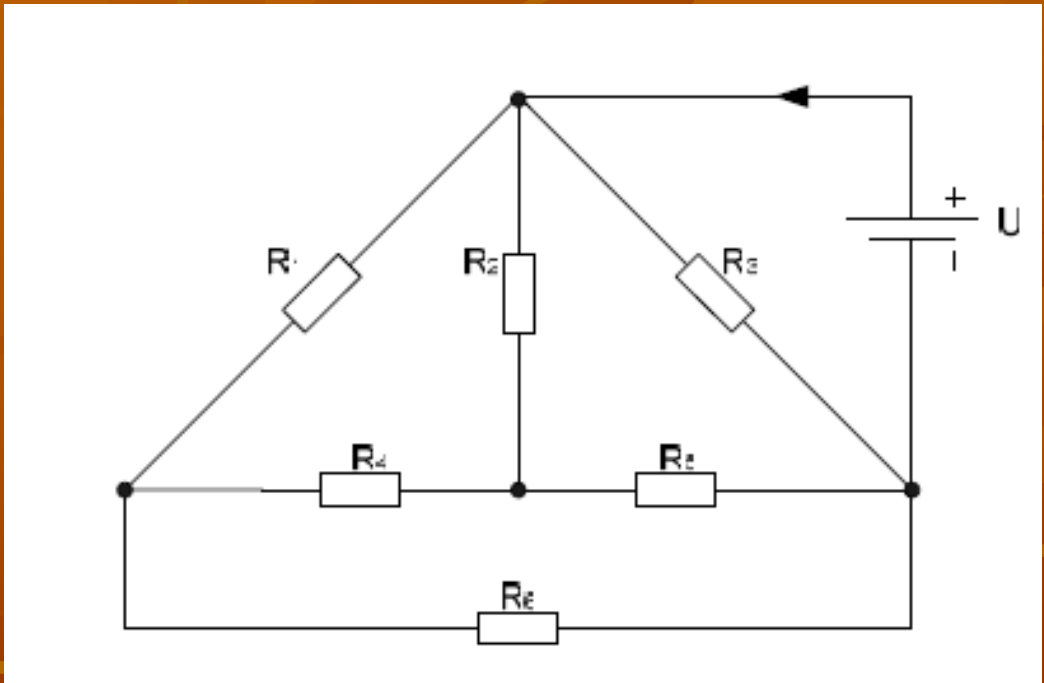
Zadaci i rješenja sa natjecanja iz Osnova elektrotehnike 2

Zadatke i rješenja pripremila škola Ruđer Bošković i škola u Daruvaru, prezentaciju izradio Zdravko Borić, prof.

- Izračunaj napon U, ako je zadano:

  $R_1=18\Omega$, $R_2=6\Omega$, $R_3=18\ \Omega$, $R_4=6\ \Omega$, $R_5=6\ \Omega$, $R_6=18\ \Omega$ i $I=2A$



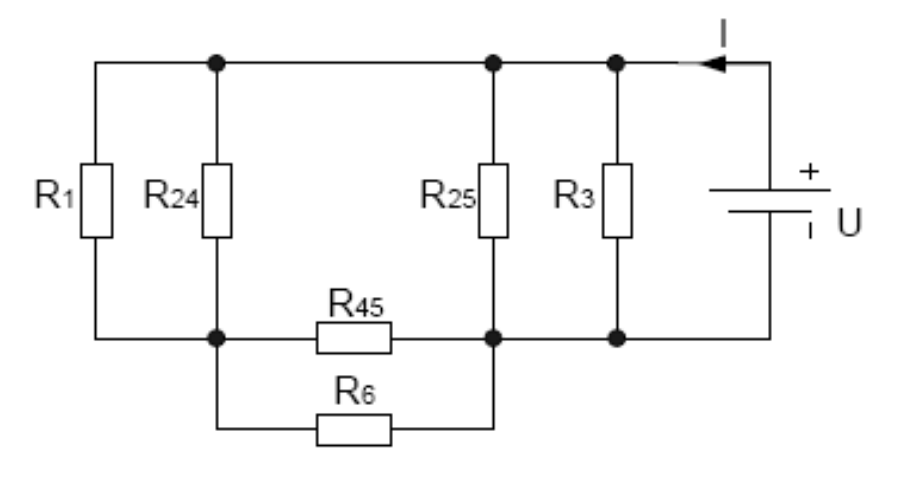
Spoj otpora R_2, R_4 i R_5 transfigurirati u spoj u trokut otpora R_{24}, R_{25} i R_{45}

$$\frac{1}{R_0}=\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}$$

$$R_0=2\Omega$$

$$R_{24}=\frac{R_2*R_4}{R_0}=18\Omega$$

$$R_{25}=R_{45}=R_{24}=18\Omega$$



$$R_{124} = \frac{R_1 * R_{24}}{R_1 + R_{24}} = 9\Omega$$

$$R_{325} = R_{645} = R_{124} = 9\Omega$$

$$R_{12456} = R_{124} + R_{645} = 18\Omega$$

$$R_{uk} = \frac{R_{325} * R_{12456}}{R_{325} * R_{12456}} = 6\Omega$$

$$U = I * R_{uk} = 12V$$

- Struja i(t)=0,5sin(*1000t*)A teče kroz serijski spoj R=30, L=10mH i C=20μF. Napiši izraze za trenutne vrijednosti $u_R(t)$,$u_C(t)$, $u_L(t)$, te $u(t)$. Prikaži vektorski struju i napone.

$$u_R(\ ) = 15\sin(1000t)\text{V}$$

$$X_L = \omega L = 10\Omega$$

$$u_L(t) = 5\sin(1000t + \frac{\pi}{2})\text{V}$$

$$X_C = \frac{1}{\omega C} = 50\Omega$$

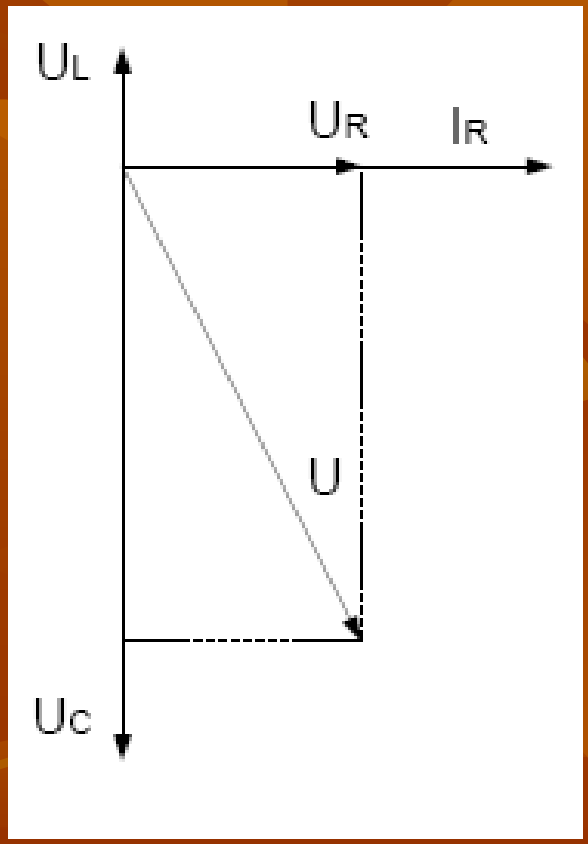
$$u_C(t) = 25\sin(1000t - \frac{\pi}{2})\text{V}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 50\Omega$$

$$\cos\varphi = \frac{R}{Z} = 0{,}6$$

$$\varphi = 53^\circ$$

$$u(t) = 25\sin(1000t - 53^\circ)\text{V}$$



- Odredi trenutne vrijednosti napona $u(t)=150\sin(628t)V$ u trenucima $\Delta t_1=0{,}5ms$, $\Delta t_2=2ms$ i $\Delta t_3=4ms$ nakon prolaza napona kroz maksimalnu vrijednost. Nacrtaj vremenski prikaz napona.

