = 2y + 13$
- c) $5t + 3 = 3t + 7$
- d) $3r - 1 = 2r + 5$
- e) $5g + 3 = 11g - 21$
- f) $10 + 11w = 60 + w$

Equations and Fractions

Learning Objective: Use the balance method to solve an equation involving fractions.
Equations & Fractions

Solve the following equations using the balance method:

- a) $\frac{6}{5} + 3 = 7$
- b) $\frac{5t - 1}{2} = 3t + 1$
- c) $\frac{7}{6} + 1 = \frac{7 - k}{4}$
- d) $\frac{7}{5} - \frac{2}{3} = 2$
- e) $\frac{7}{2} + \frac{7}{5} = 21$
- f) $\frac{3(f + 1)}{8} = \frac{2(f - 3)}{3}$

Complex Linear Equations

Learning Objective: Set up and solve equations involving fractions and brackets.
Solving Complex Equations

Solve these equations.

- ANS a) $\frac{6}{x} = 18$
- ANS b) $\frac{30}{2y} = 20$
- ANS c) $\frac{6}{w + 4} = 1$
- ANS d) $\frac{5h}{h + 6} = 2$
- ANS e) $\frac{3 + 8}{x - 10} = 3$
- ANS f) $\frac{5}{x + 5} = \frac{15}{x + 7}$

For each ratio set up an equation to solve the value of x.

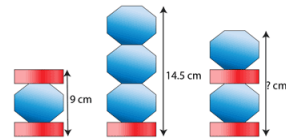
- ANS g) $x + 2 : 6x = 1 : 3$
- ANS h) $3x - 3 : 4 = 1$
- ANS i) $x - 2 : 3x + 8 = 1 : 5$
- ANS j) $4x : 6x - 3 = 3 : 4$

mr-mathematics.com

Solving Simultaneous Equations using the Elimination Method

Learning Objective: Solve a pair of Equations Simultaneously using a method of Elimination.
Simultaneous Equations

These towers are made of identical octagons and identical rectangles.

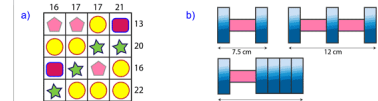


Calculate the height of the third tower.

Problems with Simultaneous Equations

Learning Objective: Derive and solve a pair of equations simultaneously using equivalent coefficients.
Simultaneous Equations with Different Coefficients

Setup pairs of equations simultaneously to solve these problems.



c) At the Post Office Simon paid £3.96 for five first class and three second class stamps. Julia paid £3.04 for four first and two second class.

What is the price for each a second and first class stamp?



Problem Solving and Revision Lessons

Solving Equations

Solving Equations with the Unknown on One Side

Solve the equations

- ANS a) $3x - 2 = 13$
- ANS b) $2x + 5 = 15$
- ANS c) $9x + 11 = 74$
- ANS d) $8x - 5 = 51$
- ANS e) $10x + 3 = 43$
- ANS f) $8x - 11 = 53$
- ANS g) $7 - 2x = 1$
- ANS h) $8 - 3x = 4$

Expand the brackets and solve the equations

- ANS i) $3(5x + 3) = 24$
- ANS j) $4(3x + 10) = 88$
- ANS k) $6(5x - 7) = 78$
- ANS l) $5(5x - 2) = 65$
- ANS m) $3(7 - 2x) = -27$
- ANS n) $9(1 - 5x) = -58.5$

mr-mathematics.com

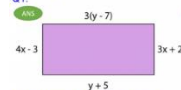
Go to lesson

Solving Equations with the Unknown on Both Sides

Equations with the Unknown on Both Sides

Exam Style Questions

Q1.



The diagram shows a rectangle.
Work out the area of the rectangle.

Q2.



