YOUNG CHILDREN UNDERSTANDING CONGRUENCE OF TRIANGLES WITHIN A DYNAMIC MULTI-TOUCH GEOMETRY ENVIRONMENT

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This study examined how small groups of second-grade children developed understandings of the concept of congruence while collaboratively exploring and solving problems with dynamic representations of triangles using Sketchpad on the iPad. One case study is presented to illustrate how young learners can infer geometrical relationships between congruent triangles and co-construct mathematical strategies to create congruent triangles using these technologies.

Keywords: Geometry and Geometrical and Spatial Thinking, Technology, Problem Solving

Introduction

Congruence is an important mathematical idea for humans to understand the structure of their environment. Congruence is embedded in young children's everyday experiences that allow them to develop intuitive senses of this geometric relationship. Understanding the concept of congruence provides strong foundations for learning more advanced mathematical processes such as area and volume measurement (Huang & Witz, 2011; Wu, 2005). However, prior research has revealed a variety of students' difficulties in learning congruence at both the elementary and secondary grades (Clements & Sarama, 2014; Wu, 2005). Wu (2005) claims that the teaching of this concept is focused on the static informal definition "congruence is same size and same shape" (p. 5), which does not relate congruence to planar transformations, while the precise mathematical definition of the concept is based on rotations, translations and reflections. Wu notes that middle-school students have difficulties in understanding the precise mathematical definition of congruence and fail to grasp how it underlays other mathematical processes. Clements and Sarama (2014) state that the natural development of congruence also represents critical challenges for young children because they tend to analyze only parts of the shapes (e.g. length of one side) but not the relationships between these parts (e.g. lengths of all the sides) and privilege aspects of the shapes that are salient perceptually (e.g. orientation) rather than aspects that are mathematically relevant (e.g. number of sides). Thus, young children fail when one of the two figures is rotated or flipped or when the figures are unusual for them (e.g. long and thin triangles, scalene triangles, hexagons). The authors suggest that traditional teaching of geometry in early grades is implemented in rigid ways, which means that children are exposed to only prototypical shapes and have little experience with non-examples or variants of shapes. Students' difficulties can endure until adolescence if not well addressed educationally, limiting their access to formal mathematics in higher grades (Clements & Sarama, 2014). Furthermore, although learning congruence is important for the growth of advanced mathematical thinking, its teaching has been traditionally relegated to middle school (Huang & Witz, 2011; Wu, 2005). However, prior research has shown that from birth to 7-8 years of age, children spontaneously develop Euclidean geometry knowledge about two-dimensional shapes including triangles (Shustermann, Lee & Spelke, 2008) as well as intuitive ideas of congruence (Clements & Sarama, 2014). This suggests that second-grade children could engage in informal reasoning about congruence and benefit from the early implementation of the concept as groundings for its future formal learning.

Researchers have stressed that utilizing digital interactive technologies in early childhood education can promote new ways of mathematical thinking in young learners (Clements & Sarama, 2014; Hegedus, 2013; Sinclair & Moss, 2012). The use of dynamic geometry software such as

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Geometer's Sketchpad® (Jackwic, 2009, hereon Sketchpad) could support young children's reasoning on properties of two-dimensional shapes and facilitate their access to more complex concepts (Sinclair, deFreitas & Ferrara, 2013; Ng & Sinclair, 2015). The addition of multi-touch devices could foster direct interaction with the mathematical configurations and collaborative behaviors that, in turn, could support the co-construction of shared mathematical meanings (Hegedus, 2013). This study posits that combining *Sketchpad* with the iPad through the application *Sketchpad®Explorer*, could enhance young children's learning experiences of congruence by helping them grasp in a dynamic way what means 'same shape and same size', so that they can link these properties to continuous geometric motions and to a variety of triangles. Moreover, children could work in small groups manipulating the dynamic shapes directly and simultaneously on the iPad, to also benefit from gestural expressivity and social interaction. Such an environment could help students overcome some of the learning challenges stated above. However, research on early learning of congruence is scarce. Furthermore, little is known about how the use of these digital multimodal technologies in small groups could benefit young children's development of foundations on congruence. This study aimed to design and implement a sequence of exploratory and problem-solving activities using *Sketchpad* on the iPad in order to examine the ways in which small groups of young learners reason about and understand congruence ideas while collaboratively working with dynamic representations of triangles. The question was: *How do small groups of second-grade children make sense of the concept of congruence within a collaborative Dynamic Multi-touch Geometry Environment?*