$$f=\frac{\omega}{2\pi}=100Hz$$

$$T=10ms$$

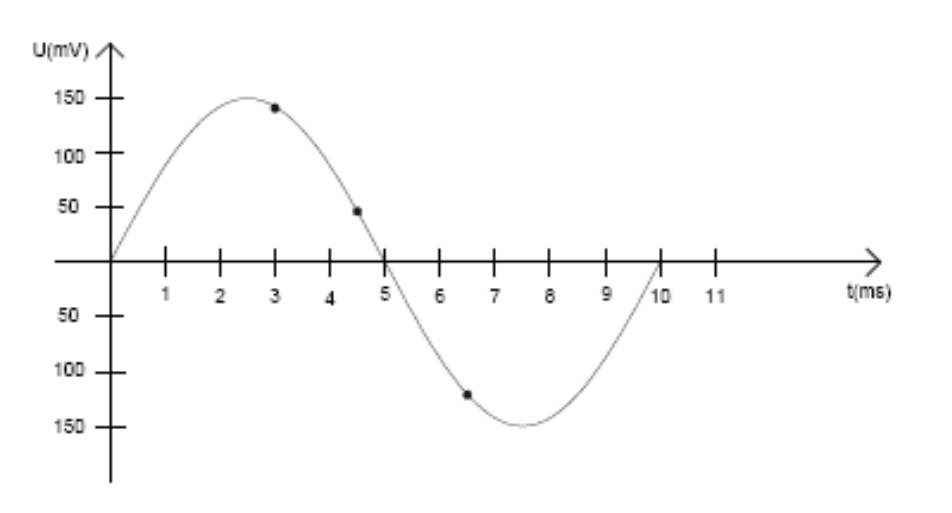
Maksimalna vrijednost napona je u trenutku

$$\frac{T}{4}=2{,}5ms$$

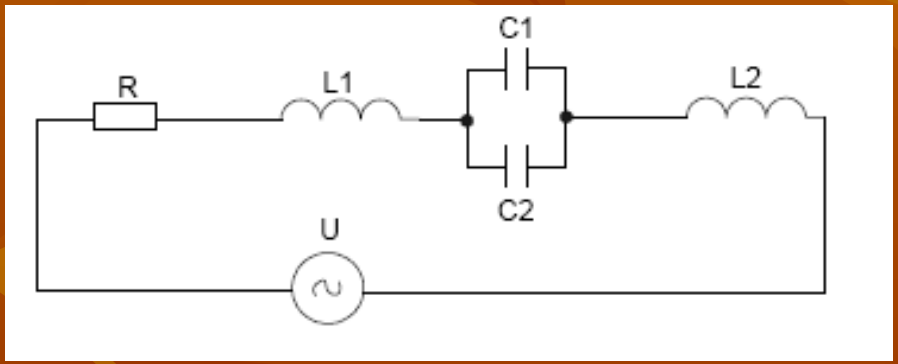
$$\Delta t_1+\frac{T}{4}=3ms \qquad u(3ms)=142{,}7V$$

$$\Delta t_2+\frac{T}{4}=4{,}5ms \qquad u(4{,}5ms)=46{,}6V$$

$$\Delta t_3+\frac{T}{4}=6{,}5ms \qquad u(6{,}5ms)=-121{,}2V$$



- Odredi frekvenciju kod koje će krug biti u rezonanciji, te izračunaj struju i prividni otpor kod :
  a) rezonantne frekvencije
  b) frekvencije $f_1=f_0/2$
  c) frekvencije $f_2=2f_0$



$R = 100\Omega$

$L_1 = 20\text{mH}$

$L_2 = 20\text{mH}$

$C_1 = 60\text{nF}$

$C_2 = 30\text{nF}$

$U = 10\text{V}$

$$C = C_1 + C_2 = 90\text{nF}$$
$$L = L_1 + L_2 = 40\text{mH}$$
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 2654\text{Hz}$$

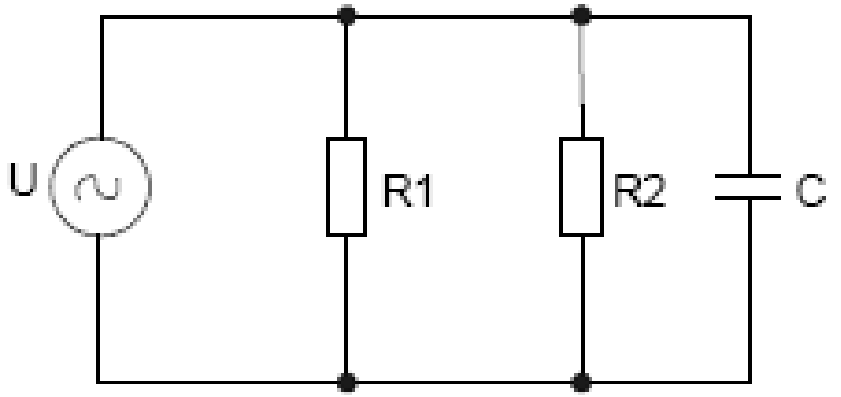
a)
$$Z = R$$
$$I = \frac{U}{R} = 0{,}1\text{A}$$

b)
$$f_1 = 1327\text{Hz}$$
$$X_L = \omega L = 333\Omega$$
$$X_C = \frac{1}{\omega C} = 1333\Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 1005\Omega$$
$$I = \frac{U}{Z} = 9{,}95\text{mA}$$

c)
$$f_2 = 5308\text{Hz}$$
$$X_L = 1333\Omega$$
$$X_C = 333\Omega$$
$$Z = 1005\Omega$$
$$I = 9{,}95\text{mA}$$

- Odredi vrijednost otpora R_1 tako da fazni kut izmenu napona i ukupne struje bude 30° .

$$U = 10\angle 0^\circ$$
$$f = 1\text{kHz}$$
$$R_2 = 47\text{k}\Omega$$
$$C = 10\text{nF}$$



$$X_C = \frac{1}{2\pi fC} = 15{,}9\text{k}\Omega$$
$$B_C = \frac{1}{X_C} = 62{,}8\mu\text{S}$$
$$\text{tg}\varphi = \frac{B_C}{G}$$

$$G = \frac{B_C}{\text{tg}\varphi} = 109\mu\text{S}$$
$$G_2 = \frac{1}{R_2} = 21\mu\text{S}$$
$$G_1 = G - G_2 = 88\mu\text{S}$$
$$R_1 = \frac{1}{G_1} = 11{,}4\text{k}\Omega$$

- Tri jednaka trošila Z=4500 Ω vezana su u trokut i priključena na trofaznu mrežu 220/380V.

  a) koliki trebaju biti otpori trošila vezanih u zvijezdu da bi linijske struje u oba slučaja bile jednake

  b) izračunaj snage za oba slučaja; spoj u trokut, spoj u zvijezdu

.a)

$$I_L = \frac{U_L}{Z} \cdot \sqrt{3} = 0{,}146\text{A}$$

$$I_L' = \frac{U_f}{Z'}$$

$$Z' = \frac{U_f}{I_L} = 1506\Omega$$

b)

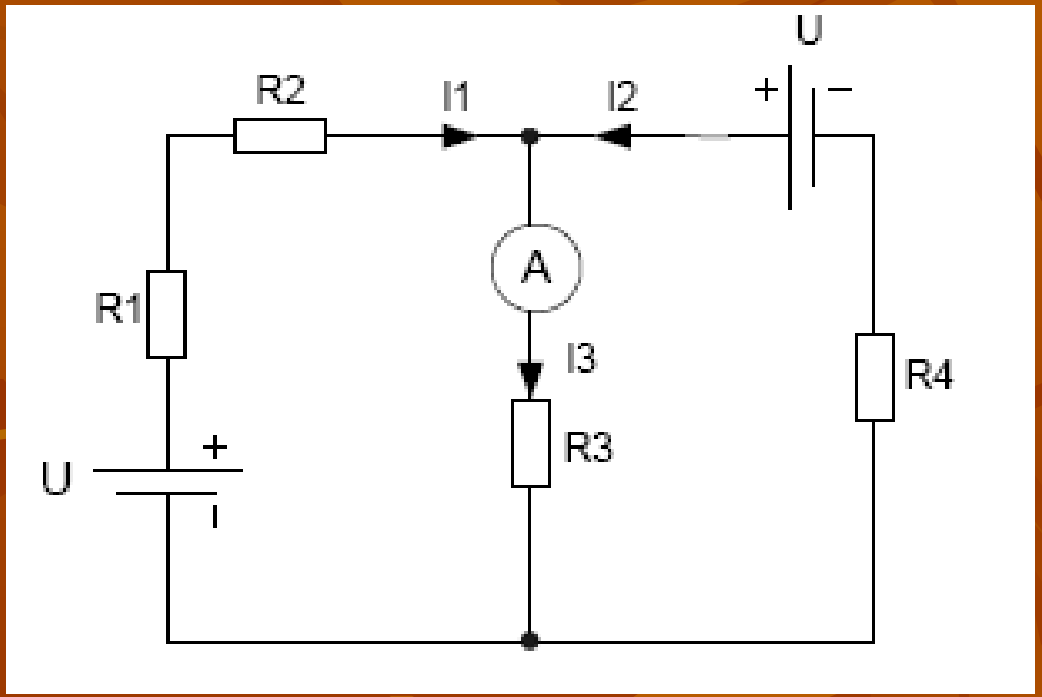
$$P = \sqrt{3} \cdot U_L \cdot I_L$$

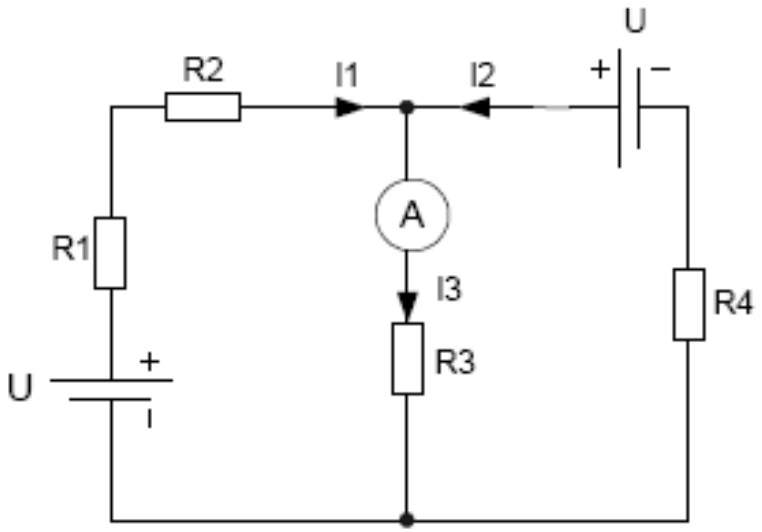
$$P_\Delta = 3I_F \cdot U_L = 3 \cdot \frac{I_L}{\sqrt{3}} \cdot U_L = \sqrt{3} \cdot 0{,}146 \cdot 380 = 96{,}1\text{W}$$

$$P_\lambda = 3 \cdot I_L \cdot U_F = 3 \cdot I_L \cdot \frac{U_L}{\sqrt{3}} = \sqrt{3} \cdot 0{,}146 \cdot 380 = 96{,}1\text{W}$$