Two parallel lines are shown.
Work out the value of w .

mr-mathematics.com

Go to lesson

Solving Equations with Fractions

Solving Equations Involving Fractions

Solve these equations

- ANS a) $\frac{x}{2} + 1 = 7$
- ANS b) $\frac{x}{3} + 1 = 9$
- ANS c) $\frac{x}{5} - 2 = 2$
- ANS d) $\frac{x}{2} + 2 = 3$
- ANS e) $\frac{x}{6} + 4 = 14$
- ANS f) $\frac{x}{6} + 7 = 2$
- ANS g) $\frac{3d}{3} = d - 10$
- ANS h) $\frac{d}{4} = a - 9$
- ANS i) $\frac{t}{5} = 2(t + 9)$

Solve these equations

- ANS j) $\frac{x}{(x+12)} = 2$
- ANS k) $\frac{3x}{x-1} = 4$
- ANS l) $1 = \frac{2x}{x-3}$
- ANS m) $\frac{5x-15}{x-2} = 4$
- ANS n) $\frac{3x-3}{x-3} = 2$
- ANS o) $\frac{2t-6x}{x-5} = -15$

mr-mathematics.com

Go to lesson

Simultaneous Equations by Elimination

Solving Simultaneous Equations

Exam Style Questions

Solve these simultaneous equations using an algebraic method.

$$\begin{aligned} 4x + 5y &= 21 \\ 2x + 3y &= 9 \end{aligned}$$

Two numbers have a sum of 50.

Three times the smaller number is the same as 10 more than the larger number.

What are the numbers?

The cost of buying a coffee and two teas in a cafe is £8.

The cost of buying two coffees and three teas in the same cafe is £10.20.

Work out the cost of buying a coffee and the cost of buying a tea.

Coffee =

Tea =

mr-mathematics.com

Problem Solving with Simultaneous Equations

Problem Solving with Simultaneous Equations

Two spheres A and B are to be melted down to make sphere C.

Sphere A has a volume of $x \text{ cm}^3$ and a density of 0.8 g/cm^3 .

Sphere B has a volume of $y \text{ cm}^3$ and a density of 1.2 g/cm^3 .

The total mass of sphere C is 9.6 grams. It has a density is 1.1 g/cm^3 .

Work out the surface area of sphere B.



mr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Scatter Graphs

Students learn how to plot and interpret a scatter graph. Learning progresses from using the line of best fit to find missing values to understanding whether correlation means causation.

Prerequisite Knowledge

- Solve comparison, sum and difference problems using information presented in a line graph
- Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.
- Work with coordinates in all four quadrants

Key Concepts

- Scatter graphs need to be drawn on graph paper or using I.C.T to ensure accuracy and help identify the line of best fit.
- Two measurements are 'associated' if the points lie approximately along a straight line. This shows a linear relationship. However, an association between two variables can exist in a non-linear relationship.
- Correlation is used to describe the strength of a linear relationship between two variables. If no correlation exists (the points do not appear to follow a trend of direction) the two variables are considered to have no linear relationship.

Success Criteria

- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation;
- draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of doing so.

Common Misconceptions

- Students often have difficulty choosing a suitable scale to use for each axis. Encourage the use of graph paper to ensure the graph is appropriately scaled.
- When drawing the line of best fit by eye it should represent the directional trend of the data. It does not have to intersect the origin or travel through every point.
- Correlation does not always imply a causal relationship since other factors could contribute.



Lessons

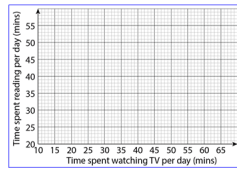
Plotting Scatter Graphs

Scatter Graphs

Learning Objective: Plot scatter graphs and identify any correlation.

Ten students took part in a survey to see how much time they spent reading and watching TV. Plot a Scatter Graph to check if the two are correlated.

| Time spent reading per day (mins) | Time spent watching TV per day (mins) |
|-----------------------------------|---------------------------------------|
| 30 | 60 |
| 30 | 55 |
| 35 | 60 |
| 40 | 40 |
| 40 | 35 |
| 45 | 30 |
| 45 | 35 |
| 50 | 25 |
| 55 | 20 |
| 55 | 20 |



Interpreting Scatter Graphs

Lines of Best Fit

Learning Objective: Use a line of best fit to describe correlation and estimate measures.