Theoretical Framework

This study is grounded on sociocultural theories of situated learning that see human activity as an integral part of the process of knowing that is mediated by both social interaction and cultural artifacts, such as digital interactive technologies. The theoretical framework of *semiotic mediation* related to the use of dynamic geometry environments and haptic technologies for the development of children's mathematical reasoning (Moreno-Armella, Hegedus, & Kaput, 2008; Hegedus, 2013; Sinclair & Moss, 2012) guided the research. The construct of *semiotic mediation* is central to understand how the use of multimodal technologies can nurture young children's co-construction of understandings about congruence. *Sketchpad* is a computer micro-world that enables users to continuously manipulate and transform, into a drawing-like space, a variety of geometrical objects that are pre-defined mathematically (Sinclair & Moss, 2012). Students can utilize the function tool *dragging* and, after any dynamic transformation, these objects preserve their defining mathematical properties, even if other characteristics vary. These affordances can mediate children's access to a variety of representations of mathematical objects and ways of thinking about the underlying properties (Hegedus, 2013; Sinclair & Moss, 2012). In this study, the dragging tool could mediate children's access to multiple representations of congruent triangles and the discovery of the underlying congruence relationship. Multi-touch horizontal tablets allow for physicality of learning, multiple inputs and co-location of students, facilitating small-group collaboration and haptic representations (Dillenbourg & Evans, 2011; Hegedus, 2013). Mediation of visual dynamic feedback and multi-touch input could foster young children's mathematical inquiry entailing reasoning and discovery, as they are able to conjecture and generalize while interacting with peers and the technology, as well as richer mathematical discourse, gestural expressivity, and understanding of geometric concepts such as congruence.

Methodology

The study entailed the design and implementation of an educational intervention strategy based on collaborative inquiry and problem solving within a dynamic multi-touch geometry environment (hereon DMGE). A sequence of seven activities was implemented in small groups of students for the

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early learning of congruence and similarity. Thirteen children (7-8 year olds) from five second-grade classrooms of a middle-SES public elementary school from Massachusetts, U.S., participated in the study. Children included girls and boys from various cultural backgrounds and were organized into five groups –two groups of two students and three of three students. This educational strategy was implemented as part of the afterschool program. A qualitative multiple-case study research approach was the method of inquiry to analyze small-group work on the tasks. This paper focuses on the three first activities of the sequence, designed to promote informal understandings of congruent triangles from a dynamic and multimodal perspective: Two exploratory activities (one task each one) and one problem-solving activity (three tasks). In Activity 1 and Activity 2, children were shown two congruent triangles of contrasting colors, and were asked to drag one of them and describe what happened with the other triangle. In Activity 1, both triangles could be continuously rotated, resized, and translated by dragging one of them, adopting different positions on the screen, but after any dynamic transformation the triangles always remained congruent (Figures 1a). In Activity 2, both triangles could be continuously transformed by dragging one of them, adopting different orientations and positions between them, but they always remained congruent (Figures 1b). In Activity 3, children were shown a referent triangle and a non-congruent triangle over a grid, and were asked to make the non-congruent triangle identical to the referent triangle (Figures 1c). This activity had three tasks with increasing degree of complexity based on the type of triangle (e.g. right, scalene). All the activities showed the area of each triangle at the top, which was called the Size Marker tool.