- Direktnom primjenom Kirchhoffovih zakona izračunaj struje I_1 i I_2 , te napone U, ako idealan ampermetar pokazuje 3A smjera prema slici.

$R_1=1k\Omega$
$R_2=1k\Omega$
$R_3=3.33k\Omega$
$R_4=1k\Omega$





$$I_1 + I_2 = I_3 \Rightarrow I_1 = 3 - I_2$$

$$U = I_1(R_1 + R_2) + I_3 R_3$$

$$U = I_2 R_4 + I_3 R_3$$

$$U = 2I_1 + 10$$

$$U = I_2 + 10$$

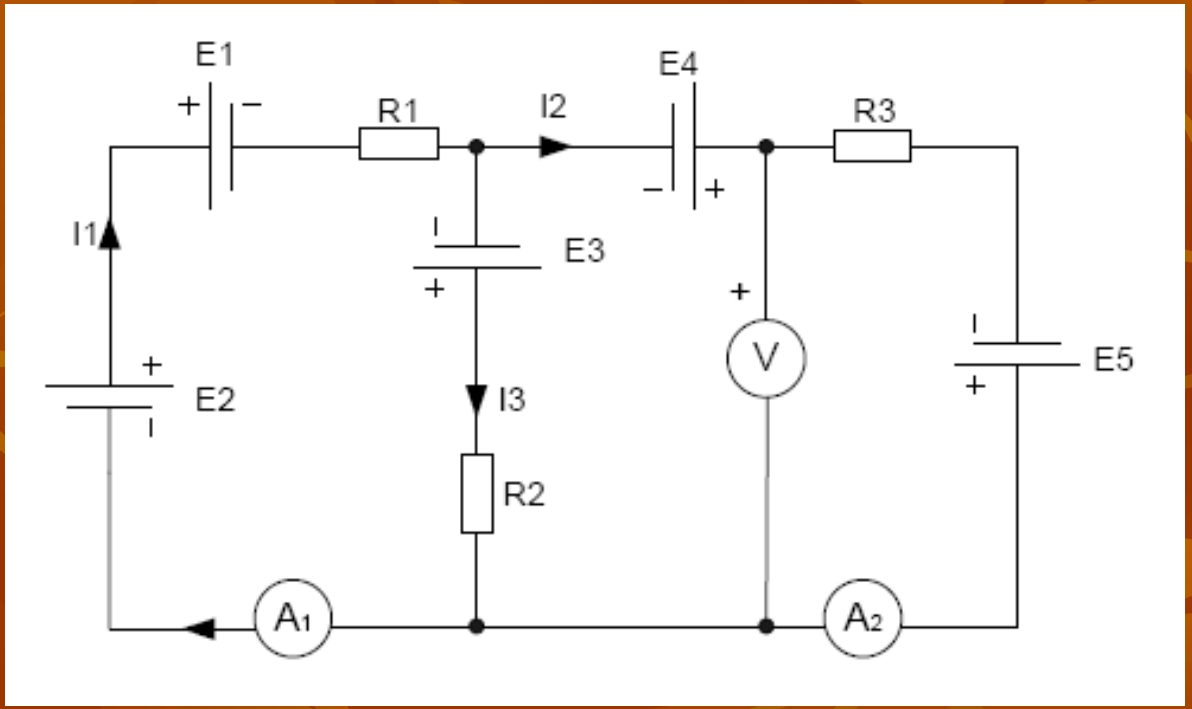
$$I_2 = 2A$$

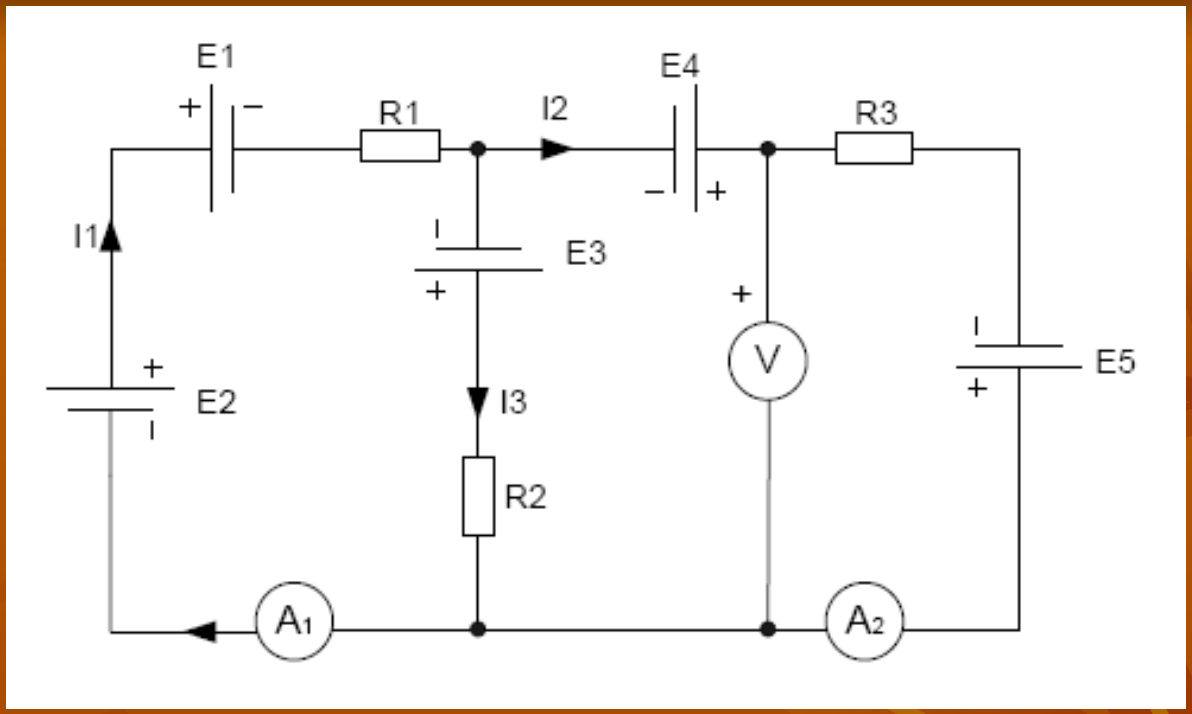
$$I_1 = 1A$$

$$U = 12V$$

- Primjenom metode konturnih struja izračunaj struju I2 te napone izvora E_1 i E_5, ako idealni ampermetar A_1 pokazuje 3A i idealni voltmetar 20V.

  Zadano je: E_2=90V, E_3=30V, E_4=40V, R_1=5Ω, R_2=10Ω, R_3=20Ω





$$I_I = 3A$$

$$U_V + E_5 = -I_{II} * 20 \Rightarrow I_{II} = -1 - \frac{E_5}{20}$$

$$E_2 - E_1 + E_3 = I_I(R_1 + R_2) + I_{II}R_2$$

$$-E_5 - E_4 + E_3 = I_{II}(R_3 + R_2) + I_I R_2$$

$$120 - E_1 = 15 * 3 + 10I_{II}$$

$$-E_5 - 10 = 30I_{II} + 10 * 3$$

$$E_5 = 20V$$

$$E_1 = 95V$$

$$I_2 = -I_{II} = 2A$$

- U paralelnom RLC spoju poznata je jalova snaga Q=200VAr, izvor napona U=50V i frekvencije 50Hz, R=5 i X_L=10. Izračunaj:

  a) sve struje

  b) kapacitivni otpor i kapacitet

  c) admitanciju

  d) fazni kut

  Nacrtaj:

  e) vektore struja

a)

$$I_R = \frac{U}{R} = 10A$$

$$I_L = \frac{U}{X_L} = 5A$$

$$Q = \frac{U^2}{X_L} - \frac{U^2}{X_C}$$

$$\frac{1}{X_C} = \frac{Q}{U^2} - \frac{1}{X_L}$$

$$I_C = \frac{U}{X_C} = 1A$$

$$I = \sqrt{I_R{}^2 + (I_L - I_C)^2} = 10,77A$$

b) $X_C = 50\Omega$

$C = 63,67\mu F$

c)