The scatter graph gives the reaction times in milliseconds for a group of adults.

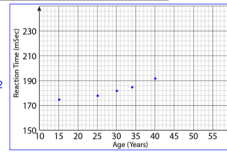
| Age (Years) | 15 | 25 | 30 | 34 | 40 | 45 | 50 | 60 |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Reaction Time (mSec) | 175 | 178 | 182 | 185 | 192 | 190 | 215 | 240 |

a) Plot the remaining points on the scatter graph.

b) Estimate the line of best fit and describe the correlation to context.

c) Estimate the reaction time for a 42 old adult.

d) Estimate the age of a person whose reaction time is 180 mSec.

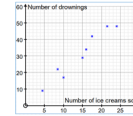


Causation Vs Correlation

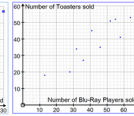
Correlation & Causal Relationships

Learning Objective: Understand the limitations of scatter graphs in identifying causation. Describe the correlation shown by each scatter graph and whether causation is indicated.

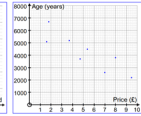
Drownings Vs Ice Creams Sold



Toasters sold Vs Blu-Ray Papers sold



Age of a car Vs Retail value



Problem Solving and Revision Lessons

Scatter Graphs and Correlation

Scatter Graphs and Correlation

The scatter graph shows the number of diving lessons and the number of tests needed to pass by 10 people.

| Age (years) | 12 | 16 | 16 | 12 | 14 | 11 |
|----------------------------|-----|-----|-----|-----|-----|-----|
| Hours of homework per week | 5.8 | 7.2 | 6.6 | 4.9 | 8.6 | 3.1 |

| Age (years) | 16 | 18 | 8 | 12 | 10 | 10 |
|----------------------------|-----|-----|-----|-----|-----|-----|
| Hours of homework per week | 7.2 | 7.8 | 2.8 | 3.9 | 2.3 | 2.8 |

a) Plot the 12 points.

b) What type of correlation is shown.

c) Draw a line of best fit.



mr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |



Constructions and Loci

Students learn how to construct angle and line bisectors using a pair of compasses. Later, students apply this knowledge to solve problems involving loci about a point and line.

Prerequisite Knowledge

- Identify and construct a radius, diameter, circumference, area, chord, tangent and arc.
- Measure and begin to record lengths and heights
- Identify acute and obtuse angles and compare and order angles up to two right angles by size

Key Concepts

- It is important for students to sketch the diagram before attempting their construction. The sketch should be drawn freehand and contain all the necessary information.
- Bisectors are used to half an angle as well as a length of a line segment.
- Constructing a 60° angle using a pair of compasses is an essential skills throughout this topic as it goes on to equilateral triangles and reflex angles.

Success Criteria

- Identify and construct a radius, diameter, circumference, area, chord, tangent and arc.
- Measure and begin to record lengths and heights
- Identify acute and obtuse angles and compare and order angles up to two right angles by size

Common Misconceptions

- Students often have difficulty constructing smooth arcs using a pair of compasses. Encourage them to try different techniques such as rotating the paper rather than the compasses.
- It is important to leave in construction lines as these form the working out.

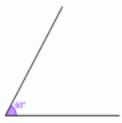

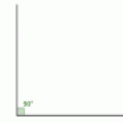


Lessons

Constructing Bisectors

Bisecting Angles

Learning Objective: Bisect an Angle using a pair of compasses and ruler.
Bisect these angles using a pair of compasses.

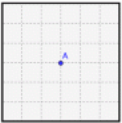
a)  b)  c) 

Locus about a Point and Line


Constructing Loci

Learning Objective: Construct the loci about a point and line using a pair of compasses and straight edge.

Shade the locus of the points that are less than 2 units from point A.



Shade the locus of the points that are less than units from line AB.




Equidistant Paths


Solving Problems with Equidistant Paths

Learning Objective: Construct a loci using compasses and a ruler to identify equidistant paths.

Show the path of a ball that is released from point A and rolls along a path that is the same distance from AC as it is AB.