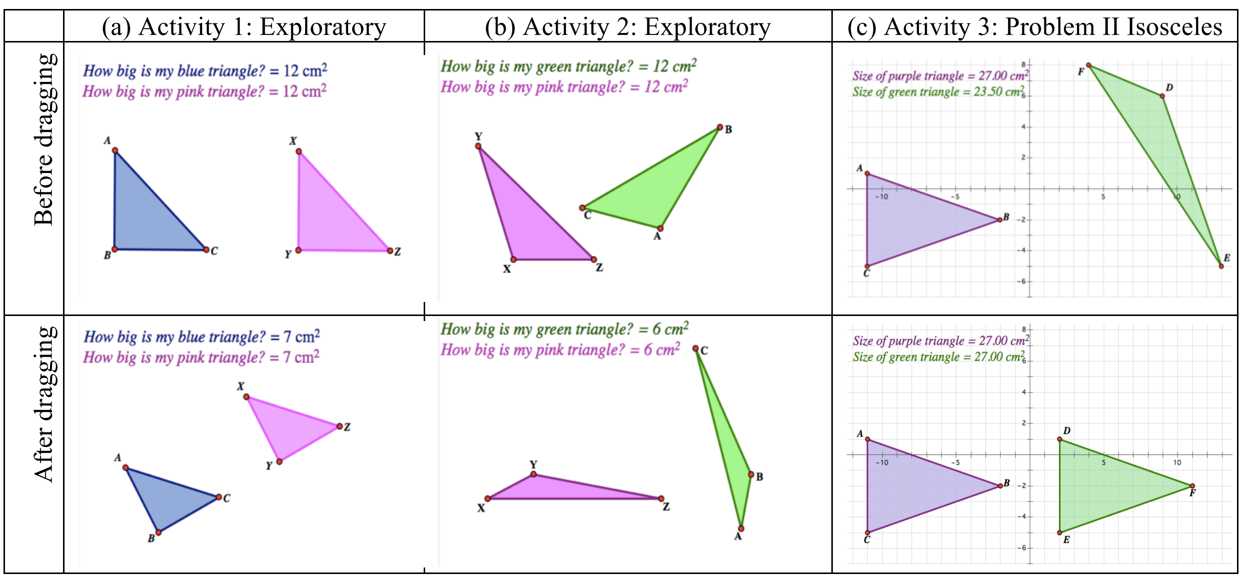

Figure 1. Sequence of exploratory and problem-solving activities for congruence.

The task-based interview with a semi-structured interview protocol was the primary data collection method. Each small group of children had one iPad with the activities developed in the DMGE and was observed and interviewed while solving each activity. The entire sequence of learning took place during four 1-hour sessions, once a week during four consecutive weeks, which were fully videotaped, transcribed and codified for analysis. Discourse analysis of children's interactions within each group was the data analysis method (Wells, 1999). The analytical framework included: (a) Children's ways of thinking about congruence (e.g. one or two relationships between attributes, discovering congruence invariance, measurement, representation of attributes), (b) Collaborative patterns, and (c) Uses of the technology. These aspects were analyzed from children's discourse -utterances, actions, gestures-. Coding consisted of a stepwise iterative process of seeking redundancy, using a first cycle-process coding method and a second cycle-pattern coding method.

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Results

Partial results from this study are presented in three sections regarding three emergent themes. These results are illustrated with excerpts from one case study correspondent to the analysis of Nathan and Kevin's discussions while interacting with each other, the researcher and the DMGE, in Activities 1 and 3. This group was selected because children planned the strategy in Activity 3 before using it, different to other groups. Actions are presented between braces, gestures underlined between brackets, and utterances are presented in normal format (between quotes only within the narrative).

Dynamism Mediated the Discovery of Congruence Relationships between Triangles

The first relevant finding of the study is that *dynamism* mediated young children's discovery of geometric relationships related to congruence of triangles within the DMGE. In Activity 1, I asked Kevin and Nathan: "I would like for you to drag the blue triangle (hereon BT) and tell me what happens with the pink triangle (hereon PT)". Initially, children showed an explorative use of the dragging function, systematically examining different continuous motions of BT such as turning around it, resizing it and, sliding it up and down, and observing the PT's behavior. When Kevin dragged BT up and down several times he began identifying *one relationship between attributes* of both dynamic triangles referred to their same movements as he said "Oh! Now when I move the triangle, if you move it up and down *{drags BT up-and-down}*, that one moves just up and down *{shows PT} [<u>moves right hand back-and-forth</u>]"*. Kevin' statement implied dynamism as he talked about the up-and-down motion of the triangles. Nathan began dragging BT, turning around several times and stretching it until the triangles got increasingly bigger or turning around and shrinking it until the triangles got increasingly smaller, while Kevin observed the screen. I had asked them to explore more, when the following discussion took place.