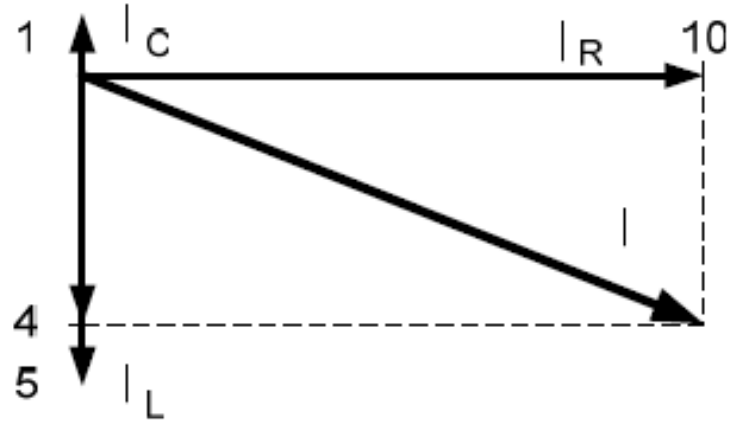
$$Y = \frac{I}{U} = 0,215s$$

d)

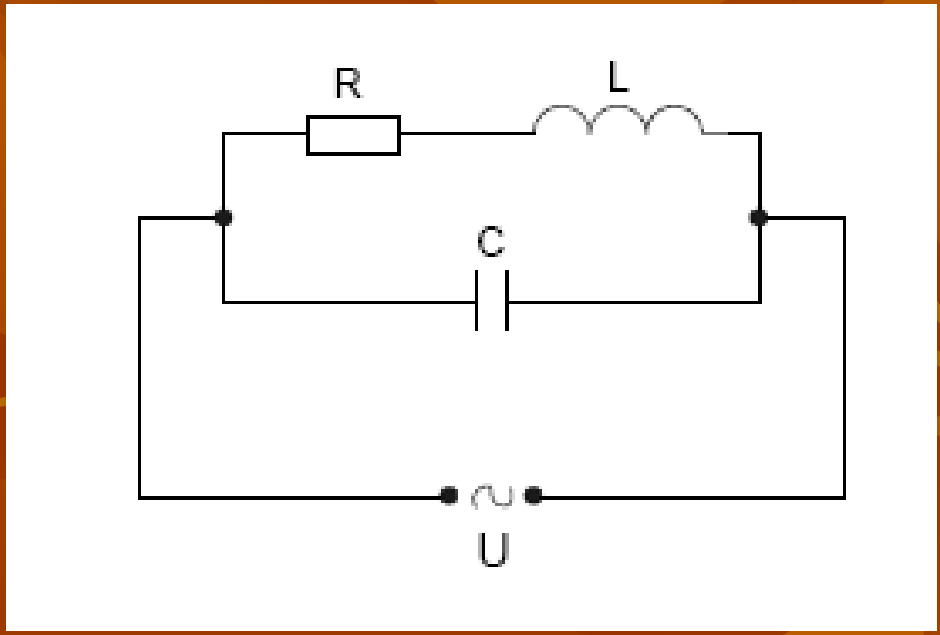
$$tg\rho = \frac{I_C - I_L}{I_R} = -0,4$$

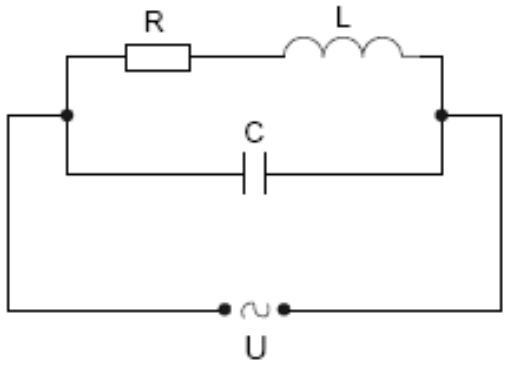
$$\rho = -21.8^\circ$$

e)



- Koliki treba biti kapacitet spojenog kondenzatora da bi krug bio u rezonanciji pri frekvenciji 500Hz, ako je R=20, L=10mH i U=220V?





$$X_L = 2\pi fL = 31{,}4\Omega$$

$$Z_{RL} = \sqrt{R^2 + X_L^2} = 37\Omega$$

$$I_{RL} = \frac{U}{Z_{LR}} = 5{,}9A$$

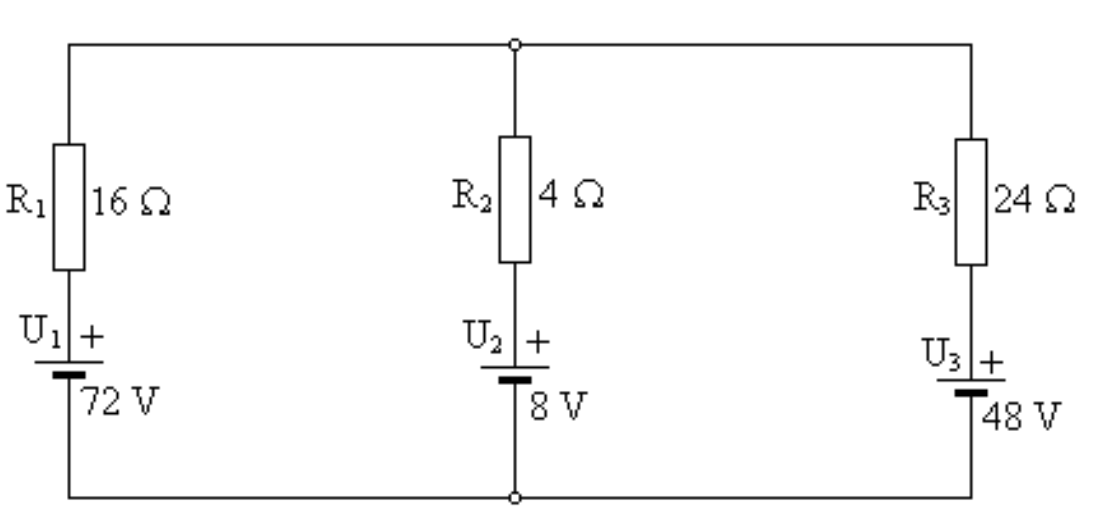
$$Q_L = I_{RL}^{\ 2} \cdot X_L = 1093VAr$$

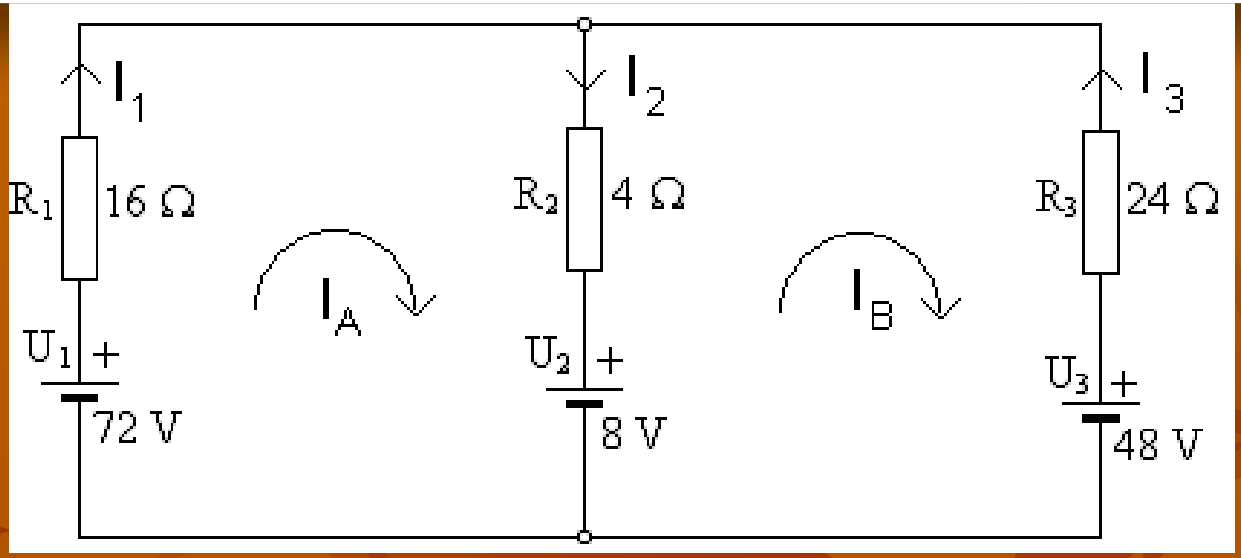
$$Q_C = Q_L$$

$$X_C = \frac{U^2}{Q_C} = 44{,}3\Omega$$

$$C = \frac{1}{\omega X_C} = 7{,}18\mu F$$

- Odredi sve struje metodom konturnih struja.