A ferry sails along an equidistant path between points A and B. Show this path.



Problem Solving and Revision Lessons

Constructing Loci

Constructing Loci

The scale drawing shows the positions of three buoys in a sea.

A shipwreck is buried
less than 400 metres from A
less than 500 metres from B
closer to A than to B.

Shade the region where the treasure could be.



Scale: 1 unit represents 100m

mr-mathematics.com



Pythagoras' Theorem

Students are guided through the discovery of Pythagoras' Theorem using Pythagorean Triples. They use Pythagoras' Theorem to find the hypotenuse and shorter side of any right-angled triangle. As learning progresses, they begin to find lengths in 3D shapes.

Prerequisite Knowledge

- derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
- calculate the perimeters of 2D shapes, including composite shapes;
- use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

Key Concepts

- Pythagoras' Theorem identifies how the three sides of a right-angled triangle are connected by the areas of shapes on each edge. To fully engage with this concept students could construct the theorem using a 3,4,5 triangle to measure the hypotenuse and calculate the area of each square. Their hypothesis can then be tested on a 5, 12, 13 triangle.
- Pythagoras' Theorem can be applied to a wide variety of geometrical and real-world problems. Students need to practise identifying when the theorem can be applied by recognising triangular components.

Success Criteria

- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the
- base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Common Misconceptions

- Students often believe that the areas of the shapes on the edges have to be squares in order for $a^2 + b^2 = c^2$ to apply. In fact, the formula applies for all shapes as long as the dimensions are in proportion to the edges of the triangle.
- Confusion often lies in identifying the Hypotenuse side of a right-angled triangle since it is not always apparent which side is longest. Encourage students to identify the hypotenuse as opposite the right angle.
- There is often difficulty when trying to calculate a shorter side of a triangle since students tend to memorise the formula with the hypotenuse as the subject.



Lessons

Introducing Pythagoras' Theorem

Pythagoras' Theorem
Learning Objective: Discover Pythagoras' Theorem using right-angled triangles.

Step 1
 Create a 3cm by 4cm Right Angled Triangle

Step 2
 Measure the length of the Hypotenuse

Step 3
 Construct a square on each of the three sides.

Step 4
 Calculate the area of each square.

Step 5
 Hypothesize

MrMathematics
 www.mr-mathematics.com

Finding the Hypotenuse

Calculating the Hypotenuse
Learning Objective: Apply Pythagoras' Theorem to solve problems involving the Hypotenuse of a Right-Angled Triangle.

Calculate the hypotenuse of each triangle.

a) b) c)

d) Simon is orienteering. He walks 4 km due East and 3 km due South. How far is he from his starting point?

Finding the Shorter Side

Calculating a Short Side of a right-angled triangle
Learning Objective: Calculate a shorter length of a right angled triangle using Pythagoras' Theorem.

Calculate the length of each triangle:

Pythagoras' Theorem in 3D

Pythagoras' Theorem in 3D
Learning Objective: Apply Pythagoras' Theorem to calculate unknown lengths in 3D shapes.

Calculate the lengths marked in each solid.

a) b) c) d)

Pythagoras' Theorem in 3D Shapes
 Calculate the length AD.

$AD^2 = 4^2 + 3^2 + 5^2$
 $AD^2 = 16 + 9 + 25$
 $AD^2 = 50$
 $AD = \sqrt{50}$
 $AD = 5\sqrt{2}$

Problem Solving and Revision Lessons

Applying Pythagoras' Theorem

Pythagoras' Theorem

Calculate the marked length in each of these right-angled triangles.

a) b) c) d)

MrMathematics
 www.mr-mathematics.com

Problem Solving with Pythagoras' Theorem

Problem Solving with Pythagoras' Theorem

The diagram shows three circles.

The circle with centre A has a radius 5 cm.
 The circle with centre B has a radius 10 cm.
 The circle with centre C has a radius 5 cm.

Show that the length of the line BC can be written in the form $a\sqrt{b}$.

MrMathematics
 www.mr-mathematics.com



Additional Departmental Resources

| | |
|--|--|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