Excerpt 1. *Case Kevin and Nathan, Activity 1 (BT: Blue Triangle; PT: Pink Triangle).*

1 *Kevin*: Ok! *{Drags BT stretching and shrinking the triangles two times}* Oh! May be, I think when you move the blue triangle that makes the blue triangle bigger and then also that makes the pink triangle bigger and it also moves?
2 *Researcher*: Yeah? What do you think Nathan?
3 *Nathan*: Whenever you make the blue triangle bigger {*drags BT stretching the triangles*} or smaller {*drags BT shrinking the triangles*}, they both are always equal, the same size {*drags BT turning around several times*}
4 *Researcher*: Yes? Can you show me that?
5 *Nathan*: *{Drags BT stretching the triangles, shrinking the triangles, turning around the triangles, stretching the triangles, shrinking the triangles, translating the triangles}*
6 *Researcher*: What do you think Kevin about what Nathan says?
7 *Kevin*: Um, well like *{observes what Nathan does on the screen},* they're, yeah, they're always like the same size *{shows the triangles}* and they're, they both have the same lengths of edges *[<u>extends two hands as horizontal parallel lines</u>]*
8 *Researcher*: Can you show me that? I want to see
9 *Kevin*: Like they both, they both have the same lengths on the sides *{shows one side in PT; then shows the correspondent side in BT; then shows another side in PT and the correspondent side in BT; then shows the last side in PT and the correspondent side in BT}*

The Excerpt 1 shows that both children began inferring *two relationships between attributes* of the dynamic triangles such as same change of size and same type of movement, for instance when Kevin said "Oh! May be I think when you move the blue triangle that makes the blue triangle bigger and then also that makes the pink triangle bigger and it also moves?". They also discovered *two invariant relationships between attributes* of the dynamic triangles such as same change of size and same size, for instance when Nathan said "Whenever you make the blue triangle bigger or smaller, they both are always equal, the same size"; or same size and same lengths of the edges as when Kevin said "they're always like the same size and they're, them both have the same lengths of edges". Kevin and Nathan' statements implied dynamism as they talked about the triangles' movements or about their changes of size, which are attributes of dynamic triangles. This dynamism mediated the discovery of the invariant relationship related to "same size", which was evidenced in the use of words such as "whenever", "always" or "also", along with the words "same" and "equal" to make explicit the condition that the triangles are always equal because of the same size, independently of the type of movement. Although they did not say "same shape", Kevin talked about the "same length of edges" as another attribute different from size. Moreover, further on Kevin was able to specify which were the equal edges by showing them by pairs of congruent sides in both triangles. The children found out what was invariance in the activity and were able to formulate these relationships in their own words as a permanent rule of the two triangles. The Excerpt 1 also shows that the use of the technology evolved from exploratory to demonstrative. The children accompanied their statements with actions of dragging and gestures (e.g. pointing out) intentionally directed to demonstrate to each other and me, as the interviewer, what they were thinking. A collaborative behavior was seen when Kevin built on and extended Nathan's idea about "same size" (Line 7).

Gestures Mediated the Co-Planning of Strategies to Create Congruent Triangles

The second relevant finding is that gestural expressivity mediated young children's collective planning of strategies to create congruent triangles within the DMGE. Children used gestures on the iPad to represent a congruent triangle and properties of congruent triangles. The Excerpt 2 is a discussion from Activity 3, Task 2 (Level II: Isosceles) in which Kevin and Nathan planned a strategy to make the green triangle (GT) identical to the purple triangle (PT, see Figure 1c).

Excerpt 2. *Case Kevin and Nathan, Activity 3a (PT: Purple Triangle; GT: Green Triangle)*

1 *Nathan*: (To Kevin) We can move the F down *[points out from GT's point F towards a place down on the grid in front of PT's point A]* and then the D, the E over here *[points out from GT's point E towards a place down on the grid in front of PT's point B]* and then the D over here *[points out from GT's point D towards a place down on the grid in front of PT's point C]*.