$U_1 - U_2 = I_A (R_1 + R_2) - I_B * R_2$

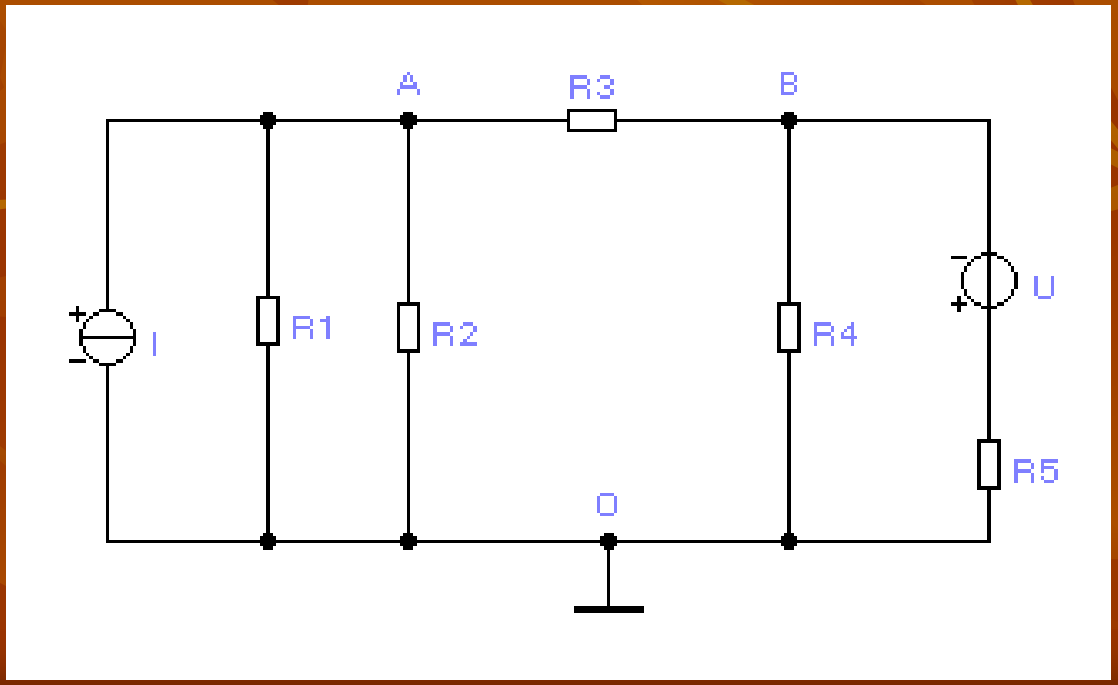
$U_2 - U_3 = -I_A * R_2 + I_B (R_2 + R_3)$

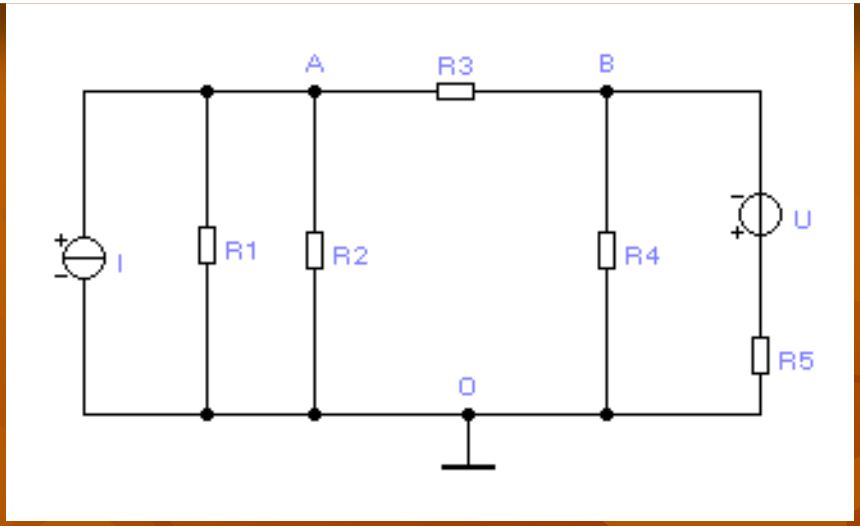
$64 = 20I_A - 4I_B$

$-40 = -4I_A + 28I_B$ $\quad I_A = 3A$ $\quad I_B = -1A$

$\underline{I_1 = I_A = 3A}$ $\quad \underline{I_3 = -I_B = 1A}$ $\quad \underline{I_2 = I_A - I_B = 4A}$

- Primjenom Theveninovog teorema odredite Theveninov napon U_T i otpor R_T između točaka A i B i jakost struje I_3 kroz otpornik R_3=300Ω. Svi ostali otpornici imaju iznos otpora 600Ω, jakost struje strujnog izvora je I=1.8A a naponski izvor ima vrijednost napona U=90V.





Theveninov otpor iznosi:

$$R_T = \frac{R_1 * R_2}{R_1 + R_2} + \frac{R_4 * R_5}{R_4 + R_5}$$

$$R_T = 300 + 300 = 600\Omega$$

Theveninov napon je napon između točaka A i B:

$$E_T = U_{AB} = U_{AO} + U_{BO}$$

$$U_{AO} = I * R_{12} = 1.8 * 300 = 540V$$

$$U_{BO} = \frac{U}{R_4 + R_5} * R_4 = \frac{90}{600 + 600} * 600 = 45V$$

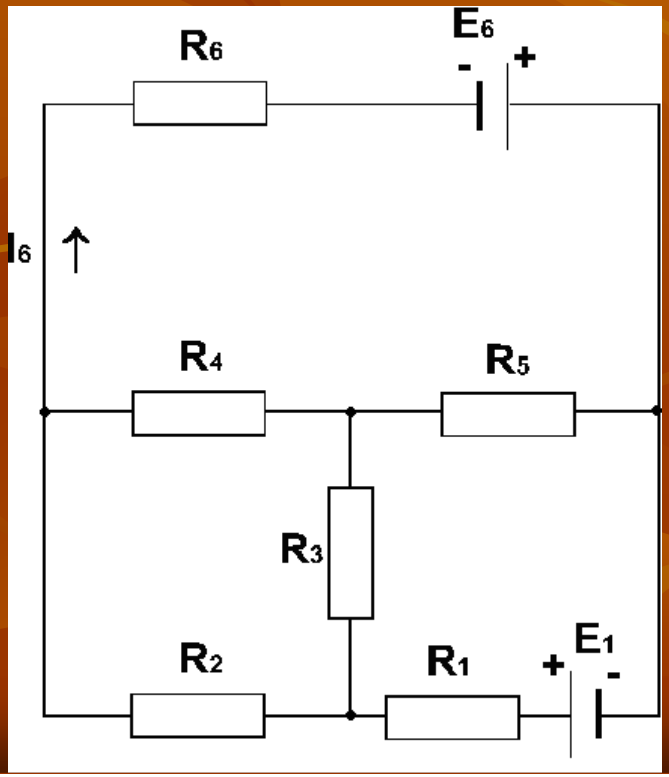
$$E_T = 540 + 45 = 585V$$

Jakost struje kroz otpornik R_3 iznosi:

$$I_3 = \frac{E_T}{R_T + R_3} = \frac{585}{600 + 300}$$

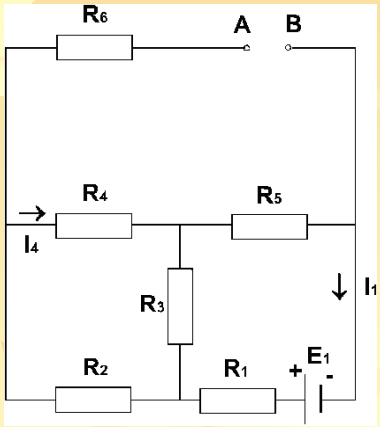
$$I_3 = 0.65A$$

- Za mrežu na slici poznato je: E_1=100 V, R_1=R_2=150 Ω; R_3=R_4=R_5=50 Ω; R_6=25 Ω I_6=0,1 A. Primjenom Theveninovog teorema potrebno je odrediti elektromotornu silu E_6, Theveninov napon U_T i Theveninov otpor R_T, gledano s priključnica izvora E_6.