2 *Kevin*: We have to equal up as there *{shows that the Size Marker of the referent triangle PT is 27 cm.}* so, that one *{shows PT}* is 27 centimeters *{shows PT' Size marker},* and that one…

3 *Nathan*: Yeah! 27, so we try to make it 27 *[nods]*

4 *Kevin*: That one is a little thinner so *{shows GT}* (Inaudible)

5 *Researcher*: How do you say Kevin?

6 *Kevin*: The purple triangle *{shows PT' Size Marker}* is like 27 centimeters *[extends 2 fingers on PT's area making a big space between fingers]* and that one is only 23.50 centimeters *[extends 2 fingers on GT's area making a smaller space between fingers]* and this one is a little skinnier *[extends 2 fingers on GT's area making a smaller space between fingers]*

7 *Researcher*: Yeah?

8 *Kevin*: Like it is skinnier *[extends two fingers on a thin area of a GT's angle leaving little space between fingers]* and then is a little fatter at the top *[extends two fingers on a wide area of a GT's angle leaving a wide space between fingers]* and then gets skinnier *[extends two fingers on another thin area of a GT's angle leaving little space between fingers]* and that one is a little fatter *[extends two fingers on a wide area of a PT's angle leaving a wide space between fingers]* than that one *[extends two fingers on a thin area of a PT's angle leaving little space between fingers]*.

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9 *Nathan*: It is skinnier over here *[extends two fingers on a thin area of a PT's angle leaving little space between fingers]* and fatter here, and then gets fatter *[extends two fingers on a wide area of another PT's angle leaving a wide space between fingers]*
10 *Kevin*: That one gets skinnier *[extends two fingers on an thin area of a GT's angle leaving little space between fingers]* and then gets skinnier again *[extends two fingers on another thin area of a GT's angle leaving little space between fingers]*. I just think that it's a little fatter *{shows PT}* than that one *{shows GT}*. We have to make it *{shows GT}* the same lengths *{shows 2 sides of PT}* and everything.
11 *Nathan*: Yeah! *[Nods]*

The Excerpt 2 shows how Nathan and Kevin explicitly discussed a strategy to create a congruent triangle before implementing it. The pre-planned strategy consisted of relocating the points of the GT and aligning them with the points of the PT (referent). Nathan used hand gestures to represent the trajectories of the three GT's points to become a congruent triangle as well as their new location on the grid just in front of the PT, displaying what I call an 'imagined' spatial representation of the new triangle. This representation implies dynamism, as it involves imagined trajectories of the points. It is also embodied, as Nathan used his fingers to show to Kevin the new positions of the points. It also implies informal processes of measuring, specifically, the estimation of distances among the points of the imagined triangle, and the alignment of the points of the two triangles. As a result, the imagined triangle had an approximate shape and size to the PT's shape and size. Kevin and Nathan also talked about another informal measuring component, equaling up the size of the two triangles through the use of tools such as the Size Markers of both triangles (Lines 2 & 3). When Kevin began talking softly about the triangles' attributes, I asked him to repeat what he said and he elaborated on his idea to explain it (Lines 4 & 5). The children began making explicit attributes of the referent and the non-congruent triangles by comparing them informally; for example, they compared their shapes using words such as "thinner", "skinnier" and "fatter". Simultaneously, they utilized hand gestures featured by the use of two fingers on the triangles and intended variation of the space between fingers, to represent the triangles' areas or the areas of their angles and to plan equaling up their sides' lengths (Lines 6, 8, 9 & 10). This process of co-planning the strategy revealed how children were aware of the relationships between attributes of the two triangles such as equal sizes and equal shapes. Uses of the technology were characterized by children's utilization of special tools to solve problems, for instance the grid to imagine the new triangle (Line 1), or the Size Markers to compare sizes (Lines 2, 3 & 6). Children were less explorative than in Activity 1; the use of their fingers was mainly demonstrative so that even without dragging they could represent both trajectories and properties of the triangles as discussed above. The Excerpt 2 also shows the emergence of a collaborative pattern to co-construct strategies. First, children proposed new ideas to each other, discussed actions of their strategy, and explained and justified their ideas demonstrating them to others through gestures (Lines 1, 6, 8, 9 & 10). Second, children built on each other's ideas. For instance, Kevin proposed an idea and Nathan adopted and extended it (Line 3 & 9), or Kevin expanded a Nathan's explanation and made a conclusion (Line10).