$$I_1 = \frac{E_1}{\frac{(R_2 + R_4)\cdot R_3}{R_2 + R_3 + R_4} + R_1 + R_5} = \frac{100}{\frac{(150+50)\cdot 50}{150+50+50} + 150 + 50} = \frac{5}{12}$$

$$I_1 \approx 0{,}42A$$



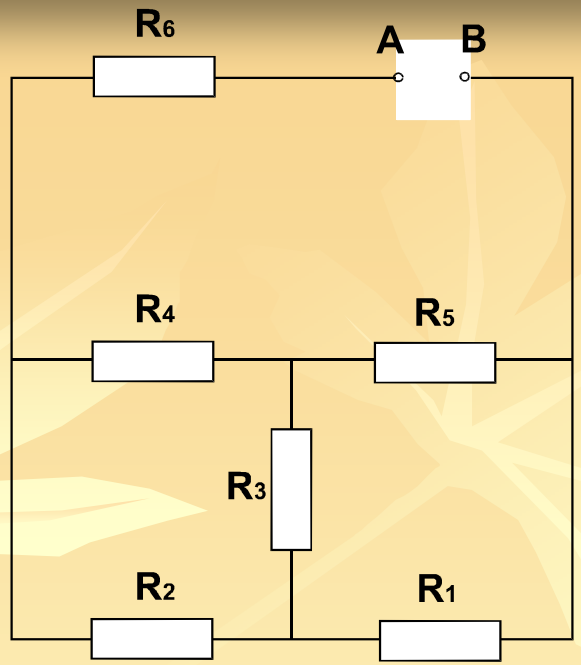
$$I_4 = \frac{R_3}{R_2 + R_4 + R_3}\cdot I_1 = \frac{50}{150+50+50}\cdot\frac{5}{12} = \frac{1}{12}$$

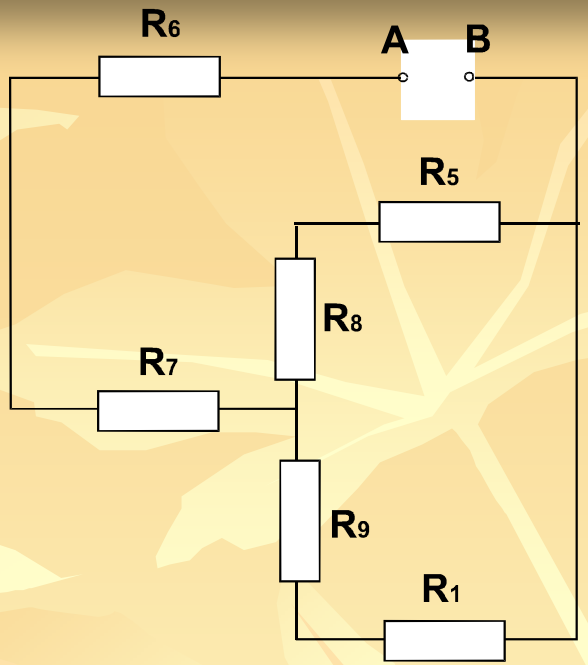
$$I_4 = 0{,}083A$$

$$U_T = U_{AB} = R_5 I_1 + R_4 I_4$$

$$U_T = U_{AB} = 50\cdot\frac{5}{12} + 50\cdot\frac{1}{12} = \frac{50}{2}$$

$$U_T = U_{AB} = 25V$$





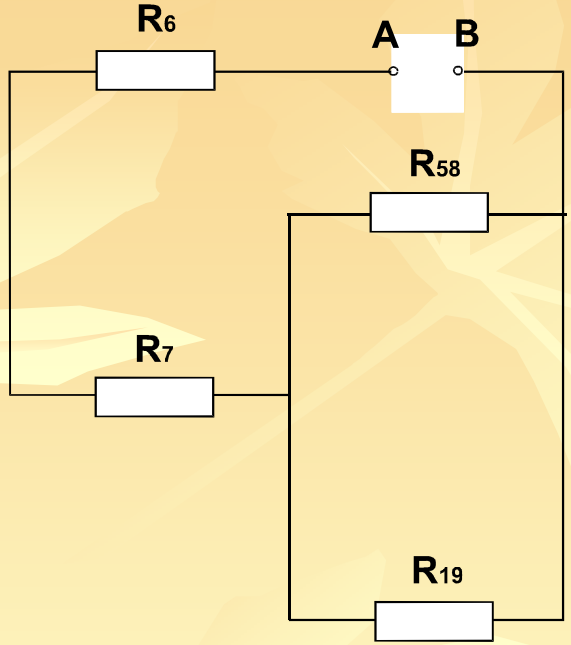
Transfiguriranjem trokuta R_2, R_3, R_4 u zvijezdu R_7, R_8, R_9 imat ćemo:

$$R_7 = \frac{R_2 R_4}{R_2 + R_3 + R_4} = \frac{150 \cdot 50}{150 + 50 + 50} = 30\Omega$$

$$R_8 = \frac{R_3 R_4}{R_2 + R_3 + R_4} = \frac{50 \cdot 50}{150 + 50 + 50} = 10\Omega$$

$$R_9 = \frac{R_2 R_3}{R_2 + R_3 + R_4} = \frac{150 \cdot 50}{150 + 50 + 50} = 30\Omega$$

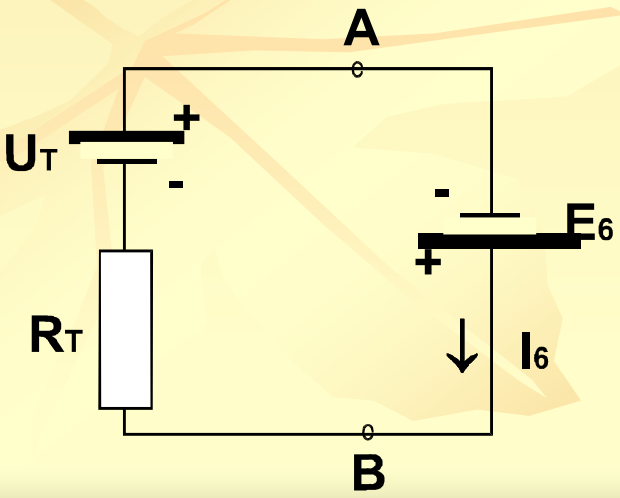
Sada je ekvivalentni Theveninov otpor:



$$R_T = R_{AB} = R_6 + R_7 + \frac{R_{58}R_{19}}{R_{58} + R_{19}} =$$

$$R_6 + R_7 + \frac{(R_5 + R_8)(R_9 + R_1)}{R_5 + R_8 + R_9 + R_1}$$

$$R_T = R_{AB} = 100\Omega$$

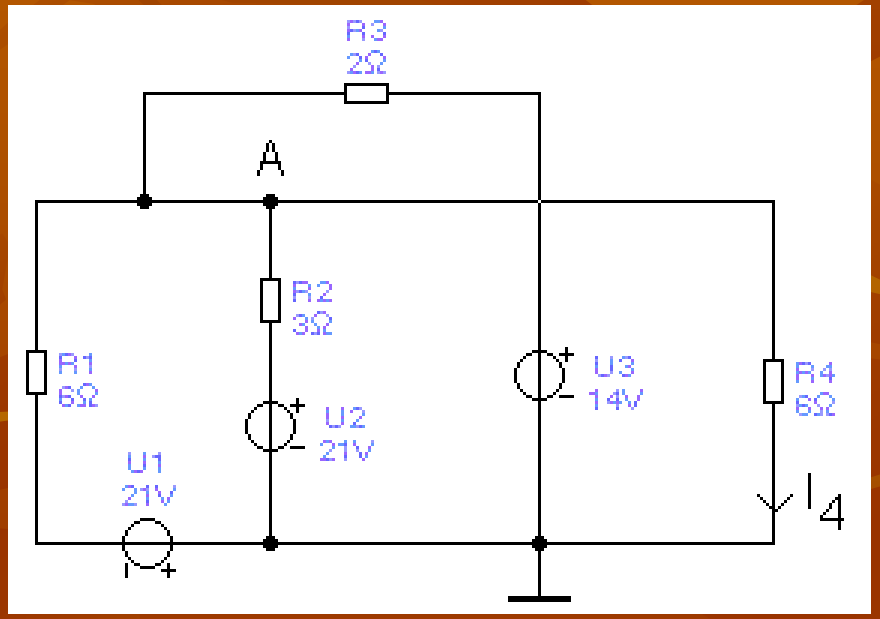


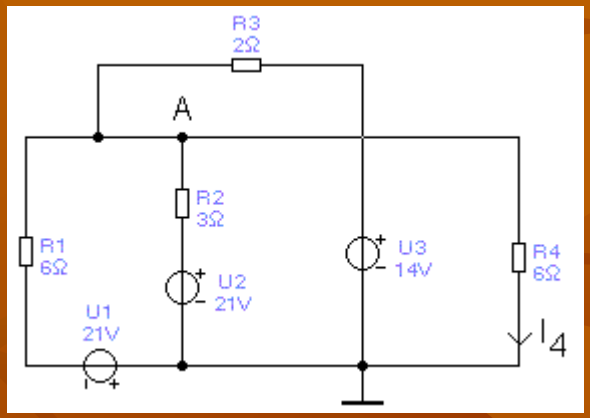
$$E_6 = R_T I_6 - U_T$$

$$E_6 = 100 \cdot 0{,}1 - 25$$

$$E_6 = -15V$$

- Metodom napona čvorova odredi potencijal φ_A točke A, struju I_4 kroz otpor R_4 u mreži prema slici.