Informal Measuring as Mathematical Focus of the Co-Creation of Congruent Triangles

The third relevant finding is that young children used emergent informal ways of measuring as the mathematical focus to assure that two triangles had the same shape and size during the collective creation of congruent triangles. The Excerpt 3 presents a discussion from the process of implementation of Kevin and Nathan' strategy during Activity 3.

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Excerpt 3. *Case Kevin and Nathan, Activity 3b (PT: Purple Triangle; GT: Green Triangle)*

1 *Nathan*: *{Re-locates GT's point F on the grid in front of PT's point A, slides GT's points D and E towards left, slides back them towards right and closes the space between them}*
2 *Kevin*: *{Looks at Size Markers}* No! Try to make them both fit *{shows Size Markers; then makes GT taller, opening space between points D and E}* (Triangle is tall but thin)
3 *Nathan*: Ok! *{Makes GT fatter opening space between point F and points E and D; then makes GT taller opening space between D and F}* (…)
4 *Kevin*: (…) We have to align all with that (to Nathan) *{shows the points in PT}*
5 *Nathan*: I'm trying to *{makes GT taller, opening space between points D and E}*
6 *Kevin*: Let's make it bigger *{shows GT}* (GT is on the top-right while PT is on the bottom-left)
7 *Nathan*: I'm doing it bigger *{smoothly slides point E up-and-down on the grid many times}*
8 *Kevin*: Move, move this one *{shows point D}*
9 *Nathan*: You can't move the D. It's a thing!
10 *Kevin*: *{Slides points F and D towards left on the grid}*
11 *Nathan*: 26 *{looks at GT, looks at Size Markers}* (PT or referent is 27 cm.)
12 *Kevin*: *{Slides points F and D back towards right}* I will move D *{Keeps sliding point D}*
13 *Nathan*: 28 *{looks at GT, looks at Size Markers, shows Size Markers}* You cannot move D!
14 *Researcher*: What are you trying to do?
15 *Kevin*: We are trying, we are trying to make it equal, so is like the same *{drags GT }*
16 *Nathan*: One more time *{superposes GT's point D on PT's point A, GT's point E on PT's point C, and GT's point F on PT's point B}*. Then I will move this one over here *{slides GT's point F on the same line of PT's point B locating F in front of B; slides GT's point D on the same line of PT's point A locating D in front of A}*
17 *Kevin*: That one, there it goes! *{Looks at Nathan moving D}* You wanna make it exactly the same?
18 *Nathan*: Yes! *{Slides GT's point E on the same line that PT's point C locating E in front of C. With finger verifies that C and E are aligned measuring distance among them, and adjusts E}*
19 *Kevin*: Yeah! *{Looks at GT and looks at Size Markers, shows Size Markers}* Not, not, move! Ok!
20 *Nathan*: *{Adjusts point F one square on the grid}* Yeah! (GT has the same shape)
21 *Kevin*: 27 centimeters! *{Looks at Size Markers}* Ok! (Size Markers show triangles are 27 cm.)

The Excerpt 3 shows that children implemented and enhanced the strategy they had planned. Their first attempt consisted of relocating the points of PT on the grid one by one. However, they aligned just one of the GT's points with one of the PT's points, using it as the only spatial reference to locate the other points (Line 1). Then they adjusted the sides while checking permanently the Size Markers (Lines 2-13). The triangles were approximate but not exact in shape and size. The first informal way of measuring emerged when Kevin proposed to align all the points of both triangles (Line 4), but children did not reach an agreement (Lines 6, 7, 8 & 9). When I asked them what they were trying to do (Line 14), Kevin had clear their goal as he said "we are trying to make it equal, so is like the same". Nathan also restarted the task suggesting that he was aware that their strategy did not assure that the triangles get the same shape and size (Line 16). Further on, three informal methods of measuring emerged. First, Nathan superposed all the points of GT onto the points of PT, which produced a congruent triangle (Line 16). Second, he dragged each point of GT towards right, one by one, using the x-axis of the grid to align the points of both triangles on the same grid lines (Lines 16 & 18). Third, he adjusted the sides, measuring with his finger the distances between the points of both triangles (Lines 18 & 20). Superposition of two triangles, alignment of the points of two triangles using the x-axis and estimation of distances with fingers between the points aligned are informal ways of measuring that allowed him to correctly equal up the length of all the sides. Kevin

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was aware of the value of aligning, so he helped giving ideas and controlling the Size Markers (Lines 17, 19 & 21). The use of informal measuring and the regular monitoring of the Size Markers helped children construct a congruent triangle. The role of the technology was focused on the constructive use of the dragging tool and other tools provided to solve the task (e.g. Size Markers, grid, x-and-y axes). The collaborative pattern consisted of distributing and coordinating two functions: Nathan superposed, aligned, verified distances amongst and adjusted the points of GT while Kevin monitored the sizes of the two triangles in the Size Markers until they had equal shape and size. They also exchanged relevant information, suggesting ideas and building on each other's ideas.

Conclusions

This paper illustrated an educational strategy envisioned to cross borders among ages/grade levels for allowing young children in early childhood education (k-2) to access to complex concepts, such as congruence, that are traditionally taught at middle grades. The implementation of the strategy involving children's participation in activities designed in a DMGE, showed three relevant findings: (a) mediation of dynamism for discovering invariant geometric relationships between congruent triangles, (b) mediation of gestural expressivity for co-planning strategies to create congruent triangles, and (c) emergence of informal ways of measuring to assure that two triangles have the same shape and size. Children's collaborative patterns included the use of varied modes for conveying ideas (e.g. dragging actions, gestures), building on each other's ideas and distribution of tasks related to the exploration and the solution of the problems. Uses of the dragging tool had three functions: explorative, demonstrative and constructive. Children also used others tools such as the Size Markers and the grid. The implementation of DMGEs has critical implications for the learning of geometric concepts in both schools and informal settings such as afterschool programs.

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References

Dillenbourg, P. & Evans, M. (2011). Interactive tabletops in education. *Computer-Supported Collaborative Learning*, *6*(4), 491–514.

Clements, D. H. & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach* (2nd ed.). New York, NY: Routledge.

Hegedus, S. (2013). Young children investigating advanced mathematical concepts with haptic technologies: Future design perspectives. *The Mathematics Enthusiast*, *10*(1&2), 87-108.

Huang, H. M. & Witz, K. (2011). Developing children's conceptual understanding of area measurement: A curriculum and teaching experiment. *Learning and Instruction*, *21*, 1-13.

Jackiw, N. (2009). The Geometer's Sketchpad computer software (v. 5). Berkeley, CA: Key Curriculum Press.

Moreno-Armella, L., Hegedus, S. & Kaput, J. (2008). From static to dynamic mathematics: Historical and representational perspectives. *Educational Studies in Mathematics*, *68*, 99–111.

Ng, O. & Sinclair, N. (2015). Young children reasoning about symmetry in a dynamic geometry environment. *ZDM Mathematics Education*, *47*(3) DOI 10.1007/s11858-014-0660-5

Shusterman, A., Lee, V. & Spelke, E. (2008). Young children's spontaneous use of geometry in maps. *Developmental Science*, *11*, F1–F7.

Sinclair, N. & Moss, J. (2012). The more it changes, the more it becomes the same: The development of the routine of shape identification in dynamic geometry environment. *International Journal of Educational Research*, *51–52*, 28–44.

Sinclair, N., deFreitas, E. & Ferrara, F. (2013). Virtual encounters: The murky and furtive world of mathematical inventiveness. *ZDM Mathematics Education*, *45*, 239-252.

Wells, G. (1999). *Dialogic inquiry: Towards a sociocultural practice and theory of education*. Port Chester, NY: Cambridge University Press.

Wu, H.S. (2005, April). *Key mathematical ideas in grades 5-8.* Paper presented at the Annual Meeting of the NCTM, Anaheim, CA. Retrieved from: http://math.berkeley.edu/~wu/NCTM2005a.pdf