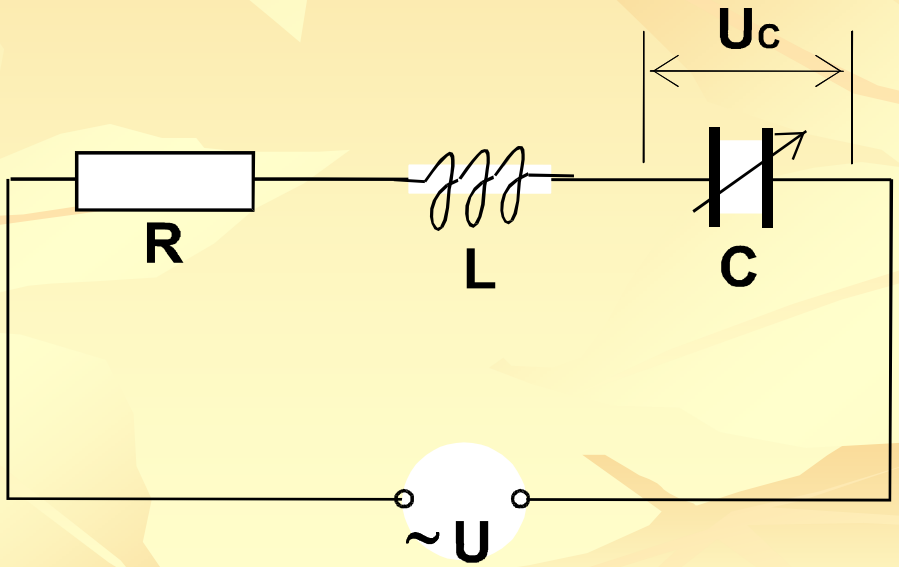


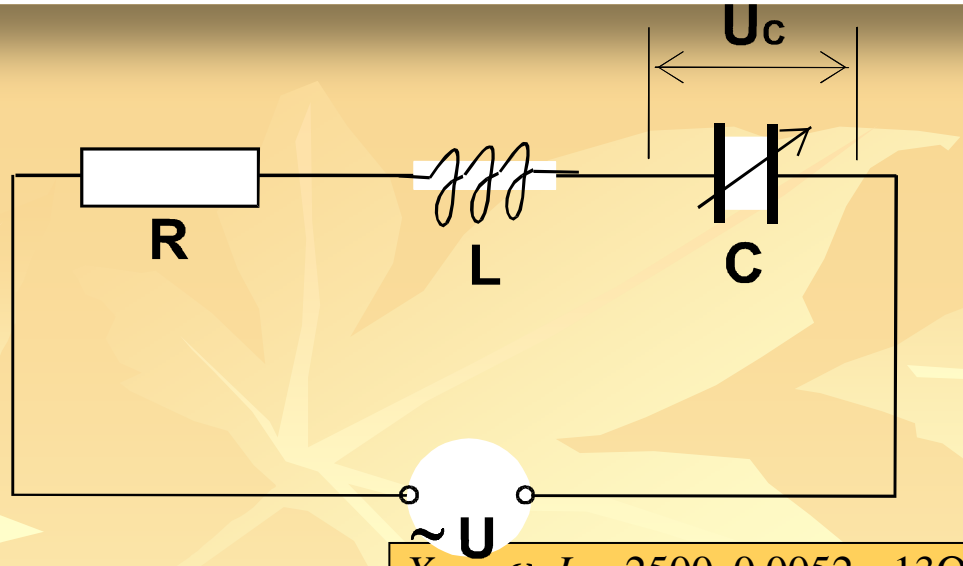
$$I_1 = \frac{\varphi + 21}{6} \qquad I_2 = \frac{21 - \varphi}{3} \qquad I_3 = \frac{14 - \varphi}{2} \qquad I_4 = \frac{\varphi}{6}$$

$I_1 + I_4 = I_2 + I_3$ $\qquad \dfrac{\varphi + 21}{6} + \dfrac{\varphi}{6} = \dfrac{21 - \varphi}{3} + \dfrac{14 - \varphi}{2}$ $\qquad \underline{\varphi = 9V}$

$I_1 = 5A$ $\quad I_2 = 4A$ $\quad I_3 = 2{,}5A$ $\quad \underline{I_4 = 1{,}5A}$

- Serijski spoj s omskim otporom R=6 Ω, induktivitetom L=5,2 mH te promjenjivim kapacitetom C služi kao djelitelj napona frekvencije ω=2500 rad/s. Kod kojeg je kapaciteta C napon na kondenzatoru jednak polovici narinutog napona?





$$U : U_C = Z : X_C$$

$$U_C = \frac{U}{2} \quad \Rightarrow \quad X_C = \frac{Z}{2}$$

$$X_L = \omega \cdot L = 2500 \cdot 0{,}0052 = 13\Omega$$

$$X_C = \frac{Z}{2} \quad \Rightarrow \quad Z = 2X_C$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$(2X_C)^2 = R^2 + X_L^2 - 2X_L X_C + X_C^2$$

$$4X_C^2 - X_C^2 = R^2 + X_L^2 - 2X_L X_C$$

$$3X_C^2 + 2X_L X_C - (R^2 + X_L^2) = 0$$

$$3X_C^2 + 2 \cdot 13 X_C - (6^2 + 13^2) = 0$$

$$3X_C^2 + 26X_C - 205 = 0$$

$$X_{C1} = \frac{-26 + \sqrt{26^2 + 4 \cdot 3 \cdot (-205)}}{2 \cdot 3} \qquad X_{C2} = \frac{-26 - \sqrt{26^2 + 4 \cdot 3 \cdot (-205)}}{2 \cdot 3}$$

$$X_{C1} = \frac{-26 + 56}{6} \qquad X_{C2} = \frac{-26 - 56}{6}$$

Rješavanjem kvadratne jednadžbe dobiju se dva rješenja:

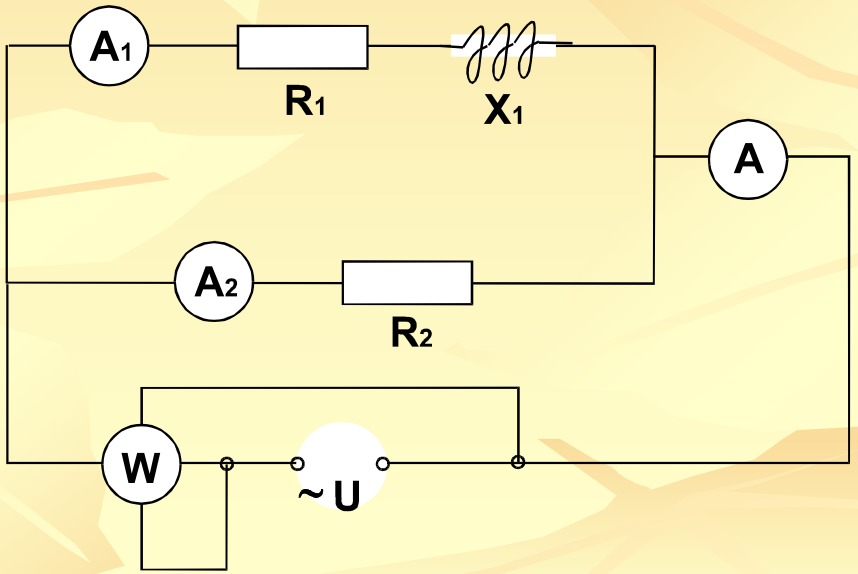
$$X_{C1} = 5\Omega$$

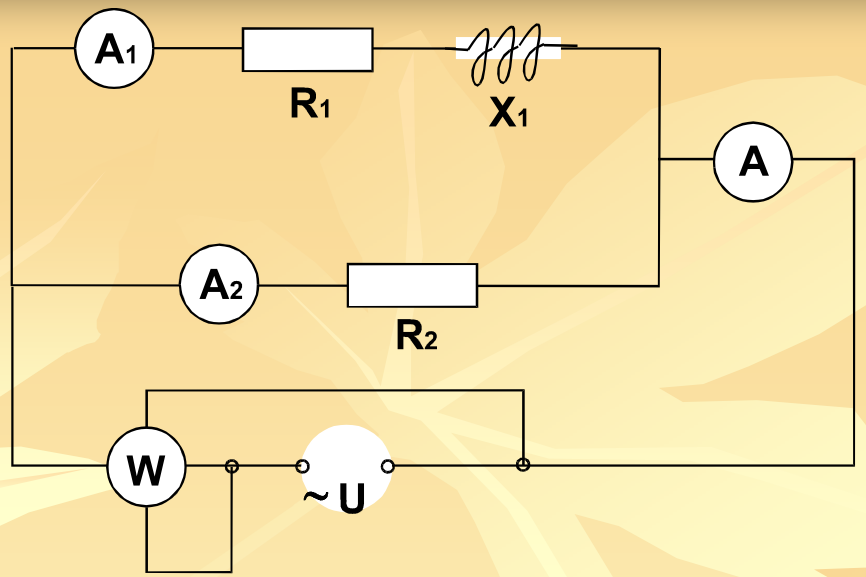
$$X_{C2} \approx -13{,}67\Omega$$

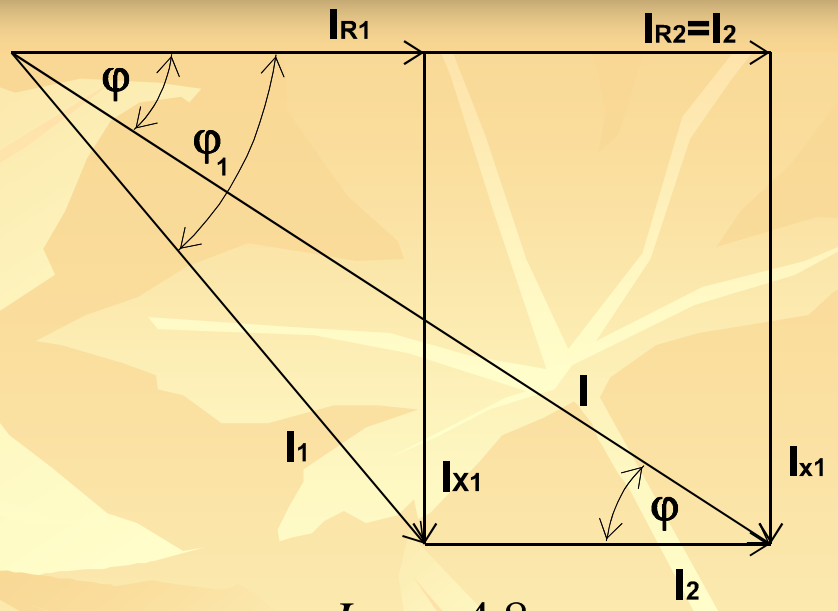
Negativni predznak drugog rješenja pokazuje da bi i induktivni otpor istog iznosa dao, također polovicu priključenog napona.

$$C = \frac{1}{\omega X_{C1}} = \frac{1}{2500 \cdot 5} = 80\mu F$$

- U zadanom paralelnom spoju izmjereno je: I_1=6 A; I_2= 2,8 A; I=8 A; cosφ=0,8; P=1344 W Potrebno je: a) odrediti faktor snage cosφ$_1$ prve grane; b) odrediti impedanciju Z_1 i njezine komponente R_1 i X_1, te omski otpor R_2







$$\frac{I_{X1}}{I} = \sin\varphi$$

$$I_{X1} = I \cdot \sin\varphi$$

$$\cos\varphi = 0{,}8 \quad \Rightarrow \quad \varphi = \arccos 0{,}8$$

$$\varphi = 36{,}87^0$$

$$\sin 36{,}87^0 = 0{,}6$$

$$I_{X1} = 8 \cdot 0{,}6 = 4{,}8A$$

$$\sin\varphi_1 = \frac{I_{X1}}{I_1} = \frac{4{,}8}{6} = 0{,}8$$

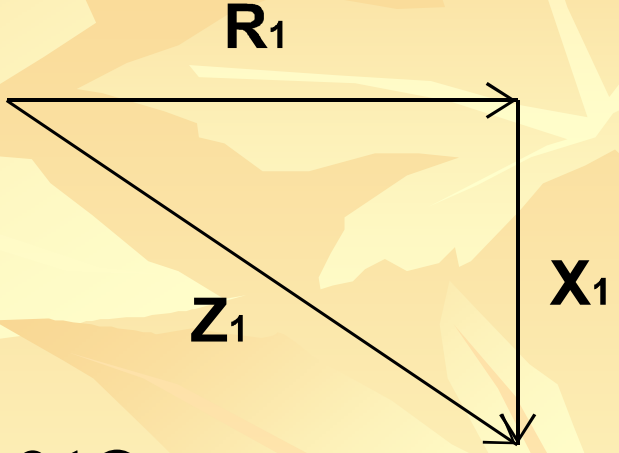
$$\varphi_1 = \arcsin 0{,}8 = 53{,}13^0$$

Faktor snage prve grane:

$$\cos\varphi_1 = \cos 53{,}13^0 = 0{,}6$$

b)

$$X_1 = Z_1 \sin\varphi_1 = 35 \cdot 0{,}8 = 28\Omega$$



$$P = UI\cos\varphi$$

$$U = \frac{P}{I\cos\varphi} = \frac{1344}{8 \cdot 0{,}8} = 210V$$

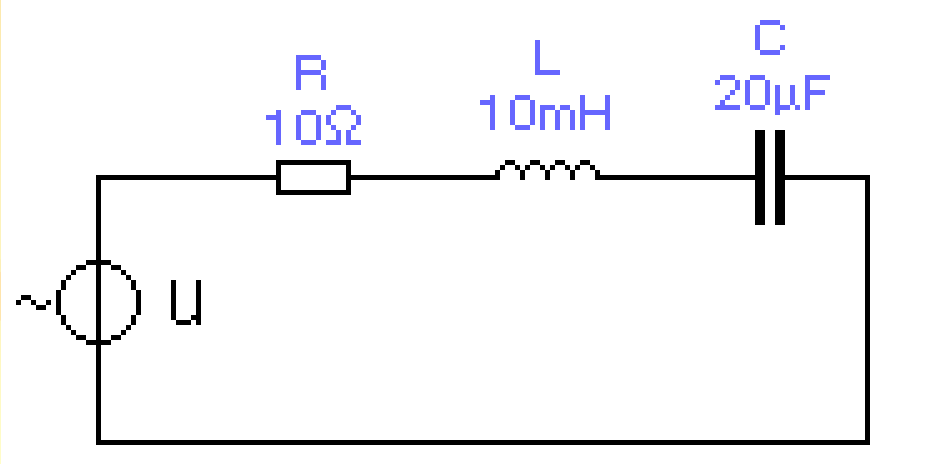
$$R_1 = Z_1 \cos\varphi_1 = 35 \cdot 0{,}6 = 21\Omega$$

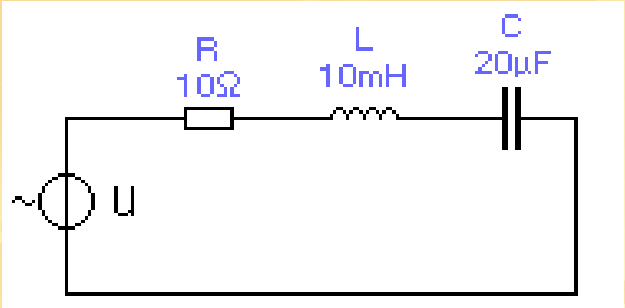
$$R_2 = \frac{U}{I_2} = \frac{210}{2{,}8} = 75\Omega$$

$$Z_1 = \frac{U}{I_1} = \frac{210}{6} = 35\Omega$$

- Na izvor serijski su spojeni R = 10Ω, L = 10 mH, C = 20 μF. Izračunaj i napiši izraze za: i(t), $u_R(t)$, $u_C(t)$ i $u_L(t)$. Skiciraj fazorski dijagram uz struju kao referentnu os.

$$u(t) = 4\sqrt{2}\sin(2000t)$$





$$u(t) = 4\sqrt{2}\sin(2000t)$$

$$X_L = \omega L = 20\ \Omega$$

$$X_C = \frac{1}{\omega C} = 25\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 11{,}18\Omega$$

$$I_{ef} = \frac{U_{ef}}{Z} = 0{,}358A$$

$$I_{\max} = I_{ef} \cdot \sqrt{2} = 0{,}506 \qquad \cos\varphi = \frac{R}{Z}$$

→ φ = 26,56° (kap)

$$i(t) = 0{,}358\sqrt{2}\sin\left(2000t + 26{,}56°\right)$$

$U_R = I \cdot R = 3{,}58V$

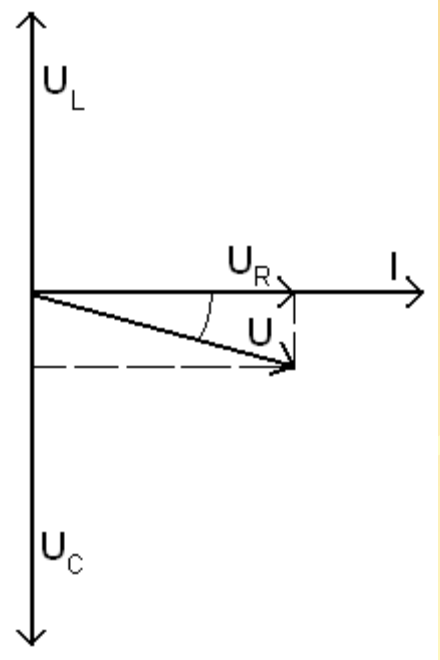
$$u_R(t) = 3{,}58\sqrt{2}\sin\left(2000t + 26{,}56°\right)$$

$U_L = I \cdot X_L = 7{,}16V$

$$u_L(t) = 7{,}16\sqrt{2}\sin\left(2000t + 116{,}56°\right)$$

$U_C = I \cdot X_C = 8{,}95V$

$$u_C(t) = 8{,}95\sqrt{2}\sin\left(2000t - 63{,}43°\right)$$



Kraj!

PITANJA